Gravity and the Dimensional Constant "G"

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INTRODUCTION

Among the many experiments performed in the field of Physics I wish to call your attention to four experiments represented by the following equations:

Newton's Second Law (1)

$$F \propto Ma \qquad F = cMa \qquad F = Ma$$

Universal Gravitation Equation (2)

$$F \propto \frac{MM'}{r^2} \qquad F = G\,\frac{MM'}{r^2}$$

Field of Electricity (3)

$$F \propto \frac{qq'}{r^2} \qquad F = k\frac{qq'}{r^2}$$

Field of Magnetism (4)

$$F \propto \frac{mm'}{r^2} \quad F = \frac{1}{\mu}\,\frac{mm'}{r^2}$$

Where,

- F = force
- M = mass
- a = acceleration
- r = separation
- G = univ. grav. const. ($M^{-1}L^3T^{-2}$)
- q, q′ = electric charges
- K = dielectric constant
- m, m′ = magnet poles
- μ = permeability

As indicated all four experiments or equations express the physical quantity of Force. The physical dimension of Force seems to be different in all four cases, however. - This inconsistency we cannot permit for the following reasons:

1. Physical dimensions are the sole agents identifying uniquely physical quantities, therefore identical quantities must have identical dimensions.

2. All four forces expressed by the respective four equations are the same, IDENTICAL physical quantities, yet have different dimensions.

3. Therefore we must conclude that some adjustments must be made so as to arrive at a true, unique dimension for Force. These adjustments, however, must be made with physical justifications. As we can see (3.)* and (4.) HAVE justifiable dimensional proportionality constants, dielectric constant and permeability respectively. A choice has to be made between equations (1.) and (2.) to decide, whether $F=MLT^{-2}$ or $F=M^2/L^2$ as indicated by (1.) and (2.) respectively. We accepted (1.) /The justification for this choice will be pointed out later/ and adjusted (2.) by G or $M^{-1}L^3T^{-2}$. This was done without any physical justification. We do not even know what these dimensions of G stand for. What if they represent the intensity of a shielding effect against the force of Gravity just like the dielectric constant and permeability represent the extensity of a shielding medium in the two analgous equations.

This is the problem that will be treated in greater detail in the following essay.

GRAVITY AND THE DIMENSIONAL CONSTANT "G"

The need for dimensional homogeneity in true physical equations cannot be overemphasized for the simple reason that no equation can express absolute truth in general unless there exists a dimensional identity besides numerical equality.† We seem always to be aware of the necessity of numerical equality while having a tendency to forget the need for dimensional homogeneity.

This existence of dimensional homogeneity can be utilized to a great extent, since with the aid of dimensional analysis we may verify developed equations, change units from one system to another, and with the combined use of experimental observation and dimensional analysis we may derive actual equations. The basis of deriving equations by this method is the guessing of the right

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* these numbers refer to equations on previous page.

†E.g. In the equation 12 $ft^3/sec = 12\ ft^3/sec$ it is just as important to have ft^3/sec on both sides, as it is important to have 12 on both sides, if the equation expresses absolute truth.

variables upon which the unknown physical quantity may depend. One equates this quantity to the product of the guessed variables, each of which is expressed in the basic units and raised to some unknown power, and one finally solves this exponential equation so as to *preserve and maintain demensional homogeneity.**

The basic units in common use are: F, force, M, mass, L, length, and T, time. Physical quantities such as volume or L^3 or velocity LT^{-1} can be expressed without any difficulty in the basic units. The problem arises when we wish to express mass, force, and time in terms of one another. Newton's second law was accepted to serve this purpose. Namely: F=ma or ($F=MLT^{-2}$).

Thus the identity of force has been established. Surely, then, whenever the physical quantity of force occurs, regardless of its origin it must have the same dimensions. Other experiments have been performed, and although they express the same force, the identity or the dimensions of the force seem to be different. We may list such relations as the force of attraction between two magnet poles, the force of attraction between two electric charges, and the force of attraction between two masses as expressed by Newton's Universal Gravitation Equation.

Naturally we cannot use four different kinds of forces on the basis of the above four equations since they are all the same. All four seemed to express the truth, however, and a choice had to be made to decide which equation was to be used in defining the basic units in terms of one another. The equation that was chosen naturally had to be considered as the one most probably true in its original form. Newton's Second Law was a good choice because we found later that magnetic force of attraction and force of attraction between two electric charges also depend respectively on permeability and the dielectric constant, besides the poles or charges and the separation between them. (The terms permeability and the dielectric constant express the nature and extensity of a shielding medium against forces of attraction.) The discovery of these two physical phenomena also tended to prove that our choice of Newton's Second Law as the fundamental true equation was a correct one, for we haven't come across any facts that might point to the application of a correction dimensional-factor to this latter equation, as in the previously mentioned magnetic and electrical equations.

We have not yet discussed the Universal Gravitation Equation. Let us observe this together with Newton's Second Law:

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* See illustrative example starting on page 21.

Universal Gravitation Equation	Newton's Second Law
$F = (m \cdot m'/r^2) \cdot (G)$	$F = m \cdot a$
where	where
F=force of attraction between masses m and m′	F=force acting on m
r =separation between m and m	m=mass of particle
	a=acceleration of particle
thus force seems to have the dimension:	thus force seems to have the dimension:
$F = M^2 . L^{-2}$	$F = M . LT^{-2}$

Surely both forces represent the same kind of force, and since we believe that the dimensions of force shown by Newton's Second Law is the correct one, $F=M^2.L^{-2}$ must be incorrect Therefore, to make it correct we must introduce a dimensional constant, namely G or the Universal Gravitational Constant having the dimensions of ($M^{-1}L^3T^{-2}$) which factor makes the equation artificially correct on the basis of the accepted dimensions, that is $F=M.LT^{-2}$.

The words "artificially correct" were used, since have we the right to multiply one side of an equation by a dimensional factor, which actually represents a physical quantity, without having a physical justification or reason to do so? Wouldn't this seem to be analogous to a 2 which we put in front of a 3 just to make it 6? In other words, if we have a case where we state that a 3=6, isn't the only right way to correct this to reexamine our original equation and our procedure in order to find our mistake, namely where that particular 2 has been omitted? If we merely multiply it by 2 without trying to find a reason for it, it is wrong, but not any more wrong than if we multiply the Universal Gravitation Equation by G or ($M^{-1}L^3T^{-2}$), without trying to find out the physical reason for it. This fact, I believe, is recognized, and we say that it is understood that this G must represent some physical quantity. We know we have to include it in our equations, we know how to use it, and we use it, but we just don't know what it is. It might be within or beyond our present physical or scientific knowledge, but the fact is that we have not been able to identify it as yet. Since it is apparent that the analogous equations in the field of magnetism and electricity do possess a factor which is connected to some shielding medium against the force of attraction, why not investigate and try to determine the possible existence or non-existence of a shielding medium against the force of Gravity, by trying to identify this G by the use of dimensional analysis and experimental observations?*

Let us now investigate G or $(M^{-1}L^{3}T^{-2})$ dimensional constant in the equation: $F = (m \cdot m' / r^2) \cdot (G)$. What might be the identities of the physical quantities or variables hidden in G? Even if we could find these physical quantities we would face two problems. Namely: 1. We would have to prove by experimental observations that F really does depend upon these newly found physical quantities. 2. We would have to show that the reason why the identities of these new variables can be hidden in G and thereby take an inactive part in the equation, is because they remain either constant or very near constant. For if this were not the case, we couldn't have arrived at the correct answers by keeping G constant as we have done in the past, yet we know that our answers were correct or very near correct. What if we could prove that the "very near constant" and "very near correct" were the case all the time, thereby implying that the variation of some physical quantity hidden in G might increase or decrease or might even completely eliminate F, the gravitational force of attraction?

We know already that the force of attraction will vary as the product of the masses or M^2 and inversely as the separation between them squared or L^{-2}, but let us see what G or $(M^{-1}L^{3}T^{-2})$

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*We have stated the procedure of deriving equations on the basis of experimental observations and dimensional analysis, but before we apply it in trying to determine G, let us present the classic example from the textbooks on dimensional analysis, merely to illustrate the procedure:

Let us assume that the period (t) of the pendulum depends on the mass (m) of the bob, the length (l) of the cord and the acceleration (g) due to gravity. That is:

$$t = f(m, l, g)$$

$$t \propto (m)^x (l)^y (lt^{-2})^z$$

$$\begin{aligned} \text{for } t, \quad & 1 = -2z \\ m, \quad & 0 = x \\ l, \quad & 0 = y + z \end{aligned}$$

$$z = -\tfrac{1}{2} \qquad y = \tfrac{1}{2} \qquad x = 0$$

$$\text{or } t \propto \sqrt{\frac{l}{g}}$$

The equation here developed by the use of dimensional analysis and correct guesses has been verified by experimental results, which would make the equation acceptable, nevertheless we know that the equation has been verified by a more complex and dependable method. Therefore this shows that it is possible to get the right answers with dimensional analysis and experimental observations.

might suggest.

The combination of the three dimensions involved might indicate innumerable possibilities. For example, it might be something per unit mass or M^{-1} or length L, or area L^2, or volume L^3 or it might be linear velocity LT^{-1} or acceleration LT^{-2}, or angular velocity T^{-1} or acceleration T^{-2}. It might also represent some function of mass density, namely ML^{-3}.

Since we do know that, (where the equation in question is applicable,) the masses in the universe do revolve about each other with a certain angular velocity, and since we hope that some shielding effect will come into the picture just as in the other two analogous equations, we might say, rather guess, that this force of attraction might also vary as the angular velocity of the masses and the density of some shielding medium. Thus if we employ dimensional analysis, and choose the variables in G as angular velocity or T^{-1} and mass density of ML^{-3}, we may proceed as follows:

$$F = f(M, r, \omega, \rho)$$

$$[MLT^{-2}] = [M]^x [L]^y [T^{-1}]^z [ML^{-3}]^{\mathfrak{z}}$$

$$\begin{aligned} \text{for } M, \quad & 1 = x + \mathfrak{z} \\ L, \quad & 1 = y - 3\mathfrak{z} \\ T, \quad & -2 = -z \end{aligned}$$

$$\begin{aligned} x &= 2 \text{ (by experiment)} \\ \therefore y &= -2 \\ z &= 2 \\ \mathfrak{z} &= -1 \end{aligned}$$

Where
M = mass
r = separation
ω = angular velocity
ρ = mass density
c = proportional constant (dimensionless)

$$F \propto [M^2 r^{-2}] [\omega]^2 [\rho]^{-1}$$

or

$$F = C \frac{mm'}{r^2} \omega^2 \frac{1}{\rho}$$

Thus dimensional analysis suggests, if our guess was right, that the force of attraction also varies as the (angular velocity squared), and inversely as some mass density, probably the mass density of some shielding medium. Let us discuss the (angular velocity squared) term before we try to verify the density factor which is supposed to be our greater concern.

First we have to prove by experimental results that what we said on the basis of dimensional analysis is true.

On the basis of Astronomical observations the following equation has been verified, where "t" stands for time of revolution:

On the basis of dimensional analysis:

$$t = \frac{r^{3/2}}{G^{1/2} m_2^{1/2}} \Phi\left(\frac{m_2}{m_1}\right)$$

or

$t \propto G^{-1/2}$

$\frac{1}{t} \propto G^{1/2}$

but

$t\omega = \theta$

$\therefore \frac{1}{t} \propto \omega$

and

$\omega \propto G^{1/2}$

or

$\omega^2 \propto G$

but

$F \propto G$

$\therefore \omega^2 \propto F$

$F \propto \omega^2$

ω = angular velocity
t = time of revolution
G = Univ. Grav. Const.
r = separation between m & m′
$m_1 = m$
$m_2 = m'$
θ = angle in radians

This analysis tends to prove that our guess was correct, since the results obtained by actual experimental observations and dimensional analysis are the same. Our second problem now is: why does not this angular velocity effect the value of the force?

The motion, and thereby the angular velocity in a circular orbit occurs because a force promotes radial acceleration. The only force which could promote this radial acceleration is the force of attraction itself. Therefore, even though the force of attraction depends on the angular velocity, rather is related to it, the fact is that the angular velocity will be a function of the force. Therefore it could afford to take an inactive part in G as long as we do not tend to change the natural value of the force of attraction. Should we introduce a positive or negative additional radial force, thereby tending to increase or decrease this natural angular velocity due to the force of attraction only, the dimensional value of (angular velocity)^2 could no longer take an inactive part in G but would very definitely have to be included separately with its proper numerical value, which at present, without any disturbances introduced, could be part of the numerical constant coefficient of G.

Since this (angular velocity)^2 factor seems to be feasible, this fact should give us some encouragement towards the possible

validity of the (1/mass density) factor, which might represent the reciprocal of the mass density of a medium with a shielding effect against Gravity. Additional facts pointing to the possible existence of a medium of this nature, are the existence of such shielding mediums against the forces of attraction in the analogous magnetic and electrical equations. Therefore, isn't it conceivable that the denser such medium gets, the smaller the force of attraction will be as indicated by our dimensional equation:

$$F = \frac{m \cdot m'}{r^2} \cdot (\text{angular velocity})^2 \cdot \frac{1}{\text{mass density}}$$

We must stop here for a moment and clear up one of the many possible arguments against the above assumptions. Someone may very well say that we are not measuring the gravitational force of attraction, for which F stands in the above equation, if we introduce an opposing force, even if this opposing force is in the form of a shielding medium, but will measure a resultant force which is the vector sum of the two. But the question now is: Whether or not we have ever measured a force due to gravity only? (Gravity of the earth for example). Isn't it true that whenever we measure the weight of a particle, which is the same as the force of attraction on the particle, we are measuring the vector sum of all the forces due to not only the gravity of the earth, but air pressure gravity of the sun, moon, stars and so on? Granted though that these latter forces are negligible and immeasurable compared to the force of the attraction due to the earth, nevertheless they are there, even though the force of attraction of the earth dominated the resultant force as we implied before. Isn't it true then, that we can never measure magnetic, electrical or gravitational force of attraction as such alone, but merely a resultant force of attraction which will be a function of many things, like permeability, dielectric constant, and possibly "gravity shielding medium" respectively, even though the major components of these resultant forces will be the respective forces of attraction measured before? Therefore, I believe we are more correct if we talk about the "apparent" or "resultant" "forces of attraction" instead of merely "forces of attraction".

Thus if we will talk about the resultant force of attraction from now in we may state the following:

Regardless of what the surrounding medium is, water or air or any gaseous or liquid matter, the denser this medium becomes the less the apparent or resultant force of attraction becomes, for the buoyant forces will tend to balance the force of attraction. But let us not consider this buoyant force as a balancing independent force, let's rather consider buoyancy itself as one of the contributing physical factors that make up the "resultant force of attraction", as indicated by Dimensional Analysis.

Also by experimental results we may prove that any mass on the surface of the earth will keep "loosing" some of its weight, or the mutual resultant force of attraction will decrease between the earth and this mass if we increase the density of the medium between them, when the density of the medium becomes the same as the density of the mass, the resultant force upon this mass will be zero. If we increase the density of this medium beyond this point, there will be a negative resultant force of attraction between the earth and this mass. Thus the resultant force of attraction must definitely be a function of the density of the medium in which the masses in question are located.

Unfortunately the above arguments cannot be applied in our dimensional equation with the present accepted scale of density. On the basis of our arguments, if the mass density of the medium equals the mass density of m′ (see equation on page 25) then F would have to be equal to zero. To satisfy this condition the assigned value of the mass density of the medium in this case would have to be infinity. This would not only be impractical but likewise inapplicable. In addition, we would face another impossible situation, if we intend to use the present system of density. This would arise if the masses were located in vacuum, the density of which is zero, and therefore the resultant force of attraction, on the basis of the equation, would turn out to be infinity. This obviously is not true. Therefore, if we wish to maintain our arguments and thereby believe that the shielding effect against gravity is a function of the density of the medium in which the masses are located, we would have to devise a new density scale*, the range of which would be different from that of our present system, and as we can see on the basis of the equation even vacuum would have to be assigned a value for density, and this value would very probably be "1".

Another possibility is the finding of some, up till now unknown medium, to which our present system of density could be applied, and the density of this medium would vary inversely as the resultant force of attraction. But even if this is the case, it is not practical when we use the medium of vacuum, in which case the resultant force of attraction would again turn out to be infinity. The only excuse we may give is, that even in vacuum this unknown shielding medium is present. Therefore we would have to assign

*Just like devising the scales of Absolute Pressure and Absolute Temperature.

a numerical value for the density of this medium present in a vacuum, which would prevent infinity from becoming the absurd outcome of force in the equation.*

CONCLUSION

Some of the applications of Dimensional Analysis to "G", seem to be impractical. Nevertheless the many possible ways of approach, and the valid explanation of some of the dimensions in "G" seem to indicate, that the key to harnessing Gravity might be found through the proper identification of "G", the Universal Gravitational Constant.

P. S. Conclusions of the talk: "How can we win the cooperation of the Scientific World?" Gravity Day 53

1. Force of Gravity, which is apparently an inexhaustible force, should be considered an asset and might eventually act as a catalist to get cheap, but not free power.

2. Further extensive studies on Gravity will not be done in vain, provided we attempt these studies with the full realization that Force of Gravity is not Energy and that the Principle of Conservation of Energy and/or Matter is an unalterable basic truth.

* We might also picture this shielding medium as something that not merely opposes gravity but at the same time furnishes an essential path along which or through which gravity acts. It might be analogous to resistance in electricity which varies inversely as the flow of current, nevertheless, without introducing some medium which has resistance, no current may flow. Similarly then, the more of this shielding medium that is present, the less the resultant force of attraction would be, as indicated by dimensional analysis, nevertheless without this shielding medium no flow of gravity could take place.