

## WHY ARE SPHERICAL STELLAR SYSTEMS RELAXED?

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## SUMMARY

Spherical stellar systems, from globular star clusters to compact galaxy clusters, appear to be dynamically relaxed. In galaxies and galaxy clusters, collisional relaxation acts too slowly to produce the observed result; and a new argument suggests that the same may be true of globular star clusters. "Violent relaxation" requires special initial conditions, and seems unable to produce sufficiently extended haloes. It is here proposed that dynamical relaxation may result from tidal perturbations by external systems. If this explanation is correct, it has important implications for the early history of galaxies and galaxy clusters.

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Although globular star clusters, spheroidal galaxies, and compact galaxy clusters span enormous ranges of mass and diameter -- nine decades in mass and six in diameter -- they are conspicuously alike in structure. All of them have smooth, symmetrical distributions of surface brightness characterized by a broad central peak and an extended halo; and systems of the same kind have closely similar brightness distributions. For example, Hubble found that the surface brightness of any spheroidal galaxy is well represented by the simple formula  $B/B_0 = (1 + r/a)^{-2}$  over the range  $0.03 \leq r/a \leq 15$ ; and King has shown that the brightness distributions of globular clusters are accurately represented by a semi-empirical formula whose three adjustable parameters represent mass, energy, and tidal radius, respectively. In the light of these and similar findings, it is difficult to avoid the conclusion that spherical stellar systems have undergone a process of dynamical relaxation that has largely obliterated the traces of individual differences in initial conditions and evolutionary histories.

The term "relaxation" was coined by Maxwell, who

showed that encounters between the molecules of an ordinary gas cause the molecular velocity distribution to assume the standard form that bears his name. The Maxwellian velocity distribution depends on a single parameter, the temperature, whose value reflects the initial value of the energy per unit volume in the gas. Maxwell showed that molecular encounters destroy all other information about the initial velocity distribution, in a time comparable to the intervals between collisions of a typical molecule.

Early in the present century, Jeans and Schwarzschild extended Maxwell's theory of collisional relaxation to stellar systems. They noticed an important difference between ordinary gases and large stellar systems. Gas-molecules move freely between encounters, and a typical encounter causes a substantial change in a molecule's momentum and energy. The motion of a star in a large stellar system is entirely different: owing to the long-range character of gravitation, numbers outweigh propinquity. A star's motion is almost wholly determined by the smoothed-out distribution of mass in the system as a whole; binary encounters produce only tiny deflections. As a result, the collisional relaxation time for a large stellar system is always much longer than a characteristic orbital period. Collisional relaxation times

for spheroidal galaxies turn out to be much longer than the age of the Universe, and for compact galaxy clusters they are longer still.

For globular star clusters, on the other hand, modern calculations yield collisional relaxation times one to two orders of magnitude *shorter* than the estimated ages of these systems. A recent study (Layzer 1976a) indicates, however, that these estimates may need to be increased by a substantial factor. The argument runs as follows. Because individual binary encounters give rise to small changes in the momentum and energy of a test star, a large number of encounters is needed to "Maxwellize" an initially nonMaxwellian velocity distribution. Let  $\langle(\Delta E)^2\rangle$  denote the average squared energy change suffered by a star in a single binary encounter. Because successive encounters are statistically independent, the expectation value of the squared energy change after  $N$  encounters is just  $N\langle(\Delta E)^2\rangle$ . The standard definition of the collisional relaxation time equates this cumulative change to the square of the mean energy itself; the collisional relaxation time is proportional to the resulting value of  $N$ . This argument tacitly assumes that -- as is true for an ordinary gas -- the higher moments of the velocity distribution (e.g.,  $\langle(\Delta E)^3\rangle$ ,  $\langle(\Delta E)^4\rangle$ , etc.) relax at the same rate as the lower moments. This

turns out not to be true for stellar systems, however. The probability distribution for the energy change in a single encounter has an odd shape: It peaks at a small value of the impact distance, then decreases exceedingly slowly with increasing impact distance -- so slowly in fact that the relaxation process is dominated by weak distant collisions. Calculation shows that the time required for an initially nonMaxwellian velocity distribution to relax to the Maxwellian form exceeds the conventional relaxation time by a factor  $4 \log_e N$ , where  $N$  is the number of stars in the system. For a typical globular cluster  $N \approx 10^6$  and this factor is around 50. In the light of these considerations, it is no longer obvious that the relaxed structure of globular clusters can be attributed to collisional relaxation.

What are the alternatives? King and Lynden-Bell have suggested that stellar velocity distributions might be relaxed by gravitational-field fluctuations accompanying the initial collapse of a newly-formed stellar system. This hypothesis encounters serious difficulties, however. It requires rather special initial conditions; and even when these conditions are assumed to prevail, recent numerical studies have shown that "violent relaxation" fails to produce sufficiently extended haloes.

Although the fluctuating gravitational fields

involved in collisional and violent relaxation arise in different ways, both relaxation mechanisms are *endogenous*: they depend on interactions within the system. According to a recent suggestion (Layzer 1976b), relaxation in spherical stellar systems may be an *exogenous* process, mediated by tidal forces exerted by external systems. For example, a star belonging to a galaxy that is itself a member of a galaxy cluster experiences a fluctuating tidal force, arising mainly from the nearest passing galaxy. Tidal forces have certain well-known long-term, essentially disruptive, effects. But they also have shorter-term effects that seem to have been largely ignored and that resemble in some ways the effects of violent relaxation. Like violent relaxation, tidal relaxation is mediated by large-scale fluctuating gravitational fields that, in a first approximation, are statistically uncorrelated with the velocities of the stars they act upon. Consequently, both violent and tidal relaxation tend to randomize the stellar velocity distribution. Unlike violent relaxation, however, tidal relaxation is especially effective in the outer regions of extended stellar systems. A recent approximate theory (Layzer 1976b) indicates that tidal interactions in galaxy clusters could well have produced galactic haloes with realistic density distributions.

There remains the problem of explaining the relaxed structure of galaxies that are not members of obvious clusters, and of globular star clusters and compact galaxy clusters. It is tempting to turn the preceding argument around: If we may posit that spherical stellar systems have been relaxed through tidal interactions, then we may infer that tidal interactions were more prevalent in the past than they are now. The hypothesis that all galaxies once belonged to clusters, some of which were subsequently disrupted, is in fact entirely consistent with the observed peculiar velocities of galaxies and with what is known about their spatial distribution. Indeed, it provides a natural explanation for the observed velocity dispersion of "field galaxies." Somewhat more boldly, one may speculate that present-day globular clusters are the survivors of a more numerous company, decimated by the very process responsible for the relaxed structure of its survivors. This hypothesis, too, is consistent with our current understanding of galactic evolution and with the observed velocity dispersion of noncluster stars. Finally, the relaxed structure of compact galaxy clusters may plausibly be attributed to tidal perturbations during their formative stages.

Thus the relaxed appearance of spherical stellar systems may be a clue to the large-scale structure of the Universe at an earlier stage of its evolution.

REFERENCES

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is professor of astronomy at Harvard University, where he received his A.B. in mathematics in 1947 and his Ph.D. in theoretical astrophysics in 1950. He writes: "As a graduate student and for several years thereafter I specialized in theoretical atomic physics. Cosmogony started as a side interest but has gradually become my major professional preoccupation. I have also published a number of theoretical papers on the physics of the earth's upper atmosphere. At the moment I am working on solar magnetism and related questions, as well as on problems connected with the origin of astronomical systems. I am also interested in the philosophy of nature, and I have developed a general education course at Harvard, called 'Space, Time and Motion,' that seeks to give a unified account of the logical, mathematical, physical, astronomical, biological and psychological aspects of its subject matter."

## V I T A

### PERSONAL:

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### REFERENCES:

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