Self Similar Spherical Gravitational Collapse and the Cosmic Censorship Hypothesis

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ABSTRACT

We show that a self-similar general relativistic spherical collapse of a perfect fluid with an adiabatic equation of state $p = (\gamma - 1)\rho$ and low enough γ values, results in a naked singularity. The singularity is tangent to an event horizon which surrounds a massive singularity and the redshift along a null geodesic from the singularity to an external observer is infinite. We believe that this is the most serious counter example to cosmic censorship that was obtained so far.

The cosmic censorship conjectures[1] is generally accepted as the most important current open question in classical general relativity[2]. This conjecture suggests that a spacetime singularity, which develops in the future of a regular Cauchy hypersurface cannot be seen by any observer (strong version) or at least by an external observer (weak version). If the weak version of the cosmic censorship conjecture is true predictability is saved, at least in the region external to the event horizon, even when singularities form. If the strong version is true, predictability is saved everywhere. By now, several authors have described counter examples[2-8] to the cosmic censorship hypothesis. Some examples result from aphysical initial data[6-8]. The other examples deal mostly with pressureless matter (either dust of null fluid) and include either shell crossing (which can also appear with nonzero but bound pressure[3]) or shell focusing singularities. The first can be disregarded if we allow δ function distribution. The second are more difficult to get rid off and Eardley[2] has recently suggested that we should either avoid pressureless matter or treat dust self-consistantly using the collisionless Boltzman equation in order to rule out these singularities.

We have investigate the self-similar spherical collapse of a perfect fluid with an adiabatic equation of state $p = (\gamma - 1)\rho$. We find that unbounded pressure (but with γ not much larger than 1) does not prevent the appearance of naked singularities . This solution increases significantly the range of matter fields that should be ruled out in order that the cosmic censorship hypothesis will hold. In this essay we describe the essential features of the solution, various details we be given elsewhere.

To obtain a self similar solution of the spherical field equations[9] we define $x \equiv r/|t|$ and look for a solution of the form: $u^r(r,t) = u^r(x)$; $g_{rr}(r,t) = g_{rr}(x)$; $g_{tt}(r,t) = g_{tt}(x)$; $\rho(r,t) = d(x)/4\pi t^2$. Einstein equations[9] become a set of ordinary differential equations which determine the initial conditions at $t_0 < 0$, that lead to a self-similar collapse, as well as the dynamics in the domain $t > t_0$.

The central density $\rho(0,t) = d_0/4\pi t^2$,($d_0 \equiv d(0)$) diverges at $t = 0$ if $d_0 \neq 0$. *This real singularity at ($r = 0, t = 0$) is a basic feature of the solution (for $d_0 \neq 0$), and it does not reflect any singularity in the solution of the self-similar collapse equations.* For some range of the parameter space, null geodesics originating at the singularity reach an observer at infinity and the singularity is naked.

The self-similar solution has a sonic point, x_s (a test particle moving on the world-line $r = |t|x_s$, moves at the speed of sound $a = (\gamma - 1)^{1/2}$ relative to the fluid), where generally the solution is discontinuous. There exist, however, a discrete set of values of d_0 , for which the solution is regular[10]. We choose to discuss here one of these discrete regular solutions, the general-relativistic equivalent of Penston's Newtonian solution, which seems to be the simplest candidate for a naked-singularity.

The solution is characterized by two parameters, γ and d_0. For a given choice of these parameters we integrate the field equations numerically, from $x = 0$ towards $x = \infty$. Fig. 1 displays a numerical solution for $\gamma = 1.01$ and $d_0 = 1.438$, which correspond to the special regular solution discussed earlier. To understand the nature of our example it is sufficient to consider the solution near the origin and at infinity. Near the origin the solution describes an almost homogeneous, $d \approx d_0 - \frac{d_0}{2(\gamma-1)}[\frac{2d_0\gamma}{3}(\frac{3}{2}\gamma - 1) - (2 - \frac{16}{9\gamma})]x^2$, uniform, $u^r \approx -\frac{2}{3\gamma}x$, and almost Newtonian, $2m/r = 1 - g_{rr}^{-1} \approx \frac{2d_0}{3}x^2$, collapse. The solution goes over asymptotically to an isothermal,$d \approx d_\infty x^{-2}$, $2m/r \approx 1/2 + d_\infty - \sqrt{(1/2 + d_\infty)^2 - 2d_\infty(1 + \gamma u_\infty^2)}$, constant velocity, $u^r \approx -u_\infty$ infall. We notice that $2m/r$ is small near the origin and that it remains less then unity, even as we approach the singularity at $t = 0$. In fact, $2m/r < 1$ for all x and hence for all r and $t < 0$. Just like the shell focusing singularities[2] this singularity has a Newtonian character. The system remains almost Newtonian and a black hole does not appear before t=0, i.e. until the singularity is reached.

The isothermal character of the asymptotic solution means that the total mass diverges ($\lim_{x\to\infty} 2m/r \neq 0$ and the spacetime is not asymptotically flat. In order to obtain an asymptotically flat spacetime, we introduce a cutoff in the density profile at $r_{c0} = |t_0|x_c$. Beyond r_{c0}, the density drops smoothly to zero in a finite range. The cutoff break the self similar nature of the solution: $r_c(t)$ divides the spacetime to an inner self-similar region and an exterior non self-similar one. If r_{c0} is large enough, the cutoff is beyond the "photonic point", x_p, (where $x_p^2 g_{rr}(x_p) = -g_{tt}(x_p)$ and the world line $r = |t|x_p$ is null). Perturbations introduced at $t = t_0$ at $r > x_p|t_0|$, cannot influence the singularity at $(r = 0, t = 0)$. The condition $r_{c0} > |t|x_p$ ensures, therefore, that the singularity, and its immediate nearby region, are in the inner self-similar part and the cutoff cannot influence the singularity and the causal structure of the spacetime near it .

To bypass a coordinate singularity (in t) on the line $t = 0$ we transform to comoving coordinates R, T in which $u^T = g_{TT}^{1/2}$ and $u^R = u^\theta = u^\phi = 0$. We map $(r = 0, t = 0)$ to $(R = 0, T = 0)$ and we transform to comoving coordinates at $t_t < 0$ and $T_t > 0$, avoiding another coordinate singularity (in T) at $T = 0$ and obtaining two overlapping, regular coordinate patches that cover all the spacetime.

In the comoving coordinates we define $y = R/T$, $D = 4\pi\rho T^2$, $\tilde{r} = r/T$, such that g_{TT}, g_{RR}, D and $\tilde{r}$ are functions of y only. Substitution of these definitions into the comoving spherical collapse equations yields the comoving self similar collapse equations. The numerical solution for the $(R \geq 0, T > 0)$ patch is shown in Fig. 2. The solution terminates at y_s $(y_s > 0)$, where there is a real spacetime singularity. Near y_s we find that $D \approx D_s(y - y_s)^{-\gamma/(2-\gamma)}$, $g_{TT} \approx g_{TTs}(y - y_s)^{(2\gamma-2)/(2-\gamma)}$, and $g_{RR} \approx g_{RRs}(y - y_s)^{-2/(6-3\gamma)}$, while $\tilde{r} \approx \tilde{r}_s(y - y_s)^{2/(6-3\gamma)}$ (D_s, g_{TTs} g_{RRs} and $\tilde{r}_s$ are related by two algebraic equations). The spherical fluid shells crash into a central $r = 0$ singularity on the world line y_s. For $T > 0$, the singularity is 'massive' and it is surrounded by an apparent horizon, (where $2m/r = 1$). Both the mass of the singularity and the size of the apparent horizon grow linearly with the comoving time T (note that $\lim_{y\to y_s}\left(\frac{2m}{T}\right) = \lim_{y\to y_s}\left(\frac{2m}{r}\tilde{r}\right) = Const$ and see Figs. 2 and 3). The singularity is spacelike for $T > 0$ and no photons can escape from it later than $T = 0$.

Apriori it is not clear whether photons can escape from the singularity at $T = 0$. To answer this question, we look for radially outgoing null geodesics of the form $R = yT$. Clearly, such a geodesics exists if and only if $F(y) \equiv y^2 g_{RR}(y)/|g_{TT}(y)| = 1$ for some $y > y_s$. The asymptotic expansions (see Fig. 2) show that $\lim_{y\to\infty} F = \lim_{y\to y_s} F = \infty$. For $\gamma < \gamma_c$ ($\gamma_c \approx 1.0105$), which we assume in the sequel, $F(y)$ has a minimal value, $F_m < 1$, and $F(y_1) = F(y_2) = 1$. *The singularity at $(R = 0, T = 0)$ is naked for $\gamma < \gamma_c$.*

$R = y_1 T$ is a null geodesics from the singularity towards infinity. $R = y_2 T$ is the event horizon. In addition to these two special null geodesics, two disjoint families of radially outgoing null geodesics emerge from the singularity (see Fig. 3): geodesics between y_1 and y_2, that reach infinity, and geodesics located between y_2 and y_s, that are trapped by the apparent horizon, and fall into the central singularity. The analysis is more complicated but the basic causal structure remains unchanged, when we consider the influence of the cutoff on the null geodesic. The main difference is that with a cutoff $R = y_2 T$ is no longer the event horizon and some geodesics of the second family reach infinity.

Before concluding, we turn to the redshift along the null geodesics that emerge from the singularity. We find that the redshift from a source located at the center (r=0, and $t < 0$) diverges like $[r_c(t=0)/|t|]^{\alpha}$ as $t \to 0$ (where $\alpha = \gamma d_{\infty} g_{rr\infty}(1 + 2u_{\infty}^2 g_{rr\infty})$). Therefore, only singularities with an infinite luminosity can be seen by a distant observer.

So far we have ignored an important fact, which should be kept in mind whenever we discuss a naked singularity solution. The dynamics of causal future of the singularity (the domain $y < y_1$), depends on the unknown boundary condition at the singularity, and is not predictable from the initial data on t_0. Our solution is, in fact, based on an analytic extension of the solution from $y > y_1$ into the domain $y < y_1$, and it is equivalent to the assumption that *no perturbations are coming out from the singularity.* Many other solutions (with different boundary conditions) are consistent with the field equations. One can imagine, for example, that the matter which falls into the central singularity is immediately reradiated (as a thermal radiation, or Hawking radiation), and the singularity remains massless. Alternatively one can conceive a solution in which a shock wave bounces from the singularity in such a way that the singularity immediately disappears and the shock wave prevents the matter from collapsing further. It should be interesting to classify systematically all these possibilities.

We have shown that there exist a family of solutions of spherical self-similar collapse of an adiabatic perfect fluid that include naked singularities. These solutions provide a new counter example to the Cosmic Censorship hypothesis. These naked singularities resembles the shell focusing naked singularities that are observed in pressureless collapse, in spite of the fact that our matter field has a small, but non vanishing and unbound pressure. Clearly this solution is not sufficient to abandon the cosmic censorship hypothesis. One can think about a few caveats before doing so. First, the redshift along any null geodesic emerging from the singularity towards an external observer is infinite, hence energy cannot escape from this singularity (unless the singularity has an infinite luminosity). Furthermore such naked singularity occurs only for a relatively low (and possibly aphysical) γ. We might find physical reasons to rule out such matter sources. Finally, it is not clear if this causal structure is stable under perturbations and it is possible that these solutions are only of 'measure zero' and might be ignored.

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Figure Captions

Fig. 1: Self similar collapse expressed in Schwarzschild coordinates for $t < 0$ ($\gamma = 1.01$ and $d_0 = 1.4377$): $|u^r|$ (solid line), $x^2 d = 4\pi\rho r^2$ (dotted line), d (short dashed line), $2m/r$ (long dashed line) and $|g_{tt}|$ (dashed dotted line). Note that $2m/r < 1$ for all x values.

Fig. 2: Self similar collapse in comoving coordinates for $T > 0$ ($\gamma = 1.01$ and $d_0 = 1.4377$): $|u^r|$ (solid line), $100 \times 4\pi\rho r^2 = 100\tilde{r}^2 \times D$, (dotted line), $1000 \times D$ (long dashed dotted line), $2m/r$ (long dashed line), $\tilde{r} = r/T$ (short dashed dotted line) and $F(y) = y^2 g_{RR}(y)/|g_{TT}(y)|$ (short dashed line). Note that $2m/r = 1$ at y slightly below y_2, where $F(y_2) = 1$.

Fig. 3: A schematic spacetime diagram of the collapse in comoving coordinates. The singularity at y_s is represented by a sawtooth-like line. The apparent horizon is denoted by 'ah'. The cutoff is denoted by a long dashed dotted line. Dashed lines denote null geodesics that are between y_1 and y_2 and escape to infinity. Dotted lines denote null geodesics that are between y_2 and y_s and fall back into the singularity. The short dashed and dotted line denotes a geodesics that is between y_2 and y_s and would have fallen into the singularity, but it escapes to infinity because of the cutoff.

Figure 1

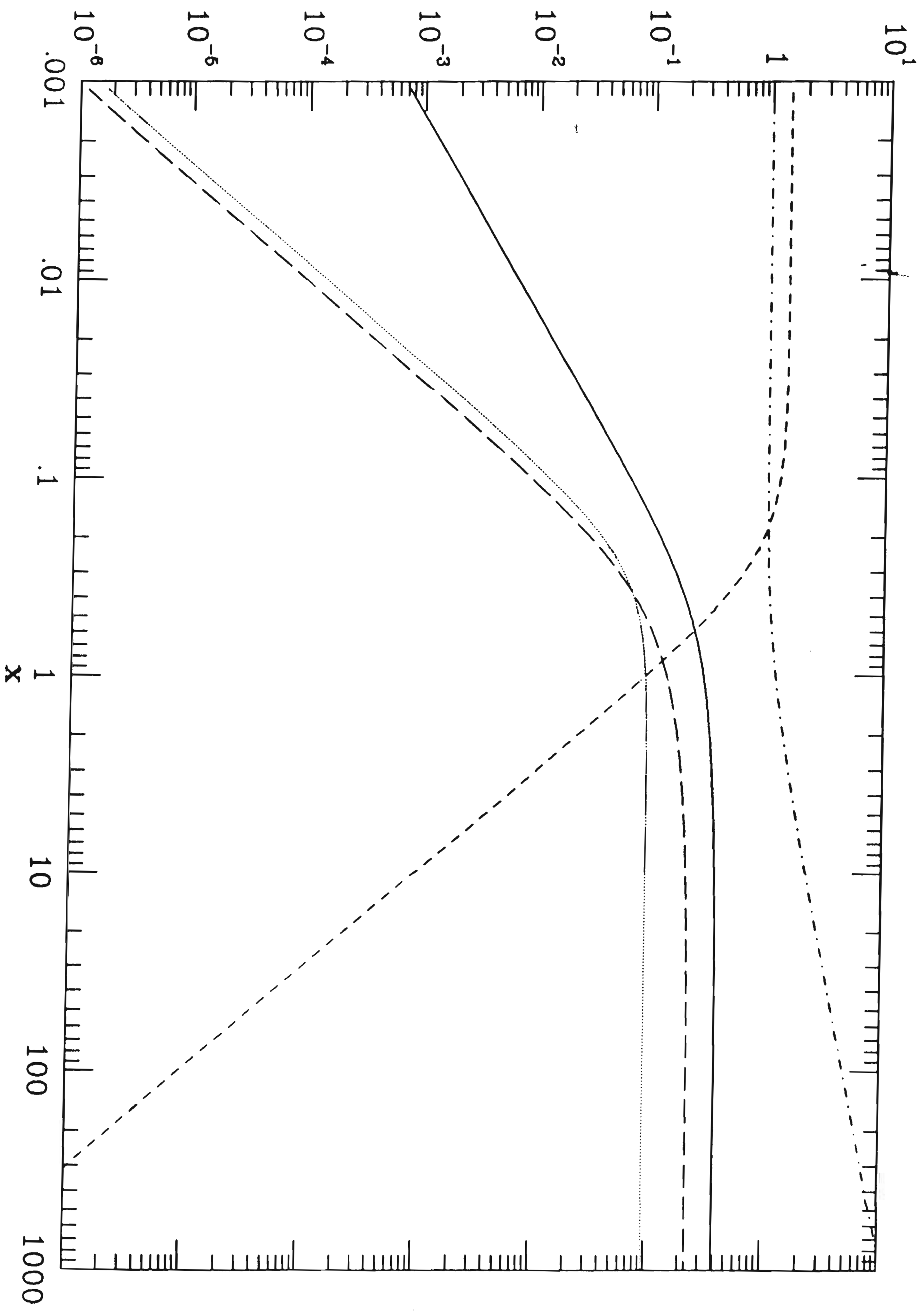

Figure 2

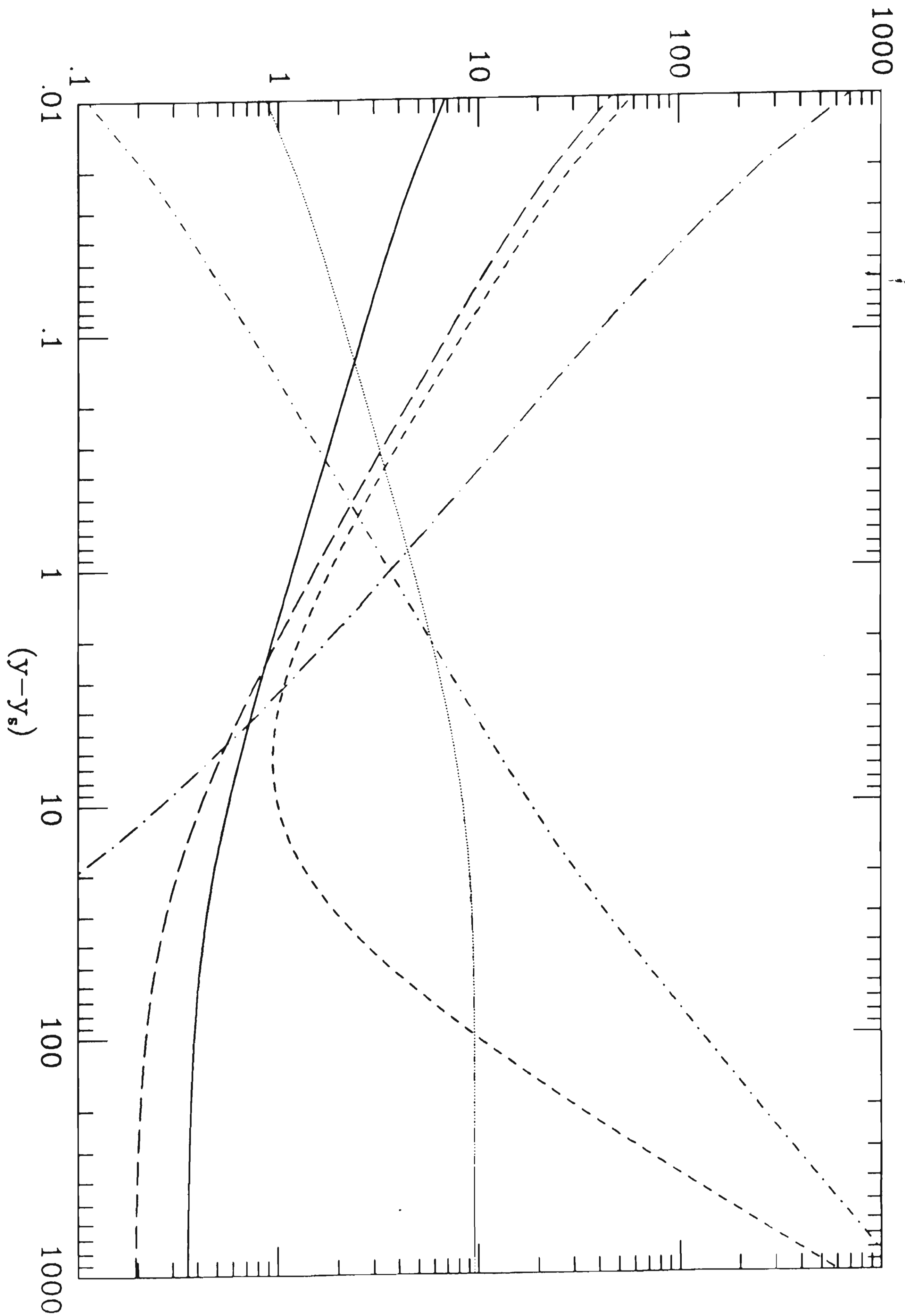

Figure 3