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PHYSICS DEPARTMENT
217 Prospect Street

March 25, 1985

President
Gravity Research Foundation

Dear Sir:

Enclosed herewith is the article "Superstring Gravity and the Early Universe", by M.J. Bowick and L.C.R. Wijewardhana. We would be grateful if you could accept this for the gravitation essay competition.

Sincerely,

L.C.R. Wijewardhana

LW/jd

Superstring Gravity and The Early Universe

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ABSTRACT

Ten-dimensional superstring theories have been proposed as candidates for a unified description of all the forces of nature. These theories reduce to Einstein gravity coupled to Yang-Mills interactions at scales small compared to the string tension. The phenomenologically promising superstring theory, the heterotic string, is investigated at the high temperatures and short distances relevant in the early universe. The massive string modes alone constitute an unstable thermodynamic system with negative specific heat. The conditions for equilibrium between the massive string modes and the massless modes (radiation) are derived. The large energy fluctuations of the system require the use of the microcanonical ensemble. There is a maximum temperature which exceeds the temperature at which the canonical partition function becomes divergent. Above a critical volume there is a phase transition during which the massive string modes must evaporate. The possibilities of spontaneous compactification, large entropy production and a solution of the horizon and flatness problems are discussed.

I Introduction

Superstring theories are one of the most original approaches to the development of a mathematically consistent quantum theory of gravity[1].They evolved from the Ramond-Neveu-Schwarz[2] spinning string theory of the strong interactions of hadrons.Along with their description of gravity these theories can naturally incorporate matter with Yang-Mills (non-abelian) gauge interactions at scales small compared to the Planck scale[3] $M_p \sim 10^{19}$Gev. Since such gauge interactions are today the favored descriptions of all non-gravitational (strong, weak and electromagnetic) processes, the superstring theories are promising candidates for the elusive unified theory of all physical interactions. The clue to their consistent quantum description of gravity lies in the short-distance behavior of string theories. A single string describes the propagation of an infinite number of degrees of freedom - its normal modes. At distances longer than the Planck length one sees only the massless modes and string theories reduce to point-like quantum field theories with which we are more familiar. In such theories the gravitational coupling strength (Newton's constant G) has dimensions of inverse energy and physical amplitudes must grow with energy to compensate the inverse powers in the coupling constant. This growth with energy leads to an uncontrollable growth in the infinities (divergences) of the quantum theory - the quantum Lagrangian requires the addition of more and more counterterms at each order in the quantum loop expansion and the theory is not renormalizable. This situation is familiar in the history of the weak interaction with the dimensionful Fermi coupling constant G_F of the four-fermion interaction the analogue of Newton's constant. The solution there was the replacement of the contact interaction with the exchange of virtual massive gauge bosons. This weak interaction theory of Glashow, Salam and Weinberg (GSW)[4] was dramatically confirmed in the last two years by the discovery of the weak interaction gauge bosons at CERN[5] and provided a description of the weak interactions in direct analogy to the very successful Q.E.D description of electromagnetic interactions. The superstring theory does something akin to the GSW theory for gravity - it softens the very short distance behavior of gravity by replacing contact interactions with the exchange of the massive modes of the string. All indications so far are that the resultant theory is not only renormalizable but in fact free of all ultraviolet divergences - it is finite. This adds calculability to the virtues of string theories, since in non-finite theories infinitely renormalized quantities are arbitrary- one simply fits them the observed values. There are very few free parameters in superstring theories. There is the string tension (which ought to be of order the Planck mass squared) and possibly a Yang-Mills coupling strength. The gravitational coupling is determined by these two and the Yang-Mills coupling may even be determined by minimizing some potential of the theory[6].

There is one furthur novelty of superstring theories that will be relevant to this paper. The closure of the Lorentz algebra necessary for a consistent quantum theory requires that the theory be formulated in ten space-time dimensions, and the absence of hexagon gauge and gravitational anomalies, which would couple unphysical currents, determines the gauge group to be SO(32) or $E_8 x E_8$[7]. At present it is only really known how to formulate the theory in ten-dimensional Minkowski space, but the gravitational interactions should dynamically determine the structure of space-time. One hopes that the final geometry is four-dimensional Minkowski space x some compact six dimensional space. These six dimensions will then be probed only at energies approaching the inverse compactification radius. This radius is not yet calculable.

II Strings and Cosmology

If the true theory of all interactions is provided by the superstring theory it is worthwhile to reexamine the standard cosmological model to see what changes. In the usual approach the matter of the very early universe is assumed to be a fixed number of species of almost free elementary particles(the known quarks, leptons, gauge-bosons etc). String theories, in contrast, predict a density of states

$$\rho(m) = m^{-a} \exp(bm) \tag{1}$$

which rises exponentially for large m. What is the behavior of such a system at the high temperatures involved in the early universe when the massive modes of the string can be excited? To answer this question, one can examine the canonical partition function

$$\ln Z = \frac{V}{(2\pi)^4} \int \rho(m)\, dm \int d^9k\, \ln \left[\frac{1 + \exp(-\beta\sqrt{k^2 + m^2})}{1 - \exp(-\beta\sqrt{k^2 + m^2})} \right] \tag{2}$$

where V is the nine dimensional volume. This gives[8] [9]

$$\begin{aligned} \ln Z &\sim \int_\eta^\infty dm\, m^{-a + 9/2} \exp[-(\beta - b) m] \\ &\sim \left(\frac{T T_0}{T_0 - T} \right)^{-a + 11/2} \Gamma\left(-a + 11/2, \eta \frac{T_0 - T}{T T_0}\right) \end{aligned} \tag{3}$$

where $\Gamma(a, x)$ is the incomplete Gamma function, $T_0 = 1/b$ and η is an infrared mass cutoff below which the asymptotic density of states $\rho(m)$ is no longer valid. The canonical partition function diverges for $T > T_0$ and thus T_0 seems to describe a maximum temperature for thermodynamic equilibrium[10]. The thermodynamic observables of interest may then be calculated from Z; $P = T\,\partial/\partial V \ln Z$, $\langle E \rangle = T^2\,\partial/\partial T \ln Z$, $C = d/dT \langle E \rangle$.

For $a \leqslant 13/2$ ($(D+3)/2$ for space-time dimension D) the pressure, energy density and specific heat diverge as the temperature approaches T_0. This behavior is consistent with T_0 being a maximum temperature of the system. As energy is pumped into this system the exponentially rising number of massive string modes are excited while the average energy per mode remains constant. For $a > 13/2$ ($(D+3)/2$), however, we see that the pressure and energy density are constant as the temperature approaches T_0 and for $a > 15/2$ ($(D+5)/2$) the specific heat is also constant as T approaches T_0. It seems that there is nothing to prevent the system from passing through $T = T_0$ as the energy is increased and yet the canonical ensemble does not provide a description of the system for temperatures exceeding T_0. The most phenomenologically promising superstring theory is the heterotic string theory[11] which has[12] $a = 9$ and $b = (2+\sqrt{2})\pi\sqrt{\alpha'}$ where α' is the Regge slope parameter(the inverse of the string tension). It thus falls into the category $a > 13/2$. Calculating the mean-square energy fluctuations we find $\frac{\langle E^2\rangle - \langle E\rangle^2}{\langle E\rangle^2} \gg 1$ when the energy density exceeds a critical value of order one in Planck units. For large energy fluctuations the canonical ensemble is no longer a good thermodynamic description of the system and one should reexamine the system using the more fundamental microcanonical ensemble. In this ensemble the total energy E is fixed and one counts the number of microstates which yield a given macrostate

$$\Omega(E,V) = \sum_{n=1}^{\infty} \left[\frac{V}{(2\pi)^4}\right]^n \frac{1}{n!} \prod_{i=1}^{n} \int \rho(m_i)\, dm_i \int d^9 p_i\, \delta(\Sigma E_j - E)\, \delta(\Sigma p_j) \qquad (4)$$

Focusing now on the ($a = 9$) heterotic string this gives

$$\Omega(E,V) \simeq V E^{-a} \exp(bE) \qquad (5)$$

reproducing the form of $\rho(m)$. It can also be shown that the favored thermodynamic configuration is for n-1 strings, where n is the most probable number of strings in a gas of strings, to carry as little energy as possible and for one string to carry the remaining energy[8 9]. The condition of large energy fluctuations is equivalent to the condition E >> $n\eta$. Thus one string carries the majority of the energy and the equilibrium system is highly inhomogeneous.

Looking further there is another striking feature of this gas of superstrings. Given the microcanonical density of states $\Omega(E,V)$ we can compute the microcanonical thermodynamic observables. The entropy S is

$$S = \ln \Omega(E,V) = -a \ln E + bE \qquad (6)$$

The temperature T is formally given by

$$T = -\frac{\partial S}{\partial E} = -\frac{a}{E} + b \qquad (7)$$

For positive energy E the temperature exceeds 1/b. The specific heat is

$$C = -\frac{1}{T^2}\left(\frac{\partial^2 S}{\partial E^2}\right)^{-1} = -\frac{1}{T^2}\left(\frac{E^2}{a}\right) \qquad (8)$$

The specific heat is negative! In the canonical ensemble the specific heat is proportional to the mean-square energy fluctuation and is thus always positive[13 14] . The negative specific heat and the inapplicability of the canonical partition function brings to mind the thermodynamics of a black hole. A four-dimensional black hole of mass M is characterized by a temperature $T = 1/(8\pi GM)$ and dM/dT is negative. A black hole (and the massive string excitations) can never be in thermal equilibrium with an infinite heat reservoir. The system is unstable to small temperature fluctuations. A system with negative specific heat can, however, be in equilibrium with another system of positive specific heat provided the net specific heat is positive. Let $S_{1,2}$ and $E_{1,2}$ denote the entropy and energy of system 1 and 2 respectively. The total number of configurations for this system is $\exp(S_1+S_2)$. The most probable values of E_1 and E_2 will be those which maximize S_1+S_2 with the constraint that the total energy is fixed. This means

$$\frac{\partial S_1}{\partial E_1} = \frac{\partial S_2}{\partial E_2} \tag{9}$$

and

$$\frac{\partial^2 S_1}{\partial E_1^2} + \frac{\partial^2 S_2}{\partial E_2^2} < 0 \tag{10}$$

For a $D(\geqslant 4)$ space-time dimensional black hole the entropy S is proportional to the area of the event horizon, which scales like R_s^{D-2} where R_s is the Schwarzschild radius. Einstein's equations for a spherically symmetric matter distribution yield $g^{00}(r) = (g^{rr}(r))^{-1}$ and $r g^{00}(r) = r + \text{const}$. In the Newtonian approximation $g^{00} = 1 + 2\phi$ where ϕ is the Newtonian potential which is $-G_D M/R^{D-3}$ in D dimensions. Therefore $g^{00} = 1 - 2G_D M/R^{D-3}$ and the Schwarzschild radius $R_s = (2G_D M)^{1/(D-3)}$. Hence the entropy scales like $M^{D-2/D-3}$. Equations (9) and (10) then yield $E_1 < [(D-3)/D]\,M$. Thus a black hole can be in equilibrium with radiation provided the radiation carries less than the fraction $(D-3/2D-3)$ of the total energy. This requires the total volume of the system to be less than a critical volume. The corresponding results for the string are that the temperature be

$$T < T_{max} = \frac{20bE - 9a \pm \sqrt{81a^2 + 40abE}}{20b(bE - a)} \tag{11}$$

The total energy in the massless excitations of the string (radiation) is then constrained by

$$E_r < (E_r)_{max} = E - (E_{string})_{min} = E + \frac{aT_{max}}{1 - bT_{max}} \tag{12}$$

and the volume for equilibrium between the massive and massless excitations is

$$V < (V)_{max} = \frac{(E_r)_{max}}{\sigma T^{10}_{max}} \tag{13}$$

where σ is the ten-dimensional analogue of the Stefan-Boltzmann constant ; $\sigma = (8\pi^5/3465) \{n_b + (1 - 1/2^9) n_f\}$ and $n_b = n_f = 4032$ for the massless modes of the heterotic string. For very large energy E, the minimum energy in the massive string excitations is proportional to $\sqrt{E}$ and most of the energy can be carried by the radiation. The above results reduce in this limit to $T_{max} \sim 1/b$, $(E_r)_{max} \sim E$ and $V < E\, b^{10}/\sigma$. Since superstring theories yield Einstein gravity only in the low energy limit we do not know if there is a singularity akin to the big bang predicted by such theories. However we expect the massive string modes to be excited and they would be in equilibrium with the massless modes for some temperature T between T_0 and T_{max}. The majority of the energy would probably be carried in radiation and the universe may be described by a generalized Friedmann-Robertson-Walker metric with the scale factor R growing like $t^{1/2}$. As the temperature dropped and the volume increased there would come a time when the volume exceeded the critical volume for equilibrium. The massive string modes would have to evaporate entirely into radiation in a phase transition which would take the system through the temperature T_0. Since there are a tremendous number of massive modes this non-equilibrium process is likely to generate a large entropy increase. The precise nature of this phase transition and the resultant effect on the evolution of the universe are not known. Perhaps this could provide an alternative to the inflationary universe's de-Sitter phase which also yields a large entropy increase and thereby solves the horizon and flatness problems[15]. Since there is a maximum temperature for thermal equilibrium of this system it may be that the horizon problem is solved by causally-disconnected regions all being at this temperature in the early universe. The large fluctuations inherent in the massive string excitations may be diluted by the predominant radiation. The physics of a gas of strings may alter when string interactions are included, particularly if the coupling is strong, although we don't expect the qualitative features of the spectrum to change. A major source of uncertainty in this discussion are the initial conditions in the early universe.

An exciting alternative to the phase transition discussed above is for the universe to spontaneously compactify[16]. We have shown that above a critical volume the massive string excitations are unstable for a > 13/2. If the geometry of space-time is $R^4 \times S^6$, for example, each mode in ten dimensions will become a tower of modes in four dimensions. There are thus more modes in the effective four-dimensional theory. Suppose the density of states is such that the effective four-dimensional "a" is lowered from 9 to something less than 7/2 ((4+3)/2). The four dimensional massive string excitations could then exist in thermodynamic equilibrium alone and thermodynamics would indicate that the system likes to dynamically compactify. Preliminary investigations of this possibility indicate that "a" is lowered by at most one, so this possibility has not yet been realised in any string theory.

Consideration of string dynamics may also elucidate some of the other problems of quantum gravity, such as what happens in the last stages of black hole evaporation[17]. If the string tension is sufficiently less than the Planck mass (of order $M_p/100$) then the entropy available to massive string excitations exceeds that available to the black hole[18]. An evaporating black hole could increase its entropy by making a quantum transition to a state corresponding to a massive string excitation.

The possibilities for novel cosmologies are immense in the new physics which the superstring theories reveal at short distances. Furthermore the evolution of the universe, and any relics of it, may very well provide the only unique signatures of superstring theories since the massive modes decouple at low energies and yield effective conventional quantum field theories.

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