THE ELECTRIC FIELD INDUCED BY A GRAVITATIONAL WAVE IN A SUPERCONDUCTOR: A PRINCIPLE FOR A NEW GRAVITATIONAL WAVE ANTENNA

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SUMMARY

In this paper we investigate the effect of a gravitational wave (GW) on a superconductor. We find that the key properties of a superconductor, namely zero resistance and perfect diamagnetism, give rise to an important new effect, the presence of an induced electric field **E**(**r**,t) in the interior of the superconductor. The **E** field reacts with the ions and superelectrons. We argue, that not only is the finding of the coupled interactions of gravitation, electromagnetism and superconductivity inherently interesting, but that the induced **E** field might provide a significantly more sensitive means of detecting gravitational waves. It appears likely that existing resonant-mass superconducting antennas with $L \simeq 3m$, $Q \simeq 10^8$ could be readily modified to detect **E** fields induced by GWs of dimensionless amplitude $h \simeq 10^{-24}$.

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Huei Peng[a,b] and Douglas G. Torr[a]

a. Physics Department, The University of Alabama in Huntsville, Huntsville, AL 35899, USA

b. Institute of Applied Mathematics, Academia Sinica, Beijing, P. R. China

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1. INTRODUCTION

In this paper we investigate the consequences of electric interactions which arise in a gravitational wave (GW) detector operated at superconducting temperatures. This work was motivated by recognition of the fact that when an antenna is cooled below the critical temperature, the superelectrons released and the ions will respond independently to the GW. Because the superelectrons and ions carry electric charges, these independent motions establish currents with a resultant electric field, which then couples the ion and superelectron motions.

In what follows we derive the equations which describe the physics of the interaction. We then provide quantitative estimates of the induced E field, and conclude that a measurement of the strength of this field could provide a significantly more sensitive means of detecting the presence of a GW than other techniques currently in use.

In order to reduce system noise, gravitational wave bar detectors are cooled to as low an antenna temperature (T_a) as possible[1-3]. One of the major goals for the third generation of resonant-mass antennas is to achieve ultralow operational temperatures of 10-50 mK[4,5,6]. At this T_a, an aluminium bar becomes a superconductor. It is hoped that this latest generation will achieve pulse sensitivities of $h \simeq 10^{-19}$–10^{-20}. Hereafter we will refer to antennas with temperatures lower and higher than the critical temperature, T_c, as superconducting and normal antennas, respectively.

A normal antenna can be treated as a structure composed of a neutral mass, because positive ions and electrons are bound and vibrate together under the influence of a GW. When one deals with a superconducting antenna it is necessary to take into account the two characteristic properties of a superconductor, namely zero electric resistance and perfect diamagnetism. As mentioned above, in a superconducting antenna, negative charges exhibit independent vibrational responses to a GW, because there is no resistance offered to superelectrons. The ions, however, still vibrate as damped harmonic oscillators. Therefore a net current arises which creates a time-dependent magnetic vector potential $\mathbf{A}(\mathbf{r},t)$. The $\mathbf{A}$ induces an electric field $\mathbf{E}(\mathbf{r},t)$ in the interior which reacts on the ions and superelectrons. In addition, there is a second argument for the presence of the internal $\mathbf{E}$ field. In order to ensure that the Meissner effect is not violated, there must be a time-dependent $\mathbf{E}$ field in the interior which generates a magnetic field which must cancel the one created by the net current induced by the GW.

Based on this effect, we suggest that instead of measuring the displacements, one may detect the $\mathbf{E}$ field induced by a GW in the interior of a superconducting antenna. In what follows we demonstrate that existing technology could, in principle, permit a sensitivity of $h \simeq 10^{-24}$. We let the speed of light $c = 1$.

2. THE GW INDUCED ELECTRIC FIELD IN THE INTERIOR

We will assume that: (1) The ion motion is still represented by a damped simple harmonic oscillator; (2) GWs will penetrate antennas; (3) Small vibrations of ions will not destroy the superconductivity of the antenna; and, (4) All time-varying quantities have the form $\mathbf{u}(\mathbf{r},t) = \mathbf{u}(\mathbf{r})e^{-i\omega t}$.

Because of the independent responses of ions and superelectrons to a GW and the effect of the induced **E** field on ions and superelectrons, we must treat ions and superelectrons separately. The equation of motion of the ions in the detector's proper reference frame is given by

$$\frac{d^2x_i}{dt^2} + \frac{dx_i}{\tau_0 dt} + \omega_0{}^2 x_i = a_{GW} + \frac{e}{m_i} E, \qquad (1)$$

and the equation of motion of the Cooper pairs is

$$\frac{d^2x_e}{dt^2} = a_{GW} - \frac{e}{m_e} E, \qquad (2)$$

where m_i and m_e are the masses of ions and electrons, respectively, and a_{GW} is the gravitational driving acceleration that results from projecting the tidal gravitational force due to a GW onto the antenna. We have also assumed that the mass of a Cooper pair is approximately equal to $2m_e$ (Ref. 7).

Then the displacements of ions and superelectrons are respectively given by

$$x_i = G(\omega)\,[\,-a_{GW} - (e/m_i)\,E\,], \tag{3}$$

$$x_e = (1/\omega^2)[\,-a_{GW} + (e/m_e)\,E\,], \tag{4}$$

where

$$G(\omega) = (\omega^2 - \omega_0{}^2 + i\omega/\tau_0)^{-1},$$

is the harmonic oscillator response function.

The ion-current and electron-current are respectively given by

$$J_i = \quad 2ne\, dx_i/dt = i2ne\omega G(\omega)a_{GW} + J_{is}(x,t), \tag{5}$$

$$J_e = -\,2ne\, dx_e/dt = -\,i2nea_{GW}/\omega \quad + J_{es}(x,t), \tag{6}$$

where n is the number of Cooper pairs per unit volume, and

$$J_{is} = -\frac{2ne^2}{m_e}\,\frac{m_e\omega^2 G(\omega)}{m_i}\,A(x,t), \tag{7}$$

$$J_{es} = -\frac{2ne^2}{m_e}\,A(x,t), \tag{8}$$

J_{es} is the London supercurrent. We refer to J_{is} as the ion-supercurrent. The J_{is} is dependent on both the frequency of the incoming GW and the quality factor $Q(\omega_0\tau_0)$ of the superconductor. Therefore, for a static vector potential A, i.e., when $\omega \to 0$, the ion-supercurrent vanishs, whereas the London supercurrent does not.

The net current induced by the incoming GW is a combination of the electron-current and ion-current,

$$J = J_e + J_i. \tag{9}$$

Now we derive the expression for the electric field E produced by the net current. Since GWs are very weak, we will not use the general relativistic Maxwell equations to derive the generalized London equations. Instead, we substitute the net current into the Maxwell equations to derive the generalized London equations which will be used to find the **E** field in the interior.

$$- \nabla \times \mathbf{J} = \mu_0^{-1} \lambda^{-2} \mathbf{B}, \tag{10}$$

$$\partial \mathbf{J}/\partial t = \mu_0^{-1} \lambda^{-2} \mathbf{E} + 2ne\,[\omega^2 G(\omega) - 1]\mathbf{a}_{GW}, \tag{11}$$

$$\nabla^2 \mathbf{J} = (\lambda')^{-2}\, \mathbf{J} + i\omega 2ne\,[\omega^2 G(\omega) - 1]\mathbf{a}_{GW}, \tag{12}$$

$$\nabla^2 \mathbf{E} = (\lambda')^{-2}\, \mathbf{E} + \mu_0 2ne\,[\omega^2 G(\omega) - 1]\mathbf{a}_{GW}, \tag{13}$$

$$\nabla^2 \mathbf{B} = (\lambda')^{-2}\, \mathbf{B}, \tag{14}$$

where

$$\lambda' \equiv \lambda\, (\, 1 - \omega^2 \lambda^2\,)^{-1/2}, \tag{15}$$

$$\lambda \equiv \lambda_L\, [\, \omega^2 G(\omega) d + 1\,]^{-1/2}, \tag{16}$$

$$d \equiv m_e/m_i, \tag{17}$$

λ_L is the London penetration depth. The real part of λ' represents the penetration depth.

If we ignore the effects of the GW, i.e., let $\omega \to 0$ and $a_{GW} \to 0$, Equations (10-16) reduce to the London equations[8]. According to the London theory, there will be no net current, electric field, and magnetic field in the interior of a superconductor. But there will be a net current and **E** field in the interior of the superconductor under the influence of a GW. In the Equations (12,13), the second term on the right hand sides result from the assumption that an incoming GW will penetrate a superconductor entirely and hence cause superelectrons and ions not only on the surface but also in the interior to vibrate.

A correct theory must be able to explain the Meissner effect. Equation (14) describes the Meissner effect, but the question is whether the magnetic field produced by the net current in the interior will be canceled or not, i.e., whether Equation (14) is consistent with Equations (12,13), or not. We start with the solutions of Equations (12,13) which consist of two parts, respectively,

$$\mathbf{J} = \mathbf{J}_s + \mathbf{J}_{int},$$

$$\mathbf{E} = \mathbf{E}_s + \mathbf{E}_{int},$$

where

$$\mathbf{J}_{int} = - i2ne\omega(\lambda')^2[\omega^2 G(\omega) - 1]\mathbf{a}_{GW}. \tag{18}$$

$$\mathbf{E}_{int} = - 2ne\mu_0(\lambda')^2[\omega^2 G(\omega) - 1]\mathbf{a}_{GW} = i\omega\mathbf{A}_{int}, \tag{19}$$

$\mathbf{J}_s$ and $\mathbf{E}_s$ are the surface terms which only exist in a very thin layer of surface; $\mathbf{E}_{int}$ and $\mathbf{J}_{int}$ are respectively the electric field and current remaining in the interior of a superconducting antenna. We are interested in what happens in the interior. Substituting Equations (18,19) into one of Maxwell's equations, we obtain

$$\nabla \times \mathbf{B}_{int} = \mu_0 \mathbf{J}_{int} + \partial \mathbf{E}_{int}/\partial t = 0. \tag{20}$$

These show that: (1) There is a net current $\mathbf{J}_{int}$ and an electric field $\mathbf{E}_{int}$, and therefore a vector potential $\mathbf{A}_{int}$, in the interior of the superconducting antenna. But the magnetic fields produced respectively by the $\mathbf{J}_{int}$ and the time dependent $\mathbf{E}_{int}$ are canceled out such that $\mathbf{B}_{int} = 0$, i.e., the vibrations of the magnetic field $\mathbf{B}$ die away within the penetration depth $\mathrm{Re}(\lambda')$; (2) Part of energy of GW will be transferred to electromagnetic energy.

The mechanism by which the induced **E** field and the net current, including both ion-supercurrent and London supercurrent, expel the magnetic field may be explained as follows. In the interior of the superconducting antenna, vibrations of ions and superelectrons will be adjusted by the induced potential $\mathbf{A}_{int}$ such that there will be no magnetic $\mathbf{B}_{int}$ field.

3. THE PRINCIPLE OF A NEW ANTENNA

As we have shown above, for a superconducting antenna an incoming GW will induce an electric field in the interior. Based on this new effect, instead of detecting the displacements of the end of an antenna, we propose that one may detect the GW induced electric field $\mathbf{E}_{int}$ (or $\mathbf{A}_{int}$) which is given by Equation (19)

$$E_{int} \simeq - a_{GW} \frac{m_e}{e} \frac{(\omega_0^2 - i\omega/\tau_0)}{[\omega^2 - \omega_0^2 + \omega^2 d + i\omega/\tau_0]}, \tag{21}$$

where we have ignored small terms. The E_{int} is a narrow-band resonant function centered at $\omega_0/\sqrt{(1 + d)}$ with a full width at half maximum of ω_0/Q. One might expect that a more significant bandwidth restriction comes from the readout system. In order to estimate a representative value for E_{int}, we need to relate the length of a superconducting cylindrical antenna to that of an idealized oscillator.

The bulk modulus of elasticity is slightly different in the superconducting and normal states[9], but the effects are extremely small. Thus the speed of sound in both states can be considered to be approximately the same. It can be shown that when one takes into account the effects of the GW induced E_{int} field, the relation between the length L of a bar in the superconducting state and the length l of an idealized oscillator is approximately the same as that derived for a bar in the normal state

$$l \simeq 4L/\pi^2.$$

The restriction on the size of a superconducting antenna is

$$\omega = \pi v_s/L,$$

which is the same as that for a normal antenna.

At resonance, and averaged over all possible directions of incoming unpolarized GW, a representative magnitude of the E_{int} field at one end of the antenna is then given by

$$E_{int} \simeq \frac{2}{\pi^2\sqrt{15}} \frac{m_e}{e} Q\omega_0{}^2 Lh, \qquad (22)$$

where the expression for a_{GW} of Ref. 10 has been used and small terms have been neglected, h is the dimensionless amplitude of incoming GW, and L is the length of the antenna. Integrating (22), from the central point at the face of one end of a cylindrical antenna to the center of mass, the sensitivity of the superconducting antenna is found to be:

$$h = \frac{2\pi^2\sqrt{15}}{Q} \frac{eV}{m_e\omega_0{}^2L^2}, \qquad (23)$$

where V is the voltage between the center of mass and the end of the superconducting antenna.

If it proves to be technically feasible to measure a voltage drop of 10^{-22} V, say by using a superconducting quantum interference device (SQUID) magnetometer[11], the sensitivity of a superconducting antenna with $Q \simeq 10^8$, $\omega_0 \simeq 10^3$Hz and $L = 3$m, is of the order

$$h \simeq 10^{-24}. \tag{24}$$

Comparing this sensitivity with that of the third generation antennas[5] for which $h \simeq 10^{-20}$, it appears to be worthwhile to investigate this new concept for the detection of GWs further. We suggest that one might measure the displacement at one end and the GW induced **E** field at the other end of the superconducting antenna.

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