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The Gravity of Cosmic Loops

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Summary:

It is demonstrated that the gravitational field of a loop of cosmic string can *repel* particles. This leads to a very interesting pattern of accretion of matter around a cosmic loop.

In recent years there has been immense interest in the possibility of cosmic line defects arising during a phase transition in the early universe[1]. These line defects, called "cosmic strings", would exist in the shape of infinite random walks or in the form of closed loops. It is speculated that the loops of cosmic string may have been the seeds for galaxy formation.

One might visualize a loop of cosmic string as a giant, stretched rubber-band that oscillates under its own tension. The equation of motion of the loop follows quite straight-forwardly from the Nambu action. In addition, matters are greatly simplified if we adopt the "conformal gauge" and consider the loop to be situated in a flat, non-expanding background. Then, in the center-of-mass frame of the loop, the loop's motion can be decomposed into left- and right- moving components. If the position of the loop is $f^{\mu}(\sigma,t)$, we can write,

$$f^{0}(\sigma,t) = t \ , \quad \vec{f}(\sigma,t) = \frac{\vec{a}(\xi) + \vec{b}(\eta)}{2} \tag{1}$$

where, $\xi = 2\pi(\sigma - t)/L$, $\eta = 2\pi(\sigma + t)/L$, and L is the length of the loop. The choice of gauge also constrains the otherwise arbitrary functions $\vec{a}$ and $\vec{b}$ so that $\vec{a}'^{\,2} = \vec{b}'^{\,2} = 1$, where the primes denote differentiation with respect to the argument. A useful way to picture these trajectories was first prescribed by Kibble and Turok[2]: the vectors $\vec{a}'$ and $-\vec{b}'$ describe two trajectories on a unit sphere. In the center-of-mass frame of the loop, the centroid of these trajectories has to be at the center of the unit sphere. It is clear, then, that unless the trajectories on the unit sphere are very convoluted, they will intersect. By differentiating equation (1), it is seen that if the trajectories intersect, there will be a point on the loop that, at one instant during the loop's period of oscillation, will move with the speed of light. Such a point on the loop is known as a "cusp". Also, since the motion of the loop is periodic, the cusp reappears in every period of the loop's motion at the same position in space.

Although cusps seem to be very likely, they are by no means inevitable. If the trajectories of $\vec{a}\,'$ and $-\vec{b}\,'$ were like the seams of a tennis ball, there would be no intersection and consequently no cusps. Another way out of cusps is to introduce the possibility of "kinks" on the string. This way, one could have breaks in the trajectories for $\vec{a}\,'$ and $\vec{b}\,'$ on the unit sphere, and so avoid cusps.

From the point of view of gravity, it is the cusps that are the most interesting features of a cosmic loop. Since a cusp moves with the speed of light, it is here that we expect the most violent things to happen.

The gravitational field of the loop can be found in the weak-field approximation. If we also adopt the harmonic gauge, Einstein's equations take on the usual form of an inhomogenous wave equation. Then the solution to Einstein's equation is the retarded potential ($\eta_{\mu\nu}$=(−,+,+,+), $c = 1$),

$$h_{\mu\nu}(\vec{x},t) = 4G\mu \int d\sigma \; \frac{\dot{f}_\mu \dot{f}_\nu - f'_\mu f'_\nu - \eta_{\mu\nu} \dot{f}^2}{|\vec{x} - \vec{f}|} \left[\frac{1}{1 - \vec{e}\cdot\dot{\vec{f}}} \right], \qquad (2)$$

where, the integrand is to be evaluated at the retarded time, $\tau \equiv t - |\vec{x} - \vec{f}(\sigma,\tau)|$, μ is the energy per unit length of the string, $\vec{e}$ is the unit vector in the direction of $\vec{x} - \vec{f}(\sigma,\tau)$, and, dots refer to time derivatives while primes refer to derivatives with respect to σ.

The factor in square brackets in equation (2), which also arises in classical electrodynamics in the Lienard-Wiechert potentials, is a consequence of having to evaluate the effect of the source at retarded times. This factor diverges at the cusp if the observation point, $\vec{x}$, is directly in the line of the cusp's velocity. So we might expect that the gravitational field will be very strong at points directly in the line of the cusp's velocity. Hence the line along the cusp's velocity and originating from the cusp is known as the beam and the factor in square brackets as the "beaming factor".

To find the gravitational field felt by a particle near the beam we need to know the derivatives of $h_{\mu\nu}$. So we must first differentiate equation (2) and then evaluate the contribution of the cusp to the resulting integral. This effort is hindered by the presence of the retarded time in the integrand. However, there is a fortuitous change of integration variables. If we expand the integrand around the cusp and change the integration variable to $u = \sigma/\tau$, the integral reduces quite dramatically and can easily be analysed.

We choose our coordinate system such that the cusp is formed at t=0, $\vec{f}$=0 and is moving with unit velocity in the +z-direction. If the observation point is exactly on the beam, $\vec{x} = (0,0,z)$, the result of the integration is,

$$h_{00,3} \approx -\frac{G\mu\ L^{4/3}}{z\ (t-z)^{4/3}}\ sign(t-z)\ . \tag{3}$$

It is clear that the weak-field approximation breaks down if we get too close to the cusp. An estimate of $h_{\mu\nu}$ at a point slightly off the beam shows that we can safely use the weak-field approximation until a distance of $\approx G\mu L$ from the beam. Since the parameter $G\mu$ is typically of the order of 10^{-6}, it is easy to confine ourselves to regions where the weak-field approximation holds. (Even if we are right on the beam, but far from the cusp, the weak-field approximation will remain valid over all but a very short time interval).

The most peculiar feature of $h_{00,3}$ in equation (3) is the sharp change in sign at t=z. To understand this sign change, we note that the string at the cusp first accelerates, reaches the velocity of light and then decelerates. The sign of $h_{00,3}$ is just the sign of the acceleration of the string. Furthermore, the sign change is sharp because the singularity in the beaming factor prevents $h_{00,3}$ from gradually going through zero and changing sign. This results in the infinite discontinuity in $h_{00,3}$ at t=z.

This behaviour of the metric leads to curious consequences. For example, in the present situation, the accleration of a non-relativistic test particle in the +z-direction is essentially given by $h_{00,3}$. So a test particle near the beam first experiences a strong repulsive force. Then, suddenly, the force switches sign and becomes attractive. However, by the time the attractive force turns on, the particle is further away and so experiences a weaker attractive force. Therefore the net effect of the cusp is to *repel* particles !

Fantastic, as this conclusion is, it seems quite reasonable from another viewpoint. The cusp is an ultra-relativistic point on the string and so it will beam enormous amounts of gravitational radiation in the direction of its motion[4]. Some of this radiation will be absorbed by the surrounding particles. Conservation of momentum then implies that these particles will be carried out by the radiation and hence will be repelled by the cusp.

The amount of energy gained by a particle depends on how long it experiences the repulsive force. This, in turn, depends on the factor $t-z(t)$, where, z(t) is the z-coordinate of the particle. The faster a particle moves in the +z direction, the slower does $t-z(t)$ change. So, a particle that is moving at a high velocity in the +z-direction will feel the repulsive force for a very long time and may be accelerated to a very high Lorentz factor. The energy that the particle gains must necessarily come from the emitted gravitational radiation. This results in the "Landau damping" of the gravitational wave.

In a cosmological setting such a "particle accelerator" is very interesting and desirable. If the string is formed during a phase transition at the Grand Unification scale the energy output of the loop is about 10^{48} ergs/sec. Even if a small fraction of this energy is absorbed by the matter surrounding the loop, it can lead to many spectacular astrophysical phenomena.

It should not be thought, however, that the only effect of the loop is to repel the surrounding matter. At large distances and in directions away from the beams, the particles feel the average field of the loop. The average field due to the loop is the usual *attractive* Newtonian field of the surface traced out by the loop during one oscillation[5]. The total mass of the surface is the mass of the loop while the mass density of the surface is proportional to $1/\sqrt{1-v^2(\sigma,t)}$, where, $v(\sigma,t)$ is the speed of the string. The amusing point here, is that, at the cusp, $v=1$. So the surface density diverges at the cusp and the cusp is, on the average, the "center of attraction". This means that if we were to look only at the average force of the loop, all surrounding matter would eventually collapse onto the cusps of the loop. But we have already seen that a cusp repels matter in the direction of its velocity. Hence, the net effect is that the particles will collect *behind* the cusps. If a particle continues to fall and gets in front of the cusp, it will be shot out in the direction of the beam with a huge Lorentz factor.

These gravitational features give very interesting structure to objects that accrete around cosmic loops. The results of a general relativistic (weak-field approximation) N-body simulation are shown in Figure 1. In the simulation, $G\mu = 2.5 \times 10^{-4}$, and both the Hubble expansion and self-gravity of the particles have been ignored. (If these factors were to be included in the simulation, it can be argued that they would only enhance the features of Figure 1[(3)].)

The accretion pattern in Figure 1 is quite striking in its basic similarity to some of the structures observed in the sky. It is tempting, though perhaps unwise, to speculate that these structures are the result of the gravitational repulsion of the cusps of a cosmic loop.

References:

1). To review the subject see, A Vilenkin, Phys. Rep., 121, (1985) 263.

2). T.W.B. Kibble and N. Turok, Phys. Lett., 116B (1982) 141.

3). T. Vachaspati, Phys. Rev. D, March 15, 1987, in press.

4). T. Vachaspati and A. Vilenkin, Phys. Rev. D31 (1985) 3052.

5). N. Turok, Phys. Lett. 123B (1983) 387.

Figure captions:

1a). The initial distribution of 441 particles in a plane containing the two cusps and their emitted beams. The positions of the cusps are indicated by circles and the beams by arrows.

1b). The accretion pattern after 40 loop oscillations. The positions of the cusps and the beams are the same as in Figure 1a). A few particles have escaped beyond the figure.

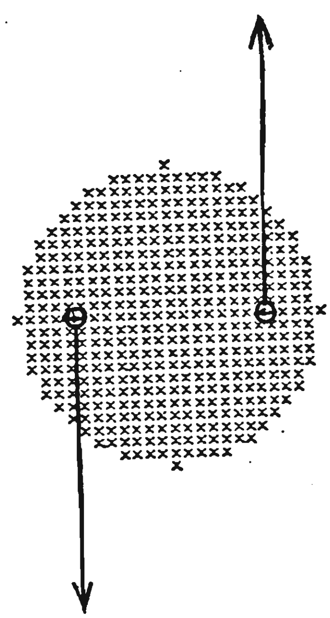

FIG 1(a)

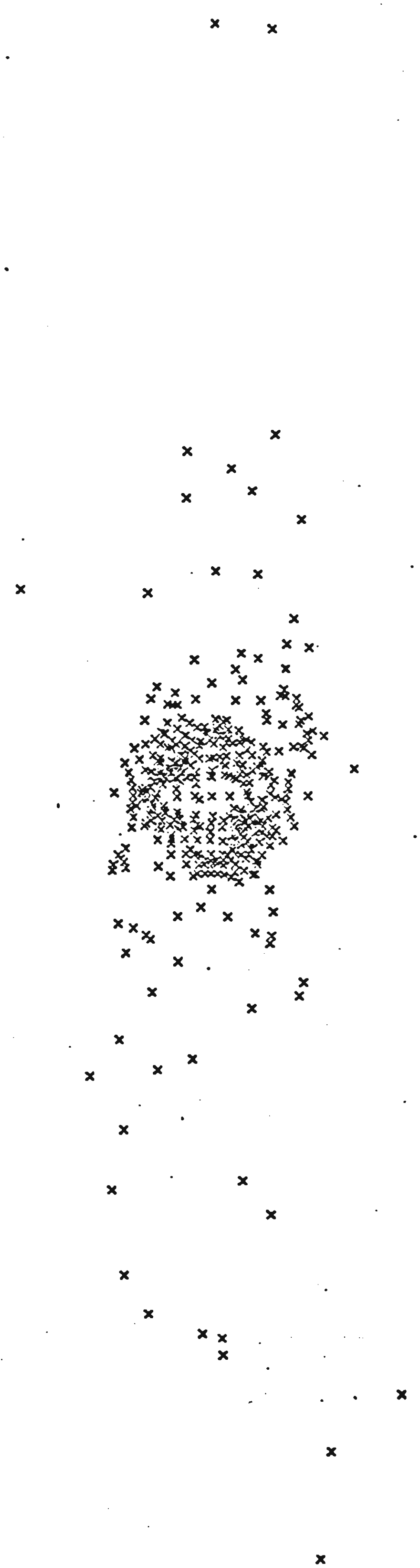

FIG 1(b)