A Secret Tunnel Through The Horizon

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Abstract

Hawking radiation is often intuitively visualized as particles that have tunneled across the horizon. Yet, at first sight, it is not apparent where the barrier is. Here I show that the barrier depends on the tunneling particle itself. The key is to implement energy conservation, so that the black hole contracts during the process of radiation. A direct consequence is that the radiation spectrum cannot be strictly thermal. The correction to the thermal spectrum is of precisely the form that one would expect from an underlying unitary quantum theory. This may have profound implications for the black hole information puzzle.

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Classically, a black hole is the ultimate prison: anything that enters is doomed; there is no escape. Moreover, since nothing can ever come out, a classical black hole can only grow bigger with time. Thus it came as a huge shock to physicists when Stephen Hawking demonstrated that, quantum mechanically, black holes could actually radiate particles. With the emission of Hawking radiation, black holes could lose energy, shrink, and eventually evaporate completely.

How does this happen? When an object that is classically stable becomes quantum-mechanically unstable, it is natural to suspect tunneling. Indeed, when Hawking first proved the existence of black hole radiation [1], he described it as tunneling triggered by vacuum fluctations near the horizon. The idea is that when a virtual particle pair is created just inside the horizon, the positive energy virtual particle can tunnel out – no classical escape route exists – where it materializes as a real particle. Alternatively, for a pair created just outside the horizon, the negative energy virtual particle, which is forbidden outside, can tunnel inwards. In either case, the negative energy particle is absorbed by the black hole, resulting in a decrease in the mass of the black hole, while the positive energy particle escapes to infinity, appearing as Hawking radiation.

This heuristic picture has obvious visual and intuitive appeal. But, oddly, actual derivations of Hawking radiation did not proceed in this way at all [1, 2]. There were two apparent hurdles. The first was technical: in order to do a tunneling computation one needed to have a coordinate system that was well-behaved at the horizon; none of the well-known coordinate systems were suitable. The second hurdle was conceptual: there didn't seem to be any barrier!

Typically, whenever a tunneling event takes place, there are two separated classical turning points which are joined by a trajectory in imaginary or complex time. In the WKB or geometrical optics limit, the probability of tunneling is related to the imaginary part of the action for the classically forbidden trajectory via

$$\Gamma \sim \exp(-2 \text{ Im } I) \ , \tag{1}$$

where I is the action for the trajectory. So, for example, in Schwinger pair production in an electric field, the action for the trajectory that takes the electron-positron pair to their required separation yields the rate of production. Now, the problem with black hole radiation is that if a particle is even infinitesimally outside the horizon, it can escape classically. The turning points therefore seem to have zero separation, and so it's not immediately clear what joining trajectory is to be considered. What sets the scale for tunneling? Where is the barrier?

In this essay, I will show that the intuitive picture is more than a picture: particles do tunnel out of a black hole, much as Hawking had first imagined. But they do this in a rather subtle way since, as just argued, there is no pre-existing barrier. Instead, what happens is that the barrier is created by the outgoing particle itself. The crucial point is that energy must be conserved [3]. As the black hole radiates, it loses energy. For black holes, the energy and radius are related, and this means that the black hole has to shrink. It is this contraction that sets the scale: the horizon recedes from its original radius to a new, smaller radius. Moreover, the amount of contraction depends on the energy of the outgoing particle so, in a sense, it is the tunneling particle itself that secretly defines the barrier.

Now, one might fear that a calculation of Hawking radiation in which energy conservation is critical would require a quantum theory of gravity because the metric must fluctuate to account for the contraction of the hole. This is true but, fortunately, there is at least one regime in which gravitational back-reaction can be accounted for reliably and that is the truncation to spherical symmetry. Intuitively, in a transition from one spherically symmetric configuration to another, no graviton is emitted because the graviton has spin two and each of the spherically symmetric configurations has spin zero. So quantizing a spherically symmetric matter-gravity system is possible because no quantization of gravitons is required. Indeed, the only degree of freedom is the position of the particle (which, being spherically symmetric, is actually a shell).

Armed with these insights, we can compute the imaginary part of the action for a particle to go from inside the black hole to outside. A convenient line element for this purpose is

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + 2\sqrt{\frac{2M}{r}} dt\ dr + dr^2 + r^2 d\Omega_2^2\ . \tag{2}$$

This line element was first written down by Painlevé and Gullstrand long ago [4, 5], but they apparently missed the significance of their discovery. What they had unwittingly found was a coordinate system that was well-behaved at the horizon. Other nice features of their coordinate system are that there is no explicit time dependence, and constant-time slices are just flat Euclidean space.

Working in these coordinates within the spherically symmetric truncation, one calculates the imaginary part of the action,

$$\mathrm{Im}\ I = \mathrm{Im} \int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}} p\ dr\ , \tag{3}$$

where p is the momentum, $r_{\mathrm{in}} = 2M$ is the initial radius of the black hole, and $r_{\mathrm{out}} = 2(M - E)$ is the final radius of the hole. Here E is the energy of the outgoing particle. Notice that this fixes the scale: the classical turning points, $2M$ and $2(M - E)$ are separated by an amount that depends on the energy of the particle. It is the forbidden region from $r = 2M$ to $r = 2(M - E)$ that the tunneling particle must traverse. That's the barrier.

One would then expect that, in the WKB limit, the probability of tunneling would take the form

$$\Gamma \sim \exp(-2\ \mathrm{Im}\ I) \approx \exp(-\beta E)\ , \tag{4}$$

where $e^{-\beta E}$ is the Boltzmann factor appropriate for an object with inverse temperature β. Indeed, this is *almost* what is found. But, remarkably, an exact calculation [3, 6] of the action for a tunneling spherically symmetric particle yields

$$\Gamma \sim \exp\left(-8\pi M E \left(1 - \frac{E}{2M}\right)\right)\ . \tag{5}$$

If one neglects the $E/2M$ term in the expression, it does take the form $e^{-\beta E}$ with precisely the inverse of the temperature that Hawking found. So at this level we have confirmed that Hawking radiation can be viewed as tunneling particles and, furthermore, we have verified Hawking's thermal formula. But, unlike traditional derivations, we have also taken into account the conservation of energy and this yields a correction, the additional term $E/2M$. Thus the spectrum is not precisely thermal!

This is exciting news because arguments that information is lost during black hole evaporation rely in part on the assumption of strict thermality of the spectrum [7]. That the spectrum is not precisely thermal may open the way to looking for information-carrying correlations in the spectrum – work on this continues. Indeed, the exact expression including the $E/2M$ term can be cast rather intriguingly in the form

$$\Gamma \sim \exp(\Delta S) \ , \tag{6}$$

where ΔS is the change in the Bekenstein-Hawking entropy of the hole. This is a very interesting form for the answer to take for a number of reasons, including the fact that it is consistent with unitarity. Put another way, our result agrees exactly with what we would expect from a quantum-mechanical microscopic theory of black holes in which there is no information loss! For quantum theory teaches us that the rate for a process is expressible as the square of the amplitude multiplied by the phase space factor. In turn, the phase space factor is obtained by summing over final states and averaging over initial states. But, for a black hole, the number of such states is just given by the exponent of the final and initial Bekenstein-Hawking entropy:

$$\Gamma = |\text{amplitude}|^2 \times (\text{phase space factor}) \sim \frac{e^{S_{\text{final}}}}{e^{S_{\text{initial}}}} = \exp(\Delta S) \ . \tag{7}$$

Quantum mechanics, we observe, is in perfect agreement with our answer.

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