Essay on Gravitation

Present Time Variation of Newton's Gravitational Constant in Superstring Theories

Yong-Shi Wu
Department of Physics, University of Utah
Salt Lake City, UT 84112, USA

and

Zi Wang
Department of Physics, Utah State University
Logan, UT 84322

Summary

Superstring theories provide an appropriate framework for studying the time variation of fundamental coupling constants. The present time-variation of coupling constants in superstring theories with currently favorable internal backgrounds critically depends on the shape of the potential for the size of internal space. If the potential is almost flat, as in perturbation theory to all orders, the present value of $|\dot{G}/G|$ for Newton's gravitational constant is calculable and estimated to be $1 \times 10^{-11\pm1}$ yr^{-1} which is just at the edge of the present observational bound for $\dot{G}/G$. If the potential has a minimum with finite curvature due to unknown nonperturbative effects, $\dot{G}/G$ will become unobservably small. The improvement of the measurement of $\dot{G}/G$ of one or two orders of magnitude would discriminate between the two situations. Problems with the time variation of other coupling constants are also discussed.

The time variation of fundamental constants may provide a connection between cosmology and particle physics. This idea can be traced back to Dirac,[1] although his original proposal for variation in Newton's gravitational constant G seems not supported by observations.[2]

Very recently superstring theories[3] appear to be promising candidates for a consistent quantum theory unifying all known interactions including gravity. They provide a suitable framework for studying the time variation of fundamental constants. The consistency of superstring theories fixes the space-time dimensionality to be ten, six of which form a very small compact manifold K($\sim 10^{-32}$cm). The metric and other bosonic backgrounds in K are constrained by string-compactification and particle-phenomology considerations.[4,5] The coupling constants in the four-dimensional world are related to those in ten dimensions by a factor of the inverse volume of K. The cosmology in the higher-dimensional universe governs the evolution of the usual three-space as well as that of K and, through the latter, dynamically determines the time variation of coupling constants in four dimensions. Generally in a higher-dimensional field-theory approach,[6] quantum effects in K[7] give rise to an effective potential which may fix the size of the internal space R_6 in vacuum and influences its cosmological evolution. but in superstring theories, Witten's nonrenormalization theorem[8] tells us that such a potential for R_6 is flat up to all orders in perturbation theory. So far, the study of nonperturbative supersymmetry-breaking effects, including world-sheet instantons, also has failed to produce a potential with a minimum at finite R_6. In this essay we will describe some recent work[9,10] on the time variation of Newton's gravitational constant in superstring theories and its critical relationship to the shape of this potential.

We find that if the potential is flat the present value of $\dot{G}/G$ is calculable; for example, for an open universe

$$(\dot{G}/G)_0 = (q_0 - 13\Omega_0 H_0^2 t_0^2/8)/t_0 \tag{1}$$

where H_0 is the Hubble constant, t_0 the age of the universe, q_0 the deceleration parameter, and $\Omega_0 \equiv 8\pi\otimes G_0\rho_0/3H_0^2$ the density parameter. Here the subscript 0 denotes the present value of the quantity. We estimate $|\dot{G}/G|_0$ to be in the range $1 \times 10^{-11\pm1}$ Yr^{-1}, which overlaps the present observational upper bound $|\dot{G}/G| \leq 1 \times 10^{-11}$ Yr^{-1}.[11] However, if the potential really has a minimum at finite R_6, $(\dot{G}/G)_0$ will be suppressed and become unobservably small. So an improvement on the measurements of $\dot{G}/G$ will give us important information about the shape of the potential. Some remarks about time variation of other coupling constants in superstring theories will be discussed at the end of the essay.

The low-energy (field-theory) limit of, say, the $E_8 \times E_8$ heterotic superstring theory,[3] which is phenomenologically promising from the point of view of particle physics, is given by N = 1 supergravity coupled to N = 1 super Yang-Mills in ten dimensions.[12] We start with the equations of motion in the bosonic sector of the theory which includes the gravitational and Yang-Mills fields as well as an antisymmetric two-rank tensor (Kalb-Ramond) field and a scalar (dilaton) field. As usual, we assume that the cosmological metric is of the Robertson-Walker form with two scale factors $R_3(t)$ and $R_6(t)$ for three-space and internal six-space K respectively. For the background bosonic fields in K, we take the well-known Candelas-Horowitz-Strominger-Witten configuration,[4] which is currently favorable by particle-physics considerations, with minimal modification in conformity to the introduction of $R_6(t)$. Two features of this configuration are essential to our discussion: 1) Because of their conformal invariance all classical equations of motion except the Einstein equations for $R_3(t)$ and $R_6(t)$ are satisfied; 2) these internal backgrounds, though nonvanishing, have no contribution to internal stress tensor and, therefore, do not provide a potential to fix the internal size. (In particular, the internal metric is a Calabi-Yau metric which is Ricci-flat.) By Witten's non-renormalization theorem, the situation remains unchanged even with higher-order perturbative quantum effects included. Since we are interested in cosmology at present times it is safe to ignore the internal components of the thermal stress tensor of matter.[13]

In an open universe ($k = -1$), the large-t asymptotic solution[9] is given by $R_3(t) \sim t$, $R_6(t) = R_{60} = \text{const}$, and is stable under perturbations $r_3(t)$ and $r_6(t)$ of $R_3(t)$ and $R_6(t)$. Assuming that matter density ρ_0 can be treated as a small quantity in the present Universe and neglecting terms of order $(t_P/t)^2$, $t_P \sim 10^{-43}$ sec being Planck time, $r_3(t)$ and $r_6(t)$ can be solved from the Einstein equation and expressed in terms of the cosmological parameters Ω_0, H_0, q_0 and t_0. The final result for $(\dot{G}/G)_0 = -6\dot{r}_6(t_0)$ is given by (1). Thus, using the most "satisfactory" set of parameters,[14] $(\Omega_0, q_0, H_0) = (0.05, 0.025, 67 \text{ km sec}^{-1} \text{ Mpc}^{-1})$ and the extreme sets of parameter, $(\Omega_0, q_0, H_0) = (0.05, -0.925, 100 \text{ km sec}^{-1} \text{ Mpc}^{-1})$ or $(1, 0.5, 40 \text{ km sec}^{-1} \text{ Mpc}^{-1})$ and $t_0 = 1.6 \times 10^{10}$ yr, we estimate the range for $|\dot{G}/G|_0$ to be $1 \times 10^{-11\pm1} \text{ yr}^{-1}$.

Rigorously speaking, if Ω_0 is close to 1, the above perturbation calculation breaks down. One needs to use a computer but this would not change the above estimation. The same estimation is expected to be true also for $k = 0$ or $k = +1$ cases. The key point here is that Einstein equations with $\rho_0 \neq 0$ does not allow $\dot{R}_6 = 0$. Thus $\dot{G}/G = -6\dot{R}_6/R_6 \neq 0$. Since t_0 is the only relevant cosmological time scale, $(\dot{G}/G)_0$ must be proportional to $1/t_0$ with a coefficient of order unity or, probably, one to two orders lower.

Now we assume that there is an effective potential for R_6 due to unknown nonperturbative quantum effects. If the potential is flat near R_{60}, the result is the same as given above. If the potential has a minimum for finite R_6, R_{60} must be located there. Assuming $(\mu t)^2 >> 1$, μ being a mass determined by the curvature of the potential at R_{60}, then

$$r_6(t) = -\frac{3}{8}\Omega_0 H_0^2 t_0^3 \frac{1}{\mu^2 t^3} + t^{-3/2} A \cos(\mu t + \delta) \tag{2}$$

The second oscillatory term vanishes after being averaged over the period $2\pi/\mu$. The first term, compared to that for flat potential, is suppressed by the factor $(\mu t_0)^{-2} = [(10^{-32} eV)/\mu]^2$. So a very tiny mass μ would make $(\dot{G}/G)_0$ in this case unobservably small. The conventional wisdom favors a not very small μ, since in four dimensions r_6 represents a Brans-Dicke-type scalar field which would compete with gravitons and would have been observed if it is massless. However, its coupling to matter might be anomalously weak; if so, a flat potential for R_6 is not in conflict with observations.

Here we have been concentrated on $\dot{G}/G$, since theoretically it is independent of the dilaton field, about which we know very little, and experimentally extracting it from data is simple and direct. In contrast, the analysis in unified string theories of the time variation of other coupling constants, such as the fine-structure constant α and strong or weak coupling constants, is much more complicated. This is because the time evolution of R_6 will lead also to a variation in the grand unification coupling constant and the latter, in turn, gives rise to a variation in almost every coupling constant and particle mass measured at low energies through renormalization-group running which depends very much on the details of the physics between the grand-unified scale ($\sim 10^{14} - 10^{15}$ GeV) and the weak scale ($\sim 10^2$ GeV). Thus the usual assumption made in previous analyses of data[15] that only one coupling constant under consideration is varying alone does not hold good in unified theories. Furthermore, all other coupling constants except G depends on the background value of the dilaton field. The time-dependence of the latter, which we have neglected in our analysis of $(\dot{G}/G)_0$, might be important in considering the variation of, e.g., α over a long period such as 5×10^9 yr, as in some previous determination of $\dot{\alpha}/\alpha$.[15]

In conclusion, further improvement in measuring $\dot{G}/G$ can discriminate between different shapes of the potential in superstring theories for the size of internal space. If the potential is almost flat or has no minimum for finite R_6, probably we are on the edge of observing $\dot{G}/G$.

Y.S. Wu was supported in part by National Science Foundation Grant No. PHY-8405648.

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