QUANTUM LINEARIZATION INSTABILITIES

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Abstract

We discuss quantizing the perturbations of a symmetrical background spacetime and are led to study the quantum analogue of linearization instabilities. We outline the derivation of the second order quantum constraints which arise whenever perturbations of symmetric spacetimes with compact Cauchy surfaces are quantized. These second order constraints require invariance of all the allowed quantum states (not just the "vacuum" state) under the symmetry group of the background spacetime. This result is discussed in light of the conclusion by Gibbons and Hawking that the thermal radiation produced by event horizons in de Sitter space is invariant under the de Sitter group and thus does not admit a semiclassical interpretation.

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I. Introduction

Hawking's[1] recent discovery of particle production by black holes has stimulated much interest in the reaction effects of the created particles upon the geometry that produces them. The standard approach to studying such reaction effects is a semiclassical one wherein the expectation value of a (suitably renormalized) stress tensor operator provides the source term in Einstein's equations for the classical gravitational field. In this essay we shall first recall some of the limitations of the semiclassical approach and then discuss an alternative in which quantized matter and gravitational perturbations are treated on an equal footing. This will lead us directly to a consideration of quantum linearization instabilities.

A fundamental limitation to the semiclassical treatment of quantum reaction effects is suggested by the recent work of Gibbons and Hawking[2] on particle production by cosmological event horizons. They study the Hawking radiation produced by event horizons in de Sitter space and find that every timelike geodesic "observer" is immersed in an isotropic bath of thermal radiation with temperature

$$T = \frac{\hbar c}{2\pi k}\sqrt{\frac{\Lambda}{3}} \qquad (1)$$

where Λ is the cosmological constant. Since these various "observers" are all boosted with respect to one another the radiation bath for one is not simply related to that of another by standard tensor transformation laws. Consequently the properties of the Hawking radiation in de Sitter space would seem to preclude a semiclassical treatment of its reaction effects. Indeed the Hawking radiation has properties so mysterious in this case that Hawking and Gibbons were led to suggest that something like the Everett-Wheeler[3] interpretation of quantum mechanics may be needed to understand it.

A tractable and fully symmetrical approach to studying the reaction effects of the Hawking process is to quantize both the matter and the gravitational perturbations about a given classical background spacetime (say empty de Sitter space). By treating the matter and the gravitational fields even handedly (though only perturbatively) one can hope to shed some light on the questions raised by Gibbons and Háwking. Though the quantization of such (first order) perturbations is superficially a straightforward matter we shall show that the occurence of quantum linearization instabilities can greatly affect the range of allowed physical states. In particular for de Sitter space we shall find that all the allowed quantum states (and not just the "vacuum" state) must be invariant under the full de Sitter symmetry group. We shall outline a derivation of this result (presented in more detail in Ref. 4) and discuss its implications.

II. Quantum Linearization Instabilities

Whenever one perturbs a solution of the Einstein equations he should be wary of the problem of linearization instabilities. This is the occurence of solutions to the linearized equations which do not extend to curves of solutions of the exact equations. Such non-integrable perturbations can always be excluded by imposing a set explicitly known second order conditions [5,6,7,8]. Linearization instabilities occur (for vacuum spacetimes with or without a cosmological constant) whenever the background spacetime admits a global Killing vector field and has a compact Cauchy surface. If sources are allowed the situation is more complicated (e.g., perfect fluids are exceptional) but for typical matter fields (e.g., electromagnetic, Yang-Mills [9]) a linearization instability arises whenever the background matter and gravitational fields admit a simultaneous symmetry.

To extend the idea of linearization instability to quantum theory requires a different viewpoint from that normally taken in the classical theory. The notion of curves of solutions of the exact equations seems to have no natural quantum analogue. A related idea which does extend to quantum theory is the approximation of a function on phase space (rather than a curve in phase space) by its Taylor series expansion. The key to reformulating linearization instability ideas for quantum theory is the recognition that certain projections of the exact constraint equations can have a Taylor series expansion which begins at second order. The anticipated first order term vanishes identically. To then avoid an undue truncation of the full set of constraints one must supplement the usual linearized constraints (the projections with non-trivial first order terms) by a set of non-linear constraints. There is one such second order constraint for each Killing field of the background solution about which the Taylor expansion is made. Applied classically this argument precisely reproduces the second order conditions previously obtained. Applied quantum mechanically it leads to new restrictions upon the physical states (additional to gauge invariance) which imply the invariance of all allowed states under the symmetry group of the background spacetime. We shall briefly sketch the argument in more detail.

An element in the function space of initial data is a pair (g, π) where g is a Riemannian metric and π is a symmetric tensor density (the gravitational momentum) defined over some compact three-manifold $\mathcal{M}$. The constraint map is

$$\Phi(g, \pi) = (\mathcal{H}(g, \pi), -2\delta\pi) \qquad (2)$$

where

$$\mathcal{H}(g, \pi) = (\det g)^{-\frac{1}{2}}\left(\pi\cdot\pi - \tfrac{1}{2}(\operatorname{tr}\pi)^2\right) - (\det g)^{1/2}\mathcal{R}(g) \qquad (4)$$

and

$$\delta\pi = \pi_{ij}{}^{|j} \qquad (5)$$

are the Hamiltonian and momentum constraints respectively. Let (g_0, π_0) be a particular solution of the constraints. This will be initial data for the background spacetime. The Taylor expansion of Φ about (g_0, π_0) is

$$\Phi(g_0+h, \pi_0+\omega) = D\Phi(g_0,\pi_0)\cdot(h,\omega) + \frac{1}{2!}D^2\Phi(g_0,\pi_0)((h,\omega),(h,\omega)) + \cdots \qquad (6)$$

where $D^n\Phi(g,\pi)$ is the n-th derivative of Φ. The projection of Φ onto any pair (C, Y) where C is a function and Y a vector field on $\mathcal{M}$ is defined by

$$\mathcal{P}_{(C,Y)}\Phi(g,\pi) = \int_{\mathcal{M}} \{ C\mathcal{H}(g,\pi) + Y\cdot(-2\delta\pi)\} . \qquad (7)$$

This projection has a trivial (i.e., identically vanishing) first order term in its Taylor series expansion about (g_0, π_0) if and only if (C, Y) are the normal and tangential projections (on the initial surface) of a Killing vector field for the Einstein spacetime determined by the data (g_0, π_0)[7,8]. Therefore the projections of the constraints onto the Killing fields of the background always begin at second order (where in fact they are known to be non-trivial[6,8]) rather than at first order as one might have expected.

The usual linearized constraints are

$$D\Phi(g_0,\pi_0)\cdot(h,\omega) = 0 . \qquad (8)$$

By the foregoing argument these should be supplemented by the second order constraints

$$\mathcal{P}_{(C,Y)}\left(D^2\Phi(g_0,\pi_0)((h,\omega),(h,\omega))\right) = 0 \qquad (9)$$

where (C, Y) are the projections of a Killing field of the background. To quantize this system we can employ Dirac's method of imposing the constraints as restrictions upon the allowed physical states. Thus with a suitable choice of operators ($\hat{h}$, $\hat{\omega}$) for the quantized perturbations one imposes

$$D\Phi(g_0,\pi_0)\cdot(\hat{h},\hat{\omega})\ |\Psi\rangle = 0 \qquad (10)$$

and

$$:\mathcal{P}_{(C,Y)}\left(D^2\Phi(g_0,\pi_0)((\hat{h},\hat{\omega}),(\hat{h},\hat{\omega}))\right):|\Psi\rangle = 0 . \qquad (11)$$

Equations (10) require gauge invariance of the allowed states and are the expected constraints of lowest order perturbation theory. Equations (11) however are new. They require invariance of the physical states under the symmetry group of the background spacetime. This conclusion follows from the observation[7,8] that the quantity

$$\rho_{(C,Y)}\left(D^2\mathcal{L}(g_0,\pi_0)((h,\omega),(h,\omega))\right) \qquad (12)$$

is precisely the constant of the motion of linearized theory associated with the Killing field (C,Y) of the background. As such it is the Hamiltonian generator of the corresponding symmetry transformation of the canonical perturbation variables (h,ω). In de Sitter space for example one has ten constraints of the type in Eq.(11). Their commutator algebra is isomorphic to the Lie algebra of the de Sitter group.

III. Discussion

How can one make sense of a quantum theory based exclusively on invariant states? Such states do not seem to allow the description of phenomena which are localized in a spacelike symmetry direction or which evolve in a timelike symmetry direction. But this rigidity of the physical states is only apparent. Spatial localization and temporal evolution must be interpreted intrinsically rather than with reference to the symmetric background spacetime. One part of the quantized system is localized or evolves only relative to another part. If timelike symmetries occur some part of the quantized system of matter and gravitational field must be used as an intrinsic clock. If spacelike symmetries occur some component of the system must be used as a benchmark for spatial localization.

This idea that temporal evolution or spatial localization must be described intrinsically is not at all new. It arises rather naturally in Wheeler's [10] geometrodynamical approach to quantization. What is new is the appearance of this viewpoint in the more conventional approach to quantization based on perturbations of a classical background.

A quantum theory based on invariant states requires for consistency that measurements be described intrinsically. The measuring apparatus must be treated as a component of the quantized system itself and not as a detached entity localized in the background spacetime. This conclusion recalls the view of Gibbons and Hawking mentioned above. They found that the "vacuum" state of de Sitter space was a group invariant--it exhibited a thermal radiation bath of fixed temperature to each of the group equivalent geodesic "observers" in the de Sitter background. We find by quite different arguments that all of the quantum states must be group invariant and suggest, as did Gibbons and Hawking, that the "observers" must for consistency be included in the description of the quantum system itself. Only in this way can one make intelligible a quantum theory whose only observables are group invariants.

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Biographical Sketch

Vincent Moncrief (b. 1/2/43 in Oklahoma City, Okla.) is a theoretical physicist who specializes in general relativity and relativistic astrophysics. He received his Ph.D. from the Univ. of Maryland in 1972 and has since worked at the Univ. of California and at the Univ. of Utah. He is currently Asst. Prof. of Physics at Yale University. His main work is the study of gravitational perturbations of known solutions of Einstein's equations. He is currently engaged in research to compute the characteristics of the gravitational radiation which is produced when a rotating star collapses to form a black hole.