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THERMODYNAMICS AND RELATIVITY

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We may well wonder whether any theory has contributed so much to humanity, in the less than fifty years since it was first published, as general relativity, a theory which may contribute still more to future mankind. Both directly and indirectly the ideas of the theory of relativity have influenced science, not only explaining the results and discrepancies of the experiments in the older Newtonian theory which called it forth, but in the profound revolution in mode of thought which it produced, preparing the way for later theories such as quantum mechanics, which had just as much or, as some believe, more effect on the details of physical theories. So general and far-reaching were the ideas expressed in relativity, touching every phase of physics in its relations to things familiar to everyone: - clocks, yardsticks, light, time and space - that there is little wonder at the profound effect exerted on even such older and seemingly unrelated fields as philosophy and art. To scientists, as to everyone, the new ideas were so novel that men were forced, for perhaps the first time, to determine the reasons for accepting and using a scientific theory in trying to understand our universe. A scientific theory should not be accepted because it seems plausible or even because it appears to be in accord with experience. It must not merely fit neatly into the niche for which it was conceived; it must suggest new fields for scientific adventure, and when these fields have been explored, must still be found applicable. It must ever serve as a source of new inspiration, a tool for new discoveries which will be of benefit to mankind. Judged on this basis, general relativity has indeed proved a successful theory, explaining at once irregularities in the motion of the planet Mercury about the sun, and at the same time helping man to understand the innermost secrets of the building stones of our universe. Applying wave mechanics, resulting from the mass energy equation, to chemistry, metallurgy, and innumerable other fields, we have gained insight into finding new drugs to cure diseases and combat pain, new alloys of greater strength for our buildings and the machines of mass production, new textiles, and many other things important in our daily life. From the principle of the equivalence of mass and energy, we have not only been able to determine the binding energy of nuclei and the constitution of elementary particles but have discovered in nuclear fission the key to new sources of power for man's needs and future dreams.

Relativity theory has, further than this, been shown to reduce to the ordinary Newtonian mechanics, when applied to systems moving at small velocities, relative to the speed of light, in limited regions of space far from any powerful gravitational fields. For all purposes of daily life, Newtonian mechanics is indistinguishable from relativity theory. Newton's apple still falls toward the center of the earth and railroad trains moving at high speed still remain the same length to an observer. Indeed, the first test applied to relativity was that it should, within the limits of then existing measuring instruments, give Newton's laws for determining the motion of falling apples and moving railroad trains. Only in the behavior of very fast electrons in "atom-smashers" or in the life-times of cosmic ray particles do we learn of relativistic effects, which in all cases have again shown relativity theory to be correct.

Although Newtonian mechanics for low velocities follows from the general theory of relativity, we have in relativity a very different way of regarding the facts of nature. The difference will be clear if we consider the case of a perfectly transparent drainpipe laid along a winding path down a hill. In Newtonian mechanics we say that water is constrained to follow the twists and turns of the pipe by "forces" exerted by the pipe on the water. However, we could equally well imagine that instead of the pipe being winding, the hill is curved and the pipe is straight. While this way of looking at the problem may seem artificial in this simple example, the method enables us to describe the flow of water as determined at each point by a property of the surrounding matter, here, the curving hill. This is the essential feature of a " field" theory such as gravity, which at no time mentions forces between particles, but instead describes the movement of the particles as being due to action of the medium on the particles. In general relativity we can then describe the motion of the planets about the sun by saying that the heavy mass of the sun combined with the mass of the earth produces a curvature of space into hills and valleys which the earth follows in its movement about the sun.

The aim of general relativity is to describe all that is known of gravity, electromagnetism, and other parts of physics by a unified field theory describing the properties of space at all points. At the present time, the theory has been applied also to the fundamental laws of thermodynamics to obtain results which describe the effects of gravity on such familiar quantities as temperature, pressure, volume, and entropy. In this discussion we shall consider the possibility of detecting gravity waves, and, in particular, we shall consider the effect of gravity waves on the temperature of bodies. However, in order to determine whether these effects will be detectable we must understand the relation of the gravitational field not only to thermodynamics but to the motion of heavy bodies about each other.

When we hear that in relativity theory we are describing the hills and valleys of space, that is, its " curvature" everywhere, it may occur to us to ask whether we can see these hills and valleys and whether they are " real" . In answer to this question the physicist tells us that we are to regard his theories as a way of describing events and are to judge his theory not by the terms used, but by its ability to explain known experimental facts and predict new results which can be checked by experience. Thus we ask not " how" , but " whether" the theory can explain known phenomena and predict the new accurately in its own fashion. This is really a necessary attitude for us to adopt when we approach a new theory which is so highly mathematical that it cannot be explained in terms to which we are accustomed. As an example, we may consider the difficulties we would have in describing the world to a person born blind. The concepts we describe will be beyond his experience. Instead of asking whether space is " real" , meaningless as phrased, since we cannot measure " reality" ; we shall ask what is meant by curvature.

This curvature of space is described by ten quantities called the field quantities and designated almost universally as g_{11}, g_{12}, g_{13}, g_{14}, g_{22}, g_{23}, g_{24}, g_{33}, g_{34}, g_{44}, which have a definite numerical value at every point of space. The value of these quantities is described by a set of ten partial differential equations which relate the density of matter in a certain region of space to the curvature of space by determining the g's. However, as is now known, the g's also determine the motion of matter, as well as being determined by it. In space unaffected by heavy masses, the values of the g's are $g_{11} = g_{22} = g_{33} = -2$; $g_{44} = 1$; $g_{12} = g_{13} = g_{14} = g_{23} = g_{24} = g_{34} = 0$, with time being measured in units of 1/c seconds where c is the velocity of light. In this space a particle, whose mass could not affect the curvature of space so that the g's had the values above, would move in a straight line as understood in ordinary Euclidean geometry. The strength of the gravitational field will be measured by the deviations of the g's from these values in empty space.

The quantity of most importance in the applications of general relativity is g_{44}, which replaces Newton's potential ψ when we attempt to describe more exactly the mechanics of bodies moving at very low velocities in the neighborhood of very heavy bodies. Indeed, g_{44} is given by a partial differential equation very similar to Newton's equation for the gravitation potential. It is:

$$\frac{\partial^2 g_{44}}{\partial x^2} + \frac{\partial^2 g_{44}}{\partial y^2} + \frac{\partial^2 g_{44}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 g_{44}}{\partial t^2} = \frac{8\pi k\rho}{c^2}$$

where ρ is the density matter and k is the gravitational constant. The equation for Newton's potential ψ is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 8\pi k\rho$$

The first difference between the equations is that g_{44} satisfies an inhomogeneous wave equation while ψ satisfies Poisson's equation. This difference is very fundamental since although the equation for g_{44} reduces approximately to Newton's equations (because of the smallness of the term $-\frac{1}{c^2}\frac{\partial^2 g_{44}}{\partial t^2}$), the changes in ψ travel with infinite velocity and appear everywhere in space instantaneously, while the changes in g_{44}, gravitational effects, travel as "gravity waves" of velocity of light. Another difference is that in the equation given here for g_{44}, ρ represents the density of matter and energy, the two being equivalent in relativity.

The preceding detailed discussion of general relativity now has placed us in a position to understand this result, which must be so unexpected to us in the presence of experience and ways of thought learned from childhood. Familiar with the sight of ripples from the splash of a stone traveling across a pond, and aware of the experience that any body released above the earth instantly begins to fall toward the center, we may mentally combine these two effects and wonder why we have never observed gravity waves. The reason for this is that, first of all, the gravitational field in which we spend our lives came into existence when the earth was formed and has remained nearly constant all our lives. Secondly, as Einstein showed, any small *changes* in space curvature generated by our earth would be completely undetectable, dependent as they would be on the rate of change with time, of distribution of matter of the earth. By this we mean that the changes occur as a result of distortions of the earth. The equation given by Einstein for the total energy carried away from a body, such as the earth, by gravitational fields is

$$S = \frac{\kappa c}{80\pi}\left[\sum_{\mu\nu}{}' \ddot{J}_{\mu\nu}^{2} - \left(\sum_{\mu\nu}{}' \ddot{J}_{\mu\nu}\right)^{2}\right]$$

where the $J_{\mu\nu}$ are moments of inertia of the body. Changes in the *shape of the earth* are nearly undetectable to physicists - while second derivatives of time rates of change are zero! The constant κ is *so small* that for *one erg per second* to be radiated from a body, the bracketed quantity would have to have the value

$$\frac{80\pi S}{\kappa c} = \frac{(80)(3.14)(1)}{(2.073\times10^{-48})(3\times10^{10})} = 4.04\times10^{39} \left(\frac{gm\ cm^2}{sec^2}\right)^2$$

If we were to imagine an inner tube to suddenly appear around the earth's equator in one second containing matter at a density of 5 gm/cm^3 , and to be placed there according to the law $m = m_0 t^2$, we can show by a simple but long calculation that the radius of the inner tube would be about 160 feet! This example illustrates the two difficulties confronting us in detecting gravity waves: (1) smallness of the gravitational fields produced by matter, and (2) necessity for detecting second order effects. By the last statement we mean that *we are concerned not merely with the magnitude of gravitational fields but with changes in the magnitude of gravitational fields.* As has been shown, these measurements must not involve the measurement of the energy carried by a gravity wave or the other features of a wave which we should usually wish to know. Instead we must look for changes in properties of matter and radiation which differ in <u>character</u> from what we should expect on the basis of pre-relativity theory. Now the magnitude of g_{44} is simply related to Newton's gravitational potential Ψ by the equation

$$g_{44} = 1 + \frac{2\Psi}{c^2}$$

so that even at the surface of the earth

$$g_{44} = 1.000000009$$

Since all the effects usually associated with gravitational fields, the slowing down of clocks, the change in length of measuring rods, and the bending of light, all depend on changes in g_{44}, we must look for effects which either depend more strongly on g_{44} or are changes in quantities which we can measure with great precision.

A seldom considered quantity, in such searches for effects of gravitational fields, is <u>temperature</u>. We ask three questions in studying the effect upon temperature of changes in g_{44}.

1) Can we expect to find a quantity which has meaning in relativity and is also simply related to the temperature of which we all speak?
2) Does this temperature depend on g_{44} ?
3) Does the temperature of an alloy depend on gravitational waves?

With this last question we come to the crux of our investigation.

The answer to our first question was found by R. C. Tolman who reformulated the entire basis of thermodynamics to satisfy requirements of relativity.

Tolman did this by using the first two laws of thermodynamics, namely

I. The energy gained by a body is equal to the work done on it and the heat gained.

$$\Delta E = \Delta Q + \Delta W$$

II. Heat flows from a body of a higher temperature to a body at a lower temperature.

The first law merely states that we cannot build a perpetual motion machine which will require no energy. The second law is familiar to anyone who has found metal drinking cups of hot coffee impossible to hold. Tolman has shown that by applying these two laws, the usual concepts of heat and temperature can be extended to apply everywhere in space. These quantities can be measured by ordinary thermometric methods. This does not mean that the quantities do not depend on gravitational fields, but that even a person in a more powerful gravitational field than that of the earth would have the same experiences with heat and temperature as we have. That is, the relations between the thermodynamic quantities are valid for every frame of reference. This is, in fact, the requirement which any quantities that appear in general relativity must satisfy. From the thermodynamic laws thus derived the variation of temperature in a system which has reached thermal equilibrium can be shown to vary from point to point in the system. The variation will be such that the temperature is given everywhere by

$$T_o \sqrt{g_{44}} = C$$

where C is constant for the system. The restriction to states of thermal equilibrium is not peculiar to relativity but merely expresses the inability of classical thermodynamics to describe certain processes called irreversible.

If we now suppose that gravitational field at a particular point is increased by a gravity wave, we see that T_o as measured by ordinary thermometric methods must change.

The dependence of temperature on the g_{44} shows, in addition, that variations in g_{44} cause changes of the same order of magnitude in T_o that would be produced in clocks or measuring rods. For this reason the only possibilities of being able to detect gravity waves by changes in temperatures would be if temperature changes could be measured much more accurately than the effects of gravity on time intervals or measuring rods - or if an alloy could be found whose temperature was more strongly affected by gravity than other matter. However, the answer to this lies in the fact that T_o depends only on g_{44}. Since g_{44} depends only on the density of matter and radiation, we conclude - temperature effects created by gravity waves are not dependent on the specific properties of the matter. Since temperature measurements to the accuracy required are as difficult as measurements of gravitational effects, we conclude the detection of the effect of gravity on the temperature of bodies is at present impossible. In particular, the discovery of an alloy whose temperature would be more strongly affected by gravity waves than common materials is impossible it would seem.

The long path we have followed through general relativity will now enable us to see clearly the grounds on which these conclusions are based. We summarize the steps taken in order to judge correctly the validity of our results.

First - We have shown that general relativity has satisfied the requirements of a successful theory and has predicted results verified by experiment. The conclusions to be drawn from it therefore deserve to be studied and experiments made to prove or disprove its conclusions.

Second - We have shown that general relativity predicts the existence of gravity waves which would produce certain effects.

Third - These effects include a change in the temperature of bodies produced by gravitational fields.

Fourth - These temperature changes are all far beyond the present limits of measurement, and, further, do not depend on the particular substance studied.

We must remember that the theory of relativity in predicting gravity waves which have not yet been observed is thus in the same position as Maxwell's theory of electromagnetism before the discovery of electromagnetic waves by Hertz. That the theory itself gives us little help in trying to discover new ways to detect gravity waves should not be regarded as a fundamental defect. The general theory does not yet include a description of electric and magnetic phenomena. It seems possible that new effects produced by the interaction of gravitational fields with powerful electric fields, such as exist in atoms, will be predicted by a unified field theory that will describe both gravitational and electric fields, just as the application of Maxwell's theory to produce dynamos, motors, new methods of transporting power, and new methods of communication was found possible - and even simple - when it was learned how to produce the strong fields necessary. A new era began of cheap power, of mass production, of labor-saving devices for the home, of radio, of television, of wonders still to come. So, too, from gravity waves we may derive future benefits for man and the small undetectable effects now predicted may be replaced by others which can be harnessed easily to drive motors and actuate new devices still undreamed of.

Our survey of the theory has shown us that if we are to succeed in this aim to apply gravity waves to help man, we are more likely to succeed by understanding better, not only the theory, but also the nature of other effects produced by gravitational fields. Whatever the final outcome may be, the contributions already made by general relativity to human knowledge are more than ample justification for the researches which should be made into the subject. The outcome of these investigations will certainly lead to an increase in our understanding of nature, and thus, benefit humanity.