TIME IN QUANTUM COSMOLOGY
FROM THE SELF–MEASUREMENT OF THE UNIVERSE

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Consideration in quantum cosmology is conventionally based on the Wheeler–DeWitt equation $H\Psi = 0$ resulting in trivial dependence of the Universe wavefunction on time. This leads to difficulties in deriving time evolution of the quantum Universe and in transition from quantum cosmology to classical one. Recently some works appeared considering self–measurement of the quantum Universe in an explicit way, with the aim to introduce classical concept of time. The purpose of the present paper is to treat self–measurement of the quantum Universe in the framework of the path–integral quantum theory of continuous measurements developed by the author. This allows one to explicitly introduce geometrically defined time into quantum cosmology. Evolution of the quantum Universe under continuous measurement of the lapse function and the scale factor, will be evaluated and analyzed in the framework of a simple minisuperspace model.

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1. Since the work by Hartle and Hawking on the Universe wavefunction [1] great interest exists to quantum cosmology. Consideration in quantum cosmology is conventionally based on the Wheeler–DeWitt equation

$$H\,\Psi = 0 \tag{1}$$

resulting in trivial dependence of the Universe wavefunction on time. This leads to difficulties in deriving time evolution of the quantum Universe and in transition from quantum cosmology to classical one. Usage of the scale factor a as an "intrinsic time" for the quantum Universe seems not quite satisfactory or insufficient.

Recently some works appeared [2] considering self–measurement of the quantum Universe in an explicit way, with the aim to introduce classical concept of time. Self-measurement means , in the present context, measurement of some degrees of freedom with the help of the other degrees of freedom, considered as a measuring device. The purpose of the present paper is to treat self–measurement of the quantum Universe in the framework of the path–integral quantum theory of continuous measurements [3,4]. This allows one not only to describe, as in [2], "decoherentization" leading to classical character of the "intrinsic time" a but also to explicitly introduce geometrically defined time into quantum cosmology. Evolution of the quantum Universe under continuous measurement of the lapse function and the scale factor, will be evaluated and analyzed in the framework of a simple minisuperspace model, generalizing the results of Halliwell [5].

2. The probability amplitude for a quantum system to propagate from the point q' to the point q'' during the time interval $[t',t'']$ can be expressed by the path integral (the units here and below are natural)

$$U(q'',q') = \int d[q] \exp\{iS[q]\} .$$

Let continuous measurement is carried out during the time interval $[t',t'']$ giving a result (reading) which we will denote by α . Then the propagator from q' to q'' can be expressed [3,4] by the integral of the same type but with an appropriate weight functional:

$$U_\alpha(q'',q') = \int d[q] \;\; w_\alpha[q] \exp\{iS[q]\} . \qquad (2)$$

Here the functional w_α expresses information, contained in the measurement result α , about the path, that the system propagates along. This means that $w_\alpha[q]$ is small or equal to zero for those paths $[q]$ that are excluded by the measurement result α , and is equal (or close) to unity for the paths $[q]$ compatible with α .

3. For description of measurement in quantum gravity one should use a functional integral over configurations of the gravitational field [4]. An amplitude describing dynamics of quantum Universe can be presented by the integral

$$U({}^3G'',{}^3G') = \int d\mu[{}^3G] \exp\{(iS[{}^3G])\} \qquad (3)$$

over paths $[{}^3G] = \{{}^3G(\tau) |\; \tau' \leq \tau \leq \tau''\}$ in the space of 3–geometries (in the superspace). The paths $[{}^3G]$ describe in fact 4–geometries.

The amplitude (3) describes dynamics of the quantum Universe in absence of measurements and satisfies the Wheeler–DeWitt equation (1). If however continuous measurement of geometry is carried out during the interval $[\tau',\tau'']$, an amplitude (propagator) should be modified by introduction a weight functional:

$$U_{[^3\mathscr{G}]}\ (^3G'',{}^3G') = \int d[^3G]\ w_{[^3\mathscr{G}]}\ [^3G]\ \exp\ (i\ S[^3G]) \tag{4}$$

Here $[^3\mathscr{G}]$ denotes the 4–geometry describing the result of measurement and $w_{[^3\mathscr{G}]}[^3G]$ expresses the information contained in this result, about actual 4–geometry $[^3G]$.

The idea of the present paper is that the physical time between the instants marked by the parameters τ', τ'' *can be evaluated from the geometry* $[^3\mathscr{G}]$ *and thus the amplitude* (4) *should depend on this physical time in nontrivial way.*

4. Halliwell evaluated in [5] an amplitude (3) for the minisuperspace model resulting from the metric of the form

$$ds^2 = -\ [N^2(\tau)/q(\tau\ )]\ d\tau^2 + q(\tau)\ d\Omega_3{}^2. \tag{5}$$

The reparametrization

$$d\tau \longrightarrow d\hat{\tau} = w(\tau)\ d\tau\ , \quad N(\tau) \longrightarrow \hat{N}(\hat{\tau}) = w^{-1}(\tau)\ N(\tau) \tag{6}$$

does not really change geometry. The time t , invariant under the reparamenrization (6), (i.e. physically significant) can be defined as $dt = N(\tau)d\tau$.

Deriving the Einstein action from the metric (5) and accepting the gauge condition $dN/d\tau = 0$, Halliwell obtained for the amplitude (3) the following integral:

$$U(q'',q') = (\tau''-\tau')\int d[N]\delta[\dot{N}]d[q]\exp[-(i/8)\int_{\tau'}^{\tau''} d\tau(\ \dot{q}^2/N-4N)] \tag{7}$$

giving the expression (with the notation $T = (\tau''-\tau')N$)

$$U(q'',q') = (8\pi i)^{-1/2} \int dT\ T^{-1/2}\exp\{-(i/8)[(q''-q')^2/T-4T]\}. \tag{8}$$

This amplitude is a solution to the Wheeler–DeWitt equation (1) taking the form

$$(1/2)(4\partial^2/\partial q''^2 - 1)\ U(q'',q') = 0 \tag{9}$$

5. Let us now modify the integral (7), taking the self–measurement of the Universe into account. In the framework of the minisuperspace model the geometry is described by the functions $[q]$ and $[N]$. The self–measurement means that some functions $[\kappa]$, $[\nu]$ are found as estimations of the functions $[q]$ and $[N]$. This can be expressed by the functionals

$$u_{[\kappa]}[q] = \exp[-\rho^{-2}\int_{\tau'}^{\tau''} d\tau\, N(\tau)\,(q-\kappa)^2]\,,\quad v_{[\nu]}[N] = \exp[-\sigma^{-2}\int_{\tau'}^{\tau''} d\tau\,|N-\nu|]\quad(10)$$

which are invariant under the reparametrization (6).

With these functionals introduced into (7), the propagator becomes (instead of (8))

$$U_{[\kappa,\nu]}(q'',q') = (8\pi i)^{-1/2}\int dT\; T^{-1/2}\; W(T/\rho)\,\Sigma_T[\nu]\; R_T[\kappa]$$

$$\times\exp\{-(i/8)[(q''-q')^2/T - 4T]\}.\qquad(11)$$

Here $W(x)$ is a known function,

$$\Sigma_T[\nu] = \exp\{-\sigma^{-2}\int_{\tau'}^{\tau''} d\tau|\nu(\tau)-T/(\tau''-\tau')|\},$$

$$R_T[\kappa] = \exp\{-(T/2\rho^2)\sum_{k=1}^{\infty}(\kappa_k-q_{0k})^2/[1-8i(T/\pi k\rho)^2]\}$$

and decomposition in a series of the type

$$z(\tau) = \sum_{k=1}^{\infty} z_k\sin\,[k\pi\tau/(\tau''-\tau')]$$

is introduced for the function $\kappa(\tau)$ and the classical trajectory

$$q_0(\tau) = (\tau-\tau')(q''-q')/(\tau''-\tau') + q'.$$

6. The functional Σ_T describes distribution of different results $[\nu]$ of the measurement of the lapse function $[N]$. It can be easily seen that $\Sigma_T[\nu]$ can be close to unity for some value of T only if the function $[\nu]$ is almost constant. If $\nu = \text{const}$, then

$$\Sigma_T[\nu] = \Sigma(T-t) = \exp\{-\sigma^{-2}|\,T-t|\}\ ,$$

where the notation $t = (\tau''-\tau')\nu$ is introduced. The value ν being an estimation of the lapse function, t is evidently an estimation of the gauge–invariant duration of the interval $[\tau',\tau'']$. Thus physically significant time emerges in our analysis.

The functional $R_T[\kappa]$ characterizes distribution of different results $[\kappa]$ of the measurement of the squared scale factor $[q]$. To obtain general notion about this functional, let us neglect the imaginary term in the denominator of its exponent describing in fact quantum effects in measurement of $[q]$ (actually this term is negligible for $t < 1$, $\sigma^2 < \rho$):

$$R_T[\kappa] \simeq \exp[-(T/2\rho^2)\sum_{k=1}^{\infty}(\kappa_k-q_{ok})^2] = \exp[-(T/\rho^2)(\tau''-\tau')^{-1}\int_{\tau'}^{\tau''} d\tau(\kappa-q_o)^2].$$

This means that the result of measurement $[\kappa]$ is close to the classical trajectory $q_o(\tau)$. Considering $R_T[\kappa]$ for a fixed function $[\kappa]$ but different q', q'' we see that these points are localized about the end points of $[\kappa]$. The quantum effects make the picture more complicated but do not violate this localization.

Let now suppose that q',q'' are close to the end points of $[\kappa]$ so that $R_T[\kappa]$ is equal to unity, and investigate dependence of the propagator on q',q'' and on t due to the other factors. For small σ, one has $\Sigma(T-t) \simeq 2\sigma^2\,\delta(T-t)$. Then the propagator is

$$U_t(q'',q') = 2\sigma^2(8\pi i)^{-1/2}t^{-1/2}\exp\{-(i/8)[(q''-q')^2/t-4t]\} \tag{12}$$

and satisfies the Schrödinger equation (instead of the Wheeler–DeWitt equation)

$$i\,\partial/\partial t\;U_t(q'',q') = H\;U_t(q'',q'). \tag{13}$$

In the opposite limit of large σ and small t,

$$\sigma >> 1, \qquad \sigma^2 >> \rho, \qquad t \lesssim \rho, \qquad \rho >> 1,$$

the functions Σ and W are equal to unity for all values of T which are essential for integration so that the propagator becomes equal to the Halliwell's amplitude (8).

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