

DARK MATTER AND INFLATION^{*}

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Abstract

I demonstrate that dark matter consisting of any type or types of stable weakly interacting elementary particle is incompatible with the minimal predictions of inflation, based on present observation of galaxy clustering, and assuming galaxies are good tracers of mass in the universe. If we wish to resolve this problem by particle physics alone, we seem to be driven to the possibility that the initial dark matter was unstable.

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Inflationary scenarios [1-3] in general make two predictions about the universe today. First, the phase of exponential de Sitter-like expansion, involving many e-foldings, assures that any initial curvature term in the Einstein equations governing the expansion quickly becomes negligible. Thus the present universe is described by a $k=0$ Einstein-de Sitter model, with an essentially exact critical density, so that the relation between the Hubble constant and mass density today is:

$$H_0^2 = 8\pi G/3\rho_0 \quad (1)$$

Next, it has recently been recognized [4-6] that quantum fluctuations in elementary particle fields in the de Sitter background phase get translated into a Harrison-Zel'dovich type [7,8] scale invariant spectrum of metric fluctuations at the time of final horizon crossing, leading to the relation:

$$\langle \delta\rho/\rho \Big|_{\text{horizon crossing}} \rangle^2 = \epsilon_H^2 = \text{constant}. \quad (2)$$

Are these two predictions compatible with observations and standard models of galaxy formation? Clearly Eq. (1) implies the need for significant dark matter since observations based on luminous matter [9] yield a density parameter $\Omega_{\text{lum}} = \rho_{\text{lum}}/\rho_0 < .01$. There is evidence for dark matter distributed on the scale of galaxies and clusters of galaxies [9]. Many different types of possible weakly interacting stable elementary particles have been proposed whose cosmological abundance and masses may be such as to result in a $\Omega=1$ universe [9]. The make up of this dark matter will determine its clustering properties, and we can distinguish two generic types, so-called "hot" or "cold" dark matter, based on differing equations of state at horizon crossing. We now demonstrate that observed clustering tends to rule out any combination of these possibilities in an $\Omega=1$ universe with a

scale invariant initial spectrum of perturbations.

(a) Hot Dark Matter

I will consider hot dark matter to consist of particles which were relativistic at the time when the scale of present day clusters of galaxies crossed the horizon. This definition is patterned after massive stable neutrinos, whose mass is constrained to be less than ~ 30 eV for an $\Omega \leq 1$ universe, if there has been no significant heating since nucleosynthesis [14]. The significance of the equation of state at the time of horizon is related to the growth of primordial fluctuations. Weakly interacting particles inside local regions of enhanced density can free-stream away from these regions, except for gravitational effects. Relativistic particles on the other hand are not gravitationally bound to any objects except for black holes. Thus any primordial fluctuations inside the horizon of weakly interacting relativistic particles are damped by this free streaming [9]. Only when the temperature drops below the mass of these particles, so that they become non-relativistic and their free-streaming length drops, can fluctuations inside the horizon grow. Once these massive particles dominate the energy density of expansion linear perturbations $\delta\rho$ can grow, $\sim t^{2/3}$ [10], or, solving (1) for a matter dominated universe $\delta\rho(t)/\rho \sim R(t)$, where R is the cosmic scale factor. Since the damping scale for 30 eV neutrinos is ~ 40 Mpsec [9] (containing a mass $\sim 10^{15} M_{\odot}$), in a neutrino dominated universe the first scale to go non-linear is the scale of superclusters. Numerical simulations [11] in the non-linear regime do indicate the formation of caustic "pancake" surfaces which can fragment into galaxies. However, the "clustering scale" L_{Cluster} today on which $\delta M/M \sim 1$ is approximately 8 Mpsec. Thus $L_{\text{Cluster}}/L_{\text{Damping}} \sim .2$ today. Simulations on the other hand indicate that at the time of galaxy formation $L_{\text{Cluster}}/L_{\text{damping}} \sim .1$ [12]. Since

$L_{\text{Cluster}}(t) \sim R(t)$ this implies that in a neutrino dominated universe scenario galaxies would have had to have formed quite recently (at redshifts $\lesssim 1$). However structures such as Quasars are observed out to redshifts of ~ 3 . Thus it is difficult to reconcile such late formation with observation. Indeed, there are other problems with this scenario with roots due to the same cause, the large initial damping scale. These include: (a) too large predicted virial velocities for clusters of galaxies [9], and (b) due to the fact that the initial scale, ϵ_H , of primordial fluctuations in Eq. (2) must be large ($\sim 10^{-4}$ - 10^{-5}) in order to get nonlinear effects on order of 40 Mpc today, the related dipole anisotropy of the microwave background due to the linear regime of perturbations may be too large [13,14].

(b) Cold Dark Matter

Cold dark matter on the other hand is made up of material (axions [15], for example) which was non-relativistic at the time the scale of galaxies crossed the horizon. Thus, primordial fluctuations on scales between galaxies and superclusters were not damped away. While the growth of fluctuations in the non-relativistic matter is greatly suppressed until the expansion becomes matter dominated [16], there is in this model a hierarchial picture, with smaller scales going nonlinear first. Thus, the problems of the hot dark matter scenario are avoided, with there now being sufficient time for galaxy size fluctuation to grow so that galaxies form at large enough redshifts, and so the scale ϵ_H in Eq. (2) need only be of order $\sim 10^{-6}$, satisfying and microwave background constraints

There is, however, another problem with cold dark matter scenarios. Since cold matter has no problem clustering on small scales, there should be some evidence on the scale of clusters of galaxies that $\Omega=1$. Indeed, the cosmic virial theorem [9,10] relates the relative peculiar velocity of

pairs of galaxies to the cosmological density parameter Ω . Using the measured galaxy correlation functions (which suggest a clustering hierarchy) one obtains the approximate relation [9]:

$$\langle v_{12}^2 \rangle^{1/2} \approx 800 \Omega^{1/2} \left(\frac{r}{\text{Mpc}} \right)^{.1} \text{ km sec}^{-1} \quad (3)$$

where $\langle v_{12}^2 \rangle$ is the mean square relative peculiar velocity of pairs of separation r .

However, the most extensive recent redshift survey [17] suggests, $\langle v_{12}^2 \rangle \approx 300 \pm 50 \text{ km sec}^{-1}$, $r \sim 1-2 \text{ Mpc}$, yielding the value $\Omega=0.1$. In general, applications of the virial theorem out to largest scales suggest $\Omega < .5$. It is difficult to understand why, if cold dark matter with $\Omega=1$ has no problem clustering on these scales, no measurements yet yield this value.

(c) Cold Dark Matter Plus Hot Dark Matter

It is natural to consider whether some combination of stable cold and hot dark matter, say axions plus neutrinos, may alleviate the problems of each separate scenario. This, however, is not the case. The discussion of the last section illustrated that the material in clusterable form appears to yield an $\Omega_{\text{clustered}}$ of $\lesssim .2$. Thus the ratio of clustered to unclustered matter, if $\Omega=1$ today, is $f \lesssim \frac{1}{4}$. This ratio remains constant as long as both species are non-relativistic, and decreased as $R(t)$ for earlier times when the hot matter was relativistic. Thus axion perturbations must grow in a background dominated by neutrinos, whose perturbations on a scale less than that of superclusters have damped away. This linear perturbation problem can be solved in the full relativistic case from the point of horizon crossing [18], but it can be shown from the Vlasov equation that well inside the horizon the evolution axion perturbations is given by [18, 19], $\delta\rho/\rho \sim t^{2k/3}$ where $k = (\sqrt{1+24f} - 1)/4$. Thus, if

$f < 1/4$, $k \lesssim 3/8$. This slow growth of axion fluctuations on scales smaller than the neutrino damping scale implies that fluctuations again go non linear first on this latter scale, and we essentially revert back to the picture of the hot dark matter scenario.

Thus, while hot dark matter seems on the most precarious footing, if galaxies provide good tracers of mass in the universe, and if measurements of the mass density of clustered matter do not increase, then any sort of dark matter made up of stable weakly interacting matter seems incompatible with the $\Omega=1$ universe and Harrison-Zel'dovich spectrum of primordial fluctuations predicted by inflation.

This conflict can be resolved if we are willing to postulate new evolutionary models of galaxy formation which suppress the formation of structure in dense regions [20], or accept the possibility of non-gaussian primordial fluctuations produced after inflation (i.e. strings, etc.) from which galaxies may form. However, within the context of standard models of galaxy clustering, if we wish to keep the minimal predictions of inflationary models--which after all provide not only the simplest initial conditions for evolution of structure but also the only ones with any real theoretical basis--then we are led to consider the possibility that the dark matter may not be stable.

Massive neutrinos are the most likely candidate for unstable dark matter [18,19]. Once they receive a non-zero mass there automatically exist channels for their decay. However, neutrinos of mass less than 1 MeV (of interest here) tend to be extremely long lived and have a dominant decay mode (radiative decay) which is ruled out for lifetimes of cosmic interest [21]. (Other decay modes to light neutrinos can be enhanced, but the radiation mode is still too large in most cases [22]).

This problem can be circumvented if we are willing to postulate new interactions which may mix different generations. Such interactions occur naturally if we wish to spontaneously break at high energies various possible global symmetries, including lepton number [13] (which must be broken if neutrinos receive Majorana masses), and family symmetry [14]. The massless goldstone bosons which result provide new, non radiative decay channels for neutrinos with lifetimes which can be computed. For the interesting case of family symmetry breaking, the neutrino lifetime (the decay $\nu_{\text{heavy}} \rightarrow \nu_{\text{light}} + \text{massless bosons}$) is [14]:

$$\tau = 3 \times 10^9 \left(\frac{F}{10^{10} \text{ GeV}} \right)^2 \left(\frac{100 \text{ keV}}{\text{mV}} \right)^3 \text{ sec.} \quad (4)$$

The scale of breaking (F) which allows the scenario described below is of the same magnitude as that already considered for axions, which are related to the breaking of a subgroup of the global family group.

Let us now consider how unstable particles can help resolve the problems outlined earlier. First, it can be seen [18] that (assuming no photons are produced in the decay of unstable particles), if we are to measure a critical density today, the period of matter domination since the time the scale of galaxies entered the horizon cannot be larger than that which occurs in the case of stable dark matter. Thus the maximum growth of fluctuations on this scale cannot be larger than the cold dark matter case. However, making neutrinos more massive and unstable allows us to change the timing of periods of matter vs. radiation domination. As we continue to make the neutrino heavier we reduce the initial damping scale. Eventually, for masses in excess of $\sim 1 \text{ keV}$, we can produce the initial phase of hierarchical clustering--allowing galaxies to go nonlinear before clusters. Even if

the neutrino mass is not this high we can alleviate problems of the pancake scenario by reducing the neutrino damping scale [25]. Also, after the particles decay, depending on the mass of the decay product, a significant portion of their mass density can go into the form of a poorly clustered background. In one of the most amusing possibilities, this background results in a radiation dominated universe today [25].

A detailed analysis of the growth of fluctuations in unstable dark matter scenarios [19] indicates a number of other possible advantages. By changing the growth pattern in the linear regime, and shifting growth to smaller scales we may reduce the dipole microwave anisotropy due to the linear regime for a given ϵ_H value. Also since, due to particle physics considerations, unlike most earlier considerations of unstable neutrinos [19] we are considering late decays (after recombination), it is possible for structure to form on a number of different scales, due to the separate potential wells of primordial light as well as heavy neutrinos.

These considerations are not meant to suggest that unstable neutrino scenarios are compelling. They do however indicate what is possible and what may be necessary to resolve the conflict between an $\Omega=1$ universe predicted by inflation and the existence of stable dark matter described in the first half of this work. In particular it is especially interesting that considerations of gravitational effects on expansion and clustering in the late universe yield such constraints on particle physics models of the very early universe. We find that observations, at present, require new physics in one area or another.

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