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New Experiments in Gravitation Physics

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Abstract

New experiments are proposed for testing the General Theory of Relativity. An antenna is described for observing gravitational radiation from the sun and stars. Apparatus is considered for possible generation of gravitational waves. Other experiments are proposed for studying the invariance of intervals in accelerated frames, and these therefore test the gravitational red shift. Results of calculations are presented, and it is shown that some experiments can be done at the present time while the others require technological advances to make them possible.

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Gravitational Wave Experiments

The General Theory of Relativity predicts that accelerated masses should radiate. Exact wave solutions are known for cylindrical waves and approximate solutions are available for the spherical waves. No gravitational radiation has ever been observed. Such waves might be generated from turbulence on the sun or by masses outside the solar system. We propose now an antenna for detection of such waves.

Assume that we have an oscillating mass quadrupole along the Z axis (see Figure 1), which is radiating gravitational waves.

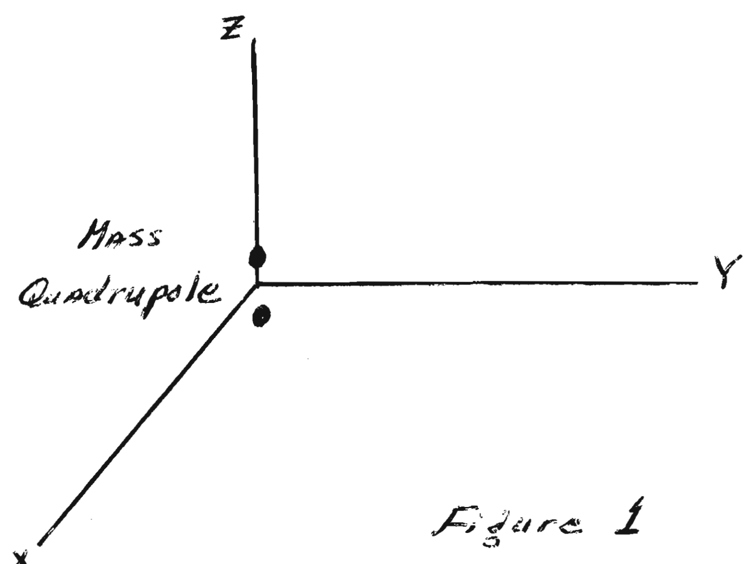

Figure 1

Let the angular frequency be ω, let the maximum value of the mass quadrupole moment be Q, let G be the constant of gravitation, and let c be the speed of light. Imagine a test mass m_1 located at point P with spherical coordinates r, θ, φ. The radial force exerted on the mass by the waves can be calculated from General Relativity and is

$$F_r = \sqrt{\frac{\pi}{2}}\left[\frac{m_1 G \omega^3 Q (3\cos^2\theta - 1)}{24\pi c^3}\right]\frac{e^{i\omega(t - \frac{r}{c})}}{r} \tag{1}$$

Expression (1) represents an outgoing spherical wave. Now imagine a second mass m_2, close to P (see Figure 2). The phase of the wave will be slightly different at m_2 from its value at m_1, and if m_2 is located as shown the phase difference can give rise to torques. If the masses are connected by a rigid rod which is suspended by a fine wire we have an oscillating torsion balance. Alternately we would mount m_1, and m_2 at

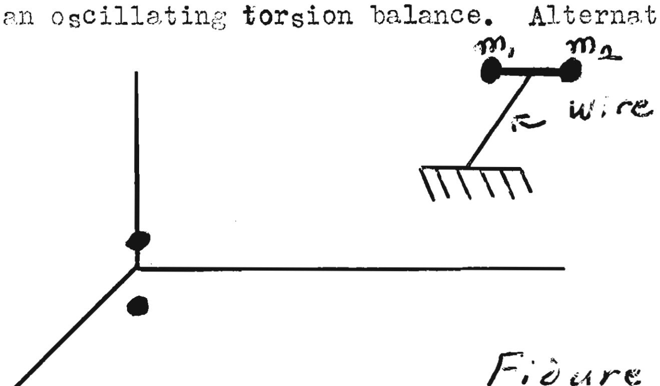

Figure 2

apposite ends of a supported spring as shown in Figure 3. The phase difference results in alternate compression and expansion of the spring, and we have here a harmonic oscillator.

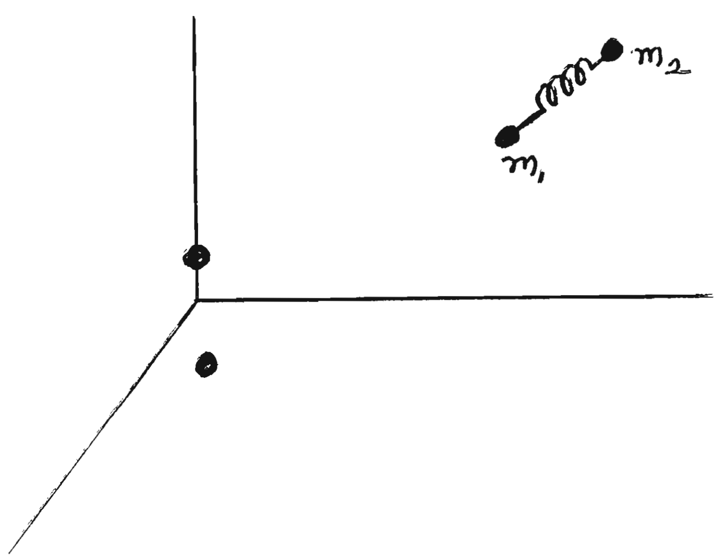

Figure 3

The energy which can be absorbed from a wave can be calculated in the following way. We imagine the antenna coupled to an energy absorber. We write a differential equation for the motion of m_1 and m_2 and from its solution we calculate the maximum absorbed power for an optimum design. The result of the lengthy calculation is that for an average orientation the absorbed power is independent of the kind of antenna and is given by

$$P_{ABSORBED} = \frac{\lambda^2}{4\pi} U \tag{2}$$

In expression (2) U is the power per unit area in the wave, λ is the wavelength. (2) states that the absorption cross section of all gravitational wave receivers averaged over all orientations is the same and equal to a circular area whose circumference is the wavelength. This result is believed to be new.

We place our receiver in a dark vacuum and cool it with liquid helium so that the thermal fluctuations will not interfere with the measurements. We average over a time T. The minimum gravitational energy flux which we can detect is obtained by setting the time averaged absorbed energy equal to the time average energy of one quantum. Accordingly we have

$$\frac{\lambda^2}{4\pi} U_{Min} = \frac{h\nu}{T} = \frac{hc}{T\lambda} \tag{3}$$

In expression (3) h is Planck's constant, ν is the frequency. Solving (3) for U_{Min} gives us

$$U_{Min} = \frac{4\pi hc}{T\lambda^3} \tag{4}$$

Assume for example that the wavelength $\lambda \doteq$ 1000 meters, and the averaging time T is one day. The minimum detectable flux is then, from (4)

$$U_{Min} \doteq 2 \times 10^{-37} \text{ WATTS PER CM}^2 \tag{5}$$

Practical considerations would make the theoretical result (5) hard to achieve. The useable sensitivity could be determined experimentally by electrically charging the masses and driving the system with electric fields. If no gravitational radiation is observed with these instruments, their known sensitivity would enable us to say that the gravitational radiation incident on earth within certain bands of frequencies does not exceed a certain limit. Future experimenters would lower this limit. Expression (5) assumes a sinusoidal wave, actually wave noise would be observed. The random motion would have to be Fourier alalyzed. This type of experiment can be done now. Either a positive or a negative result would give useful information.

It would be much better if we could generate gravitational waves in the laboratory. When an ammonia molecule undergoes an inversion transition there will be mass quadrupole radiation at 24000 megacycles. A rotating molecule will also radiate gravitational waves. Unfortunately the effects are very small.

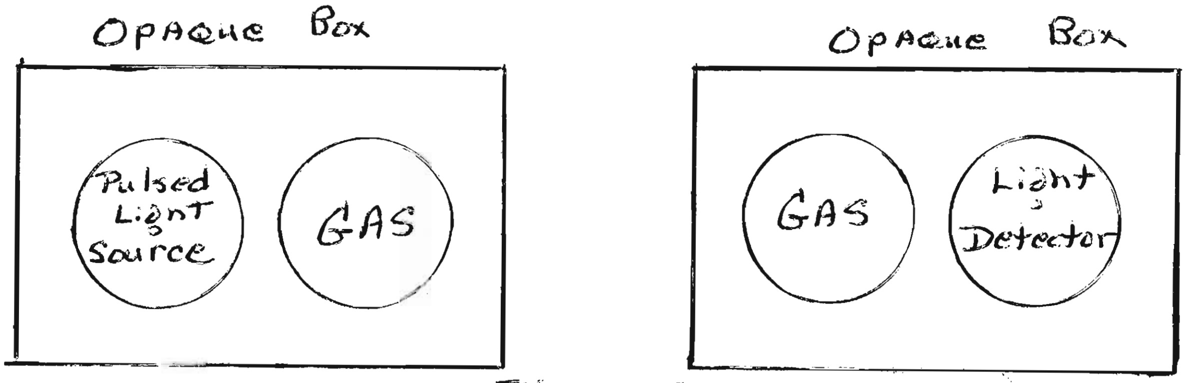

Figure 4

Consider Figure 4. We have two sets of apparatus, each one enclosed in an opaque box. A pulsed light source excites molecular rotations or vibrations at the left. In these transitions some gravitons may be emitted. These may penetrate the opaque box at the right and excite molecular rotations or vibrations. These in turn may cause light to be emitted and detected by the detector. It is difficult to accurately predict the result because coherent quantum states may be excited. The most pessimistic calculation assumes no coherence. At optical frequencies one graviton every 200 years would be emitted. However if coherence can be achieved the result would be much larger, perhaps several per second. Coherence effects at lower (microwave) frequencies have been achieved by Dicke and Hahn using electromagnetic waves. If their methods can be extended to the optical region this will become a practical experiment. Coherence in emission and absorption will be needed. We are studying it further.

Mass quadrupole transitions also occur in nuclear physics. We calculate that nuclear transitions of energy ten million electron volts might give one graviton per minute per gram molecular weight (incoherent emission). This appears promising, but an experiment would be extremely difficult to do. We are considering a kind of Cowan Rienes experiment in which gravitons from a nuclear reactor make the reaction which produced them go backwards.

Red Shift Experiments

We consider now apparatus for observing the General Relativity red shift. It has been suggested that satellites be equipped with atomic clocks for such measurements. Terrestrial measurements are more desirable and we therefore consider experiments which wou would be done in the laboratory. The equivalence principle guarantees that an accelerated frame is equivalent to a certain gravitational field. Suppose we carry out experiments on a rotating platform. We then have accelerations and if the law of invariance of four dimensional intervals is valid, we should expect red shifts.

It is worth noting that verification of the red shift only tells us that atoms are natural clocks. It may be that the quantization of the interacting matter, electromagnetic gravitational fields, and the pair fields will predict that an atom is not exactly a natural clock. If a red shift experiment disagrees with Einstein's predictions, this cannot be regarded as crucial until the quantization of relativity is better understood. Nonetheless such experiments are worth doing.

First consider the arrangement of Figure 5. Here we have an atomic clock C which is mounted on a turntable which can be rotated. The output of the clock is a periodic electrical signal which is transmitted to an antenna or slip rings at the center of the

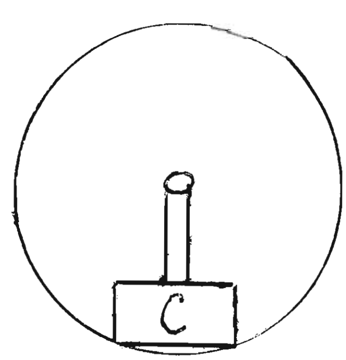

Figure 5

wheel. Let the wheel rotate with angular velocity Ω . Then the line element can be shown to be (cylindrical coordinates)

$$ds^2 = (c^2 - \Omega^2 r^2)\, dt^2 - 2\,\Omega r^2 d\phi\, dt - dr^2 - r^2 d\phi^2 - dr^2 - dz^2 \tag{6}$$

In expression (6), ϕ, r and z are measured in the rotating frame. The symbol t refers to coordinate time which is measured by synchronized clocks which remain fixed in the fixed frame.

Let the frequency with the turntable at rest be ν_1 . General Relativity asserts that the four dimensional interval ds does not change when the clock is rotated. In this case we are concerned with the interval between oscillations. It therefore follows from (6) that the new frequency ν_2 is given by

$$\nu_2 = \nu_1 \sqrt{1 - \frac{\Omega^2 r^2}{c^2}} \doteq \nu_1 \left[1 - \frac{\Omega^2 r^2}{2c^2}\right] \tag{7}$$

According to (7) the fractional change in frequency is

$$\frac{\nu_2 - \nu_1}{\nu_1} = -\frac{\Omega r^2}{2c^2} \tag{8}$$

If we assume $\Omega = 120\pi$ radians per second (3600 rpm.) and r = 100 cms., (8) becomes

$$\frac{\nu_2 - \nu_1}{\nu_1} \approx 10^{-12} \tag{9}$$

Expression (9) states that in a practical experiment a frequency shift of 1 part in 10^{12} can be obtained. During the next ten years we can expect frequency standards to become available which will be stable enough to do this.

A different experiment which gives a much larger effect is the following. See Figure 6. Here again we have an electrical oscillator C , mounted on a turntable. However the period of this clock is controlled by a feedback loop and energy circulates around the loop in one direction only. The period of the oscillator is controlled by the propagation around the loop. In this case when we rotate the turntable we do not assume $d\phi = 0$ in expression (6). We are dealing with null intervals since the propa-

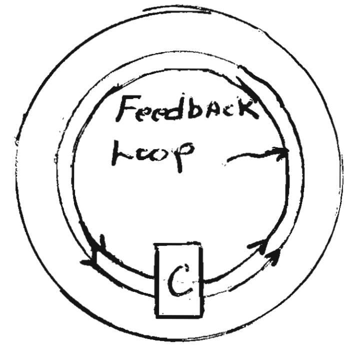

Figure 6

gation of electromagnetic waves around the loop for a change in ϕ of 2π controls the oscillator. We can therefore write, integrating (6), that

$$0 = c^2 dt^2 - \Omega^2 r^2 dt^2 - 2\Omega r^2 (2\pi)\, dt - 4\pi^2 r^2 \tag{10}$$

solving (10) for dt gives

$$dt = \frac{4\pi\Omega r^2 + \sqrt{16\pi^2 r^2 c^2}}{2(c^2 - \Omega^2 r^2)} \tag{11}$$

Expression (11) gives rise to a frequency shift which is approximately

$$\frac{\nu_2 - \nu_1}{\nu_1} \simeq -\frac{\Omega r}{c} \tag{12}$$

In (12) we note that the peripheral velocity v is equal to Ωr so (12) becomes

$$\frac{\nu_2 - \nu_1}{\nu_1} \simeq -\frac{v}{c} \tag{13}$$

We consider expression (13) and this experiment to be important. We are saying that an experiment can be done which tests the ideas of invariance of intervals in accelerated frames. It gives observable frequency shifts which are linear in $\frac{v}{c}$. This is a consequence of the fact that we are sending radiation in one direction only, not having it reflected back. If $\Omega = 120\pi$ radians per second (3600 rpm.) and r = 100 cms. (13) becomes

$$\frac{\nu_2 - \nu_1}{\nu_1} \doteq 10^{-6} \tag{14}$$

Oscillators can be constructed now with stability of 1 part in 10^9. Expression (14) predicts readily observable effects with present techniques. In this experiment also we are assuming that the clock's signals are delivered to an antenna or slip rings at the center of the wheel. A number of modifications of this experiment are possible. For example we can build a "high Q" toroidal resonator and excite it by a "Bethe Hole Coupler" to microwave travelling wave modes in one direction only. When the apparatus is rotated we should get a resonant frequency shift.

Conclusion

We conclude that it is possible, at present, to test the invariance of intervals in accelerated fromes in the laboratory by at least one method, and to get an upper limit to the gravitational wave flux incident on earth from outer space. The other experiments are not unpromising and may become feasible in the near future. We hope to start these experiments soon.