

ABSTRACT

A Determination of the Rate of Change of G. T. C. Van Flandern,
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1955-1973 utilizing Atomic Time gives a value for the empirical part of
the secular deceleration of the Moon's mean longitude of $(-80 \pm 10)''/\text{century}^2$.
This differs significantly from the portion due to tidal friction of
 $(-42 \pm 4)''/\text{century}^2$. Attributing the difference to a changing gravitational
constant, as suggested by Hoyle, the implied rate is $\dot{G}/G = (-1.1 \pm 0.3) \times 10^{-10}/\text{year}$.
This interpretation is supported by the fact that conservation of angular
momentum for tidal forces in the Earth-Moon system would be implied, and
by other astrophysical and geophysical data.

A DETERMINATION OF THE RATE OF CHANGE OF G

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Introduction

No variation in a fundamental constant of physics has yet been demonstrated. However, the expansion of the universe has suggested to many theoreticians that the Universal Gravitational Constant, G, may be decreasing, either as a cause or an effect of the expansion. Indeed, at least two currently plausible cosmological theories, Brans-Dicke (1961) and Hoyle-Narlikar (1972), demand that G be decreasing. In the widely-accepted general relativity theory, however, G remains constant. This paper deals with observational evidence that G is, in fact, decreasing at a rate of about one part in ten billion per year. It also summarizes some implications of this result, and other possible interpretations of the observational data.

Principle Behind Determination

An immediate consequence of a decreasing gravitational constant is a slow, adiabatic expansion of all orbits in the solar system about their primaries. Vinti (1974) has shown that, for two-body motion with G varying inversely with time, then

$$[1] \quad \frac{\dot{G}}{G} = - \frac{\dot{a}}{a} = \frac{1}{2} \frac{\dot{n}}{n} = - \frac{1}{2} \frac{\dot{P}}{P},$$

where dot denotes the time derivative, a = semi-major axis, n = mean motion, P = period of revolution. Hoyle (1972) has shown how such a change in the mean motion of the Moon about the Earth can be distinguished from similar

changes due to other causes, such as tidal friction. Since eq. [1] holds for the Earth's orbit around the Sun, as well as for the Moon's orbit around the Earth, it follows that an astronomical time scale derived from the observed motion of the Sun about the Earth, such as Ephemeris Time, will suffer a slowing at the same proportional rate, \dot{G}/G . Hence, when we measure the deceleration of the Moon's mean longitude utilizing Atomic Time (presumed uniform), we obtain the total deceleration from all causes. But when we measure the same deceleration utilizing Ephemeris Time, any contribution due to \dot{G} would be excluded, since it would be absorbed into the time scale. The difference between the two deceleration values would then indicate the amount due exclusively to a decreasing gravitational constant.

Observational Data Utilizing Ephemeris Time

There have been three determinations of the deceleration of the Moon's mean longitude utilizing Ephemeris Time. The first is due to Spencer-Jones (1939), who analyzed observations of the Sun, Moon, Mercury, and Venus. His results were converted to their Ephemeris Time equivalent by Clemence (1948), and the mean error was estimated by Morrison (1972a), obtaining a lunar deceleration, over and above the acceleration predicted by gravitational theory, of $(-22'' \pm 7'')/\text{cy}^2$. The second determination is due to Newton (1969), who analyzed ancient observations, primarily of solar eclipses, to obtain independent estimates of $(-41''.6 \pm 4''.3)/\text{cy}^2$ and $(-42''.3 \pm 6''.1)/\text{cy}^2$. The third determination was made by Oesterwinter & Cohen (1972), who analyzed meridian circle observations of the Sun, Moon, and planets since 1913 to obtain $(-38'' \pm 8'')/\text{cy}^2$.^{1/}

^{1/} The statement in this last paper that the Atomic Time scale was extrapolated backwards to 1913 is incorrect in the context of the present discussion--if G changes, then the time scale which was extrapolated backwards was an Ephemeris Time scale, adjusted in epoch and rate to fit the Atomic Time scale from 1955-1969. Figure 8 of that paper should be re-labelled, for present purposes, as "New Ephemeris Time minus old Ephemeris Time,"

Of these three determinations, the most suspect is the Spencer-Jones value, which, as has been pointed out by Morrison (1972b), is influenced greatly by seventeenth century observations of transits of Mercury across the Sun. Omitting this first determination and averaging the others gives

$$[2] \quad (-41'' \pm 3'')/\text{cy}^2$$

as the acceleration of the Moon's mean longitude, utilizing Ephemeris Time. Adopting any value closer to the Spencer-Jones result will make the derived rate of decrease of G larger.

Observational Data Utilizing Atomic Time

The analysis of lunar occultation data utilizing Atomic Time to determine the Moon's mean longitude deceleration has been discussed earlier by Van Flandern (1970), obtaining $(-52'' \pm 16'')/\text{cy}^2$, and by Morrison (1973), who obtained $(-42'' \pm 6'')/\text{cy}^2$. The results of a new analysis, presented here, differ from both earlier results primarily because of the use of a numerically-integrated lunar ephemeris by Garthwaite et al. (1970), instead of the Brown-Eckert analytical theory. The bias in the integration minus theory differences between 1955 and 1973 which caused this change is of unknown origin, but is readily evident in Figure 1 of the Garthwaite et al. paper, where it is seen to be actually periodic on a longer time scale.

The occultation observations in the new analysis cover the period 1955-1973, during which Atomic Time was available. To guard against an obvious source of systematic error, separate solutions were performed for

photoelectric timings (1612 observations) and for visual timings (32000 observations, three-fourths of them during 1970-73), because the effects of the reaction time of the visual observers cannot be completely removed. Also, the effect of the very non-uniform distribution of the observations was removed by a simple weighting scheme, such that the total weight of all observations within each year was kept nearly the same. The total deceleration not accounted for by ordinary gravitational theory is determined to be $(-86'' \pm 10'')/\text{cy}^2$ from the photoelectric observations, and $(-74'' \pm 6'')/\text{cy}^2$ from the visual observations. (Standard errors are quoted.) Each of these results was subjected to significance testing, and vulnerability to various anticipatable sources of systematic error. As a result of such testing, more realistic confidence levels were determined than the formal mean errors. The combined results, with an *error* estimate intended to be realistic, is then

$$[3] \quad (-80'' \pm 10'')/\text{cy}^2 .$$

The Rate of Change of G

Differencing the lunar deceleration utilizing Atomic Time, eq. [3], and the deceleration utilizing Ephemeris Time, eq. [2], we obtain the deceleration due to a changing gravitational constant, $(-39'' \pm 11'')/\text{cy}^2$. Substituting this result for \dot{n} together with the lunar mean motion of $17''33 \times 10^8/\text{cy}$ into eq. [1] gives

$$[4] \quad \frac{\dot{G}}{G} = (-1.1 \pm 0.3) \times 10^{-10}/\text{yr}.$$

Examination of Alternative Hypotheses

Examining the variation of Kepler's third law,

$$[5] \quad 2 \frac{\dot{n}}{n} + 3 \frac{\dot{a}}{a} = \frac{\dot{G}}{G} + \frac{\dot{M}}{M} \quad ,$$

where M is the mass of the two-body system being considered, we see that there are several possible interpretations of an observed non-zero value for $\frac{\dot{n}}{n}$. The preferred interpretation with $\frac{\dot{G}}{G} \neq 0$ and $\frac{\dot{M}}{M} = 0$ is given by eq. [1], and will subsequently be referred to as "Hypothesis G". However, there are three distinct interpretations possible in which $\frac{\dot{G}}{G}$ remains zero. To discuss these, let A be defined to be the observed numerical rate given by eq. [4].

Let us first consider the case for which $\frac{\dot{G}}{G} = 0$ and $\frac{\dot{M}}{M} = A$. If only the mass of the Sun decreases at the rate A, the orbits of all the major planets expand, and their periods increase, as before; except that the orbit of the Moon about the Earth would not expand, since the Earth's mass would not be changing. Under this assumption, all of the deceleration in eq. [3] must be ascribed to tidal friction, or some such mechanism; and the deceleration in eq. [2] is smaller because the Ephemeris Time scale, given by the motion of the Sun about the Earth, is decelerating with respect to Atomic Time. Let us call this "Hypothesis M (mass)". It has the attractive

feature that it would explain the discordance of the Spencer-Jones value for the lunar deceleration, to the extent that it is actually dependent upon seventeenth century transits of Mercury, as discussed by Morrison (1973). To test Hypothesis M, consider the possible ways in which the Sun's mass could be decreasing. From $E = mc^2$ and the solar constant, we can compute that the rate of conversion of matter to energy in the Sun causes a mass loss of about 7×10^{-16} /yr. Assuming that the solar wind is isotropic, the mass transported away from the Sun by the wind is at most about 3×10^{-13} solar masses per year. Using recent Skylab results that solar flares which remove 10^{16} g from the Sun per event occur with a frequency of one per 100 hours (Newkirk, 1974), the loss rate might be 10^{-15} /yr. Even the largest solar prominences on record are unlikely to remove more than about 10^{-16} solar mass. Therefore it would seem that we can safely rule out Hypothesis M.

The second possible interpretation for which $\frac{\dot{G}}{G} = 0$ and $\frac{\dot{M}}{M} = 0$ is $\frac{\dot{a}}{a} = -\frac{4}{3} A$. Call this "Hypothesis S (space)". This implies a uniform expansion of all space. If we change the units of $-\frac{4}{3} A$ for comparison with the Hubble constant, we get 143 km/S/MPC.

In support of this hypothesis, Wesson (1973) has summarized observational data which suggest that the Earth has been expanding at about the Hubble rate throughout its existence. The hypothesis is made more attractive by the prediction that the Earth would originally have had only half its present radius, and that all of the present continents would then have formed a covering over the entire Earth. Sea-floor spreading is readily

understandable with such an expanding model for the Earth. To test Hypothesis S, we need a direct measure of the rate of expansion of some orbit in the solar system. At the present time, we cannot observationally distinguish between Hypotheses G and S. The principal reasons for preferring Hypothesis G are the lack of a theoretical basis for Hypothesis S, the implausibly large Hubble rate which it requires, and the continuous increase in angular momentum of all orbiting and rotating bodies which it implies.

Discussion of Supporting Evidence

Hoyle (1972) has reviewed much of the evidence in support of Hypothesis G, that the gravitational constant is decreasing at about a part in ten billion per year. Perhaps the most persuasive implication is that of conservation of angular momentum in the Earth-Moon system. If we take Newton's (1973)

results, for example, then $10^9 \frac{\dot{\omega}}{\omega}$ for the Earth's rotation is about $(-25 \pm 3)/\text{cy}$, when measured utilizing Ephemeris Time. This breaks down into a frictional part of $(-48 \pm 4)/\text{cy}$ from the tidal action of the Sun and Moon, and a non-frictional part of $(+23 \pm 5)/\text{cy}$ of unknown origin. When Atomic time is utilized instead, observed values of $\frac{\dot{\omega}}{\omega}$ utilizing Ephemeris Time must be corrected by $2A$, where A is given by eq. [4]. Hence, Newton's results ~~from~~ ^{for} $10^9 \frac{\dot{\omega}}{\omega}$ would become $(-47 \pm 3)/\text{cy}$, in excellent agreement with the frictional part of $(-48 \pm 4)/\text{cy}$, as required by eq. [2] from angular momentum conservation considerations alone.

Other arguments cited by Hoyle in support of Hypothesis G include the thermal history of the Earth, which would have been much hotter in its early history than at present, consistent with survival temperatures for the most primitive life forms on Earth; the calculation that the initial helium content of the Sun would have to be about 15% rather than about 25% to reach its present-day luminosity, consistent with solar wind measures that the outer solar layers, which are believed to have remained essentially unchanged over the lifetime of the solar system, contain 15% helium; and the prediction of a blue, starlike class of quasistellar galaxies, such as recently found by Sandage, due to an enhancement of the rate of stellar evolution as a function of mass.

Wesson (1973) cites further geophysical evidence, including an illuminating discussion of continental drift and sea-floor spreading. If G is decreasing, the Earth must expand somewhat due to the decreased weight of the surface layers. Wesson points out that the expanding globe model makes it easier to understand

how a mid-oceanic ridge system can almost surround a continent like Antarctica.

Morganstern (1972) has examined a pressure-filled curved-space cosmological situation, and finds that positively-curved spaces with pressure predict a $\frac{\dot{G}}{G}$ rate in agreement with eq. [4].

Newton (1968) also came to the conclusion that the Earth's polar moment of inertia is increasing and that G is decreasing at rates quite consistent with eq. [4], and with the rate of upward transport of material at the mid-ocean ridges.

The principal cosmological theory consistent with Hypothesis G is that of Hoyle-Narlikar (1972). This theory also has the important features that it is consistent with general relativity insofar as the solar light bending and Mercury perihelion advance tests are concerned; and it is characterized by a properly integrated form of Mach's principle. In this theory, eq. [4] gives twice the Hubble expansion rate. Therefore, the Hubble constant implied by eq. [4] would be (54±15) km/S/MPC--in good agreement with the most recent determinations of that parameter. The Hubble age of the universe would be (9±3) aeons.

Future Observational Data

Independent determinations of the rate of change of G are now nearly possible from planetary radar ranging results. The latest determination by Shapiro et al. (1971) set an upper limit of $\dot{G}/G < 4 \times 10^{-9}/\text{yr}$; and this limit will decrease with time squared, commencing from the first planetary radar observations in the early 1960's. A determination from Mercury observations may already be possible. The lunar laser ranging program will surely result in information on this question by the end of this decade. And a spacecraft orbiter about Venus could do the job in just a couple of years. One thing seems certain -- the question of a decreasing gravitational constant will be definitively decided within the next ten years.

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BIOGRAPHICAL SKETCH

Thomas C. Van Flandern. Born June 26, 1940, Cleveland, Ohio. Undergraduate work at Xavier University in Cincinnati under General Motors Scholarship plan. B.S. in Mathematics, June 1962. Teaching fellowship at Georgetown University, 1962-3. Ph.D. in Astronomy from Yale University, June 1969. Dissertation adviser was Prof. G. M. Clemence, an earlier Gravity Research Foundation prize winner. Research astronomer at U.S. Naval Observatory since February, 1963. Research consultant at Jet Propulsion Lab, January-April 1971. Member International Astronomical Union, American Astronomical Society, American Geophysical Union, American Association for Advancement of Science, Sigma Xi. Principal research activity: Celestial mechanics, especially lunar motion.