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A PROPOSED LABORATORY MEASUREMENT OF THE VELOCITY OF PROPAGATION OF GRAVITY

QUENTIN A. KERNS – 7 Corte Bombero
Orinda, Calif.

APRIL 9, 1958

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ABSTRACT

Laboratory measurements involving gravity are limited by the small value of the gravitational coupling constant. The range of possible measurements can be extended by specialized electronics techniques developed by the author. There is described here for the first time a laboratory method for measuring the propagation velocity of gravitational interaction. The velocity measurement is achieved by a precise determination of the phase difference between electrical signals derived from a pair of electromechanical transducers. The techniques described are applicable to more general problems in the laboratory; this paper, however, deals specifically with a propagation velocity experiment.

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INTRODUCTION

Newton's law of universal gravitation gives the attractive force F between masses M_1 and M_2 a distance r apart:

$$F = \frac{G M_1 M_2}{r^2} , \quad G = 6.67 \times 10^{-8} \text{ cgs units}$$

In the proposed experiment, r varies periodically and the masses experience oscillating forces. In this general situation, the use of resonance to enhance the amplitude of motion in an indicating instrument suggests itself. We recall some early proposals of resonant devices for the determination of the gravitation constant G; Joly suggested a pendulum arrangement$^{(1)}$, while von Eötvös described a torsion instrument, the "Gravitationmultiplicator"$^{(2)}$.

The resonant device proposed here is a vibrator, termed the receiver. It is described in Section I.

A rotating "transmitter" is described in Section II. It is brought up to spped by a conventional motor, and maintained at a constant rotational speed from then on by a small servo motor supplying correcting torque to maintain the rotation.

An electronic phase-measuring arrangement is described in Section III.

Some calibration and testing procedures are suggested in Section IV, an Appendix.

SECTION I

A receiver, (electromechanical transducer) for oscillating gravitational forces.

Geometry:

Figure I shows the receiver. The gravitational attraction is primarily between the receiver mass M_2 and a transmitter Mass M_1. The receiver is to be highly evacuated, sealed off and gettered, the electrical connections being fed out through vacuum tight seals. A small adjustment of resonant frequency is possible in the sealed-off unit since the upper stem will vibrate slightly and its mass is adjustable externally. The mass center of the upper stem is to be relatively far from M_1, so that the upper stem does not experience accelerations of the magnitude of those on M_2. As M_1 moves with the appropriate period, M_2 will be set in motion relative to the walls of the enclosing quartz tube. A suggested resonant frequency is 50 cycles/second.

Amplitude:

Assume a transmitter mass M_1 of 20 kilograms, a receiver mass M_2 of 10 grams, and a minimum separation r of 20 cm. The peak force F_{max} will be (in cgs units):

$$F_{max} = \frac{6.67\times10^{-8}\times20\times10^{3}\times10}{20\times20} \approx 34\times10^{-6} \text{ dynes.}$$

The maximum value of r is large compared to 20 cm, hence the peak-to-peak value of F will be nearly the value above of 34 microdynes. Assume a sinusoidal component of 5 microdynes peak at 50 cps. This

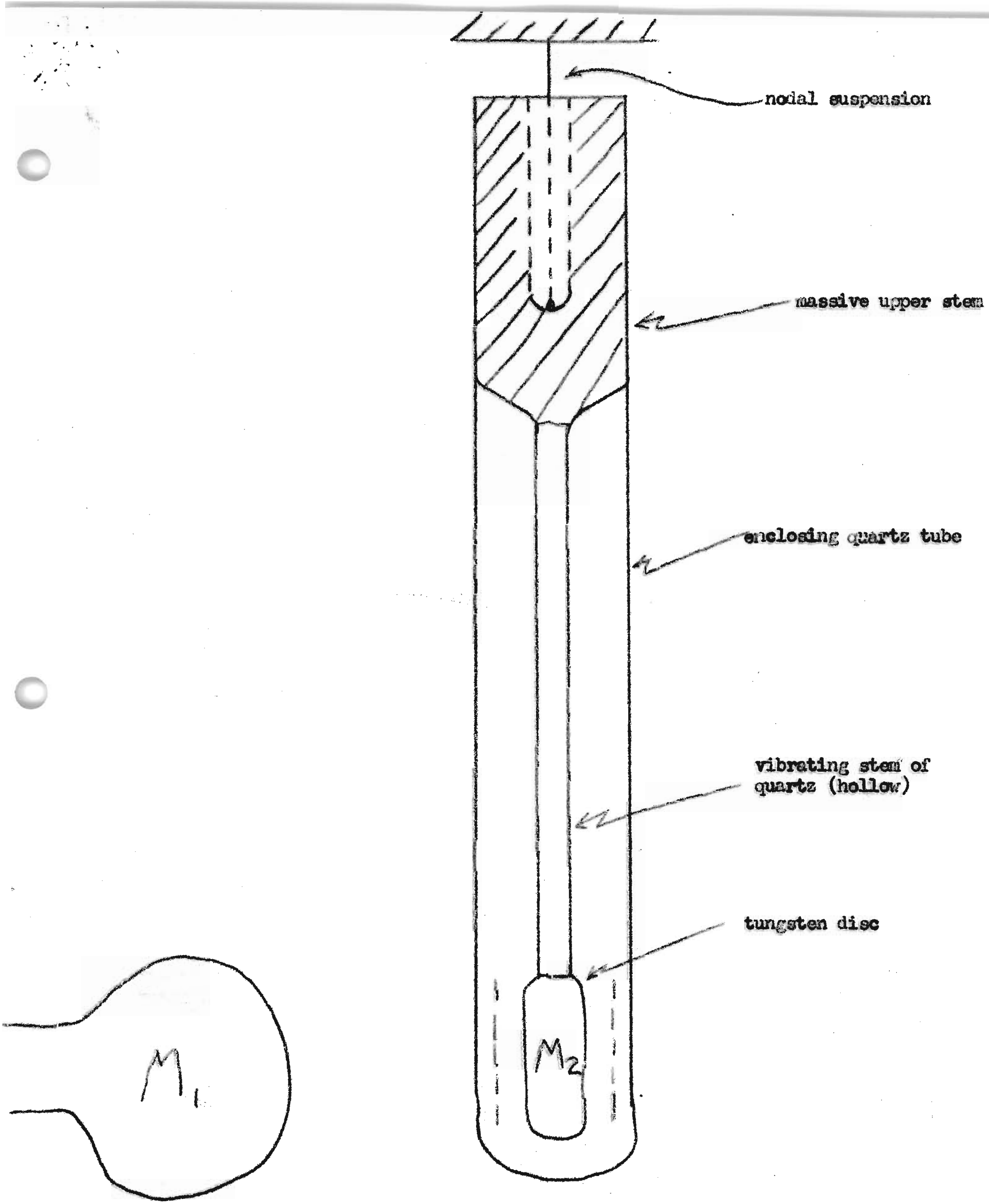


Fig. 1 Receiver geometry. Receiver shown adjacent to transmitter M_1.

force acting on a free mass of 10 grams gives a peak deflection x_{max} of:

$$x_{max} = \frac{F_{max}}{M_2 w^2} = \frac{5 \times 10^{-6}}{10 \times (2\pi 50)^2} \approx 5 \times 10^{-12} \text{ cm.}$$

A benefit of resonance is this: the deflection amplitude of the resonant instrument will be multiplied by the factor Q, where

$$Q = \frac{\text{stored energy in resonator}}{\text{energy loss/ radian}},$$

the usual engineering definition. The viscosity of fused quartz is small$^{(3)}$, hence one anticipates a high Q. Also, stability of the properties of annealed quartz is good. A Q of > 5000 was measured for a simple Pyrex resonator (87 cps) operating at room temperature. Assuming that one can reach a Q ≈ 5000 with a quartz resonator at low absolute temperature, the amplitude becomes

$$5 \times 10^{-12} \text{ cm} \times 5000 = 2.5 \text{ Å} .$$

Capacitor plates:

The relative motion of M_2 may be converted into an electrical signal by making it part of some electrical circuit; an electrostatic system is described here. The tungsten block M_2 is to be ground and polished optically flat on two parallel faces, which see corresponding flat faces on the inner wall of the outer quartz tube. The faces on the quartz tube, when metallized, form the outer plates of a capacitor of which M_2 is the center (moving) member. The quiescent position of M_2 is to be midway between the outer capacitor plates, with a gap of .001 inches.

One outer plate is connected to a fixed source of plus 100 volts D. C. and the other to minus 100 volts D. C. The block M_2, via a metallized line on the inner quartz stem, is connected to a high-impedance amplifier. The electric field between stationary and moving members is 100 volts/.001 inches or 400 microvolts/angstrom; this is the output signal obtained under motion, if the amplifier input impedance is essentially infinite. The expected electrical signal is thus 400 microvolts/angstrom x 2.5 angstroms $\simeq$ 1 millivolt.

Amplifier:

An electrometer tube is chosen as the input stage of the amplifier. The miniature tube CK5886 (Raytheon) is a possible choice. Suppose M_2 is a disc of tungsten .25 centimeters thick; the faces will have an area of 2 cm^2 for a .5 cm^3 volume, which weighs approximately 10 grams. The capacitance of each face to a stationary plate will be (MKS units)

$$C = \frac{K_0 A}{d} = \frac{8.85 \times 10^{-12} \times 2 \times 10^{-4}}{2.54 \times 10^{-5}} \approx 70 \times 10^{-12} \text{ farad};$$

the total capacitance presented to the amplifier input circuit will be 140 micromicrofarads. It will be advantageous to adjust the average potential of M_2 to such a value that it is under zero electrostatic force (the possible D. C. electrostatic force with unbalance greatly exceeds the oscillating gravitational forces). Therefore, we choose a suitable grid resistor, based on noise considerations. The signal is developed by a generator having an internal impedance equal to that of a 140 uuf capacitor; the open-circuit resistor noise is shunted by this capacitance C. In this case, a high value of resistance R does not impose the thermal-noise penalty of the open-circuit resistor, for which the mean square noise emf is:

$$\overline{V^2} = 4kTR\,df$$

where df is the bandwidth.

Instead, the noise for increasingly large R is asymptotic to a value depending upon C.(4) One expects in addition a noise contribution by the tube, from the flicker effect and shot noise. In general, flicker noise is greater at low frequencies f and may follow a $\frac{1}{f}$ law, but should be measured for a particular tube. This has been done for the CK 5886. Figure 2 shows the result. A 140 micromicrofarad capacitor shunted by a 10^{10} ohm resistor comprises the input circuit.

The total amplifier noise is about .1 microvolt for a bandwidth of 1 cycle/second; considering the phase detector properties, and thermal fluctuations in the position of M_2, one might choose a bandwidth of $\sim$ .01 cycles/second. A corresponding measuring time would be several hundred seconds.

Receiver Mounting:

It is proposed to use a pair of identical receivers and associated amplifiers; the amplifier is physically small and may be attached to the receiver and powered by integral batteries. The output signal would be electromagnetically coupled from this amplifier to the external phase-measuring circuit, hence there exists only 1 mechanical suspension point of the entire assembly. The problem of isolating the receiver unit from spurious vibration is reduced to the problem of a single suspension. (It could be a magnetic suspension. See Appendix).

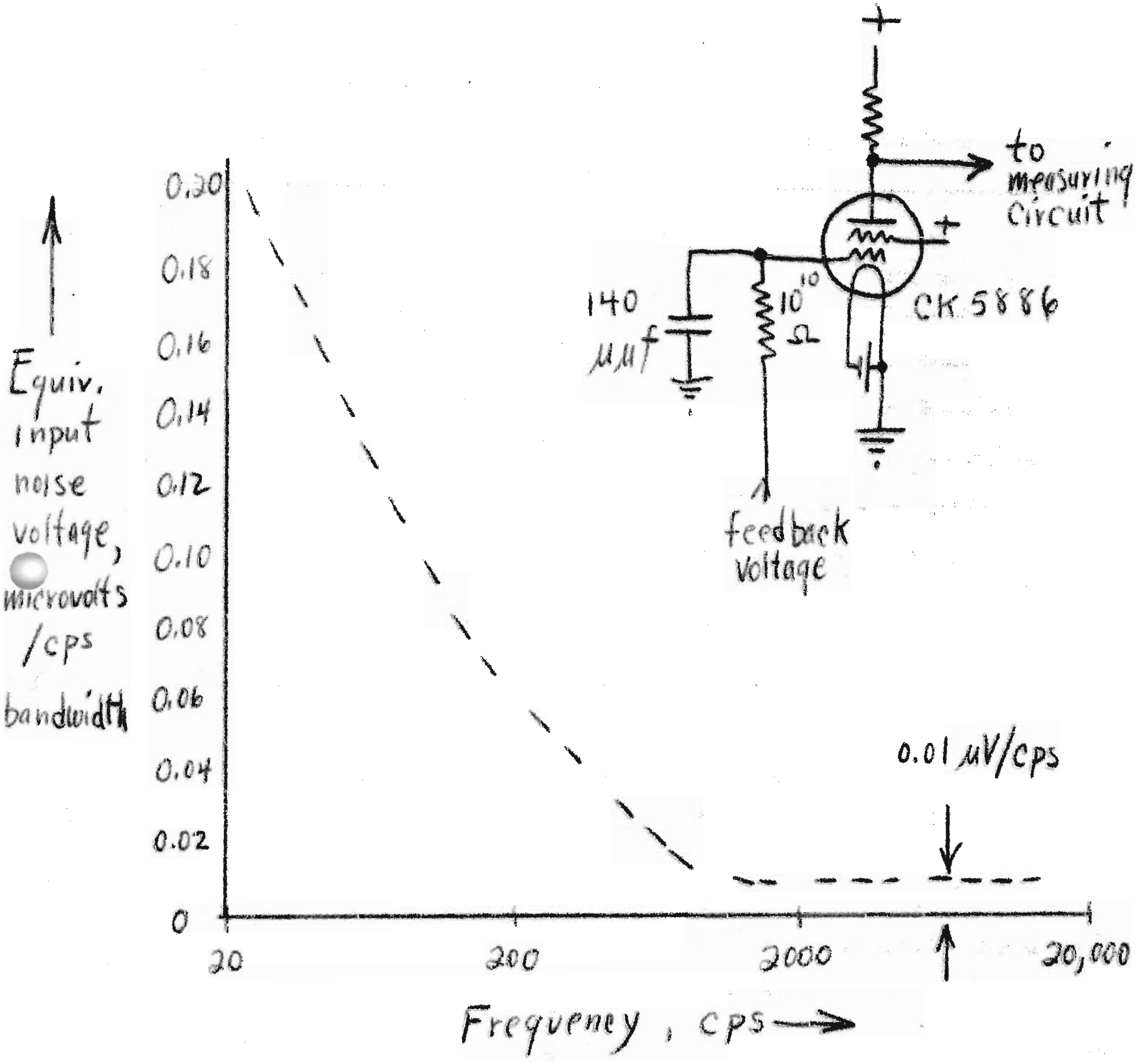


Fig. 2 Measured equivalent input noise as a function of frequency for a particular tube of the type # CK5886. The band width is 1 cps for this measurement.

SECTION II

A transmitter for audio frequency gravitational forces.

Transmitter Geometry:

The transmitter consists of a pair of 20 kg. masses connected by a cable 2 meters long and about 3 centimeters in diameter. The assembly is rotated at a frequency of 25 revs/sec to excite the receivers tuned to 50 cycles/sec. There are no appreciable alternating stresses in the material, and the problem of decoupling the transmitter and receiver for spurious mechanical signals is simplified, since the fundamental frequency of the transmitter does not excite the receiver. Some properties of the "mass-dipole" transmitter are listed in Table I. A proposed experimental layout is shown in Figure 3.

Frequency Control:

The transmitter $M_1 - M_1$ is servo-controlled to run at a frequency determined by one of the receivers, say R_1. Since the transmitter will be run in vacuum, the necessary correcting torques will be those required to overcome bearing friction, and compensate for slow creep in cable length under tension. Temperature can be controlled to eliminate thermal effects as a cause of frequency drift.

Optical Reference System:

The axes x and y shown are to be maintained by an optical system. Receiver R_2 can be arranged to move precisely on a radius through R_1 and the center of mass of $M_1 - M_1$. A small flat on the extremities of $M_1 - M_1$ contains a fiducial mark. The instantaneous position of this mark, observed as it crosses near the x and y axes by a short pulsed

General	Proposed values
MASS $= M_1$	$M_1 \approx 20$ Kg
CABLE LENGTH $= 2\rho$	$\rho \approx 1$ meter
CABLE TENSION $= T = M_1 \omega^2 \rho$	$T \approx 50$ tons
ANGULAR FREQUENCY $= \omega$	$\omega \approx 50\pi$ radians/sec
ANGULAR MOMENTUM $= 2M_1 \rho^2 \omega$	$I\omega \approx 10^4$ Kg meter^2/sec
KINETIC ENERGY $= T\rho$	$T\rho \approx \frac{1}{2}$ megajoule
PERIPHERAL SPEED $= \omega\rho$	$\omega\rho \approx$ Mach 1/2

Table 1. Dipole transmitter properties

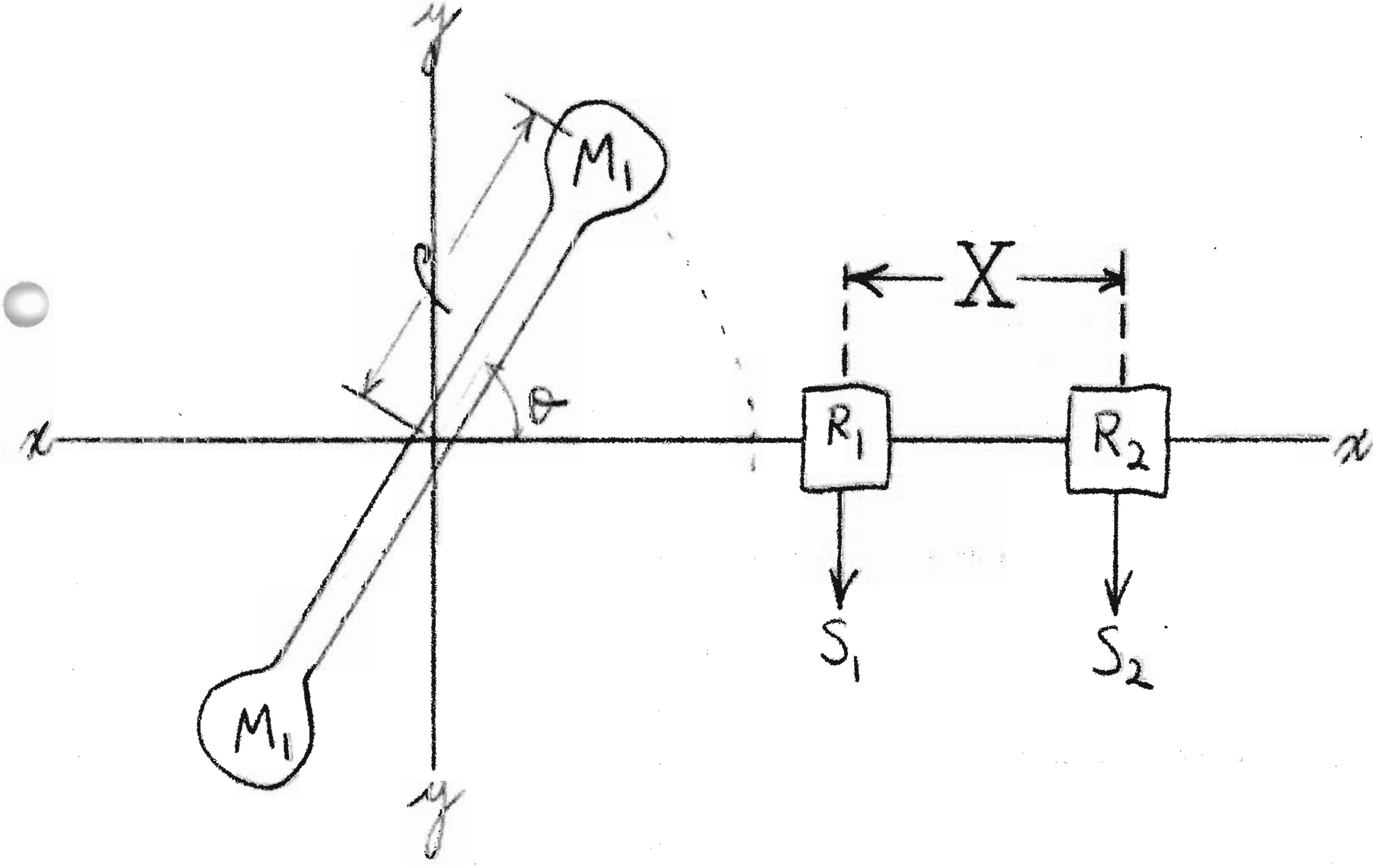


Fig. 3 Proposed experimental layout.

light and phototube combination,(5) provides the control information to the servo maintaining the angular velocity and position of $M_1 - M_1$ relative to R_1 and R_2.

Receiver Modes:

The receivers R_1 and R_2 may be arranged to respond to either the x or y component of the force; by suitably choosing the cross-section of the stem supporting M_2 in the receiver, the x and y modes can be given frequencies differing by more than 1/Q, and hence be excited separately. The x and y forces as a function of time have the form shown in Figure 4.

SECTION III

Method for measuring ultra-small phase angles.

Figure 5 is a block diagram.

$\vec{S}_1$ and $\vec{S}_2$ are sine wave signals of the same frequency but slightly different in phase and amplitude. The difference $\vec{D}$ has components D_x and D_y in phase and out of phase with respect to $\vec{S}_2$. By amplitude modulation of one of the signals, say $\vec{S}_1$, (with no accompanying phase shift), the signals can be made to differ only in phase. A subtraction will then give a signal whose amplitude is directly proportional to phase difference. In general, the amplitudes will vary with time. In order not to saturate the amplifier k_3 with the amplitude difference signal, its presence is noted as an error signal by the phase detector #1, and the amplitude of S_1 is continuously adjusted so that D_x is small. D_y can be determined in the presence of a reasonable or zero amount of

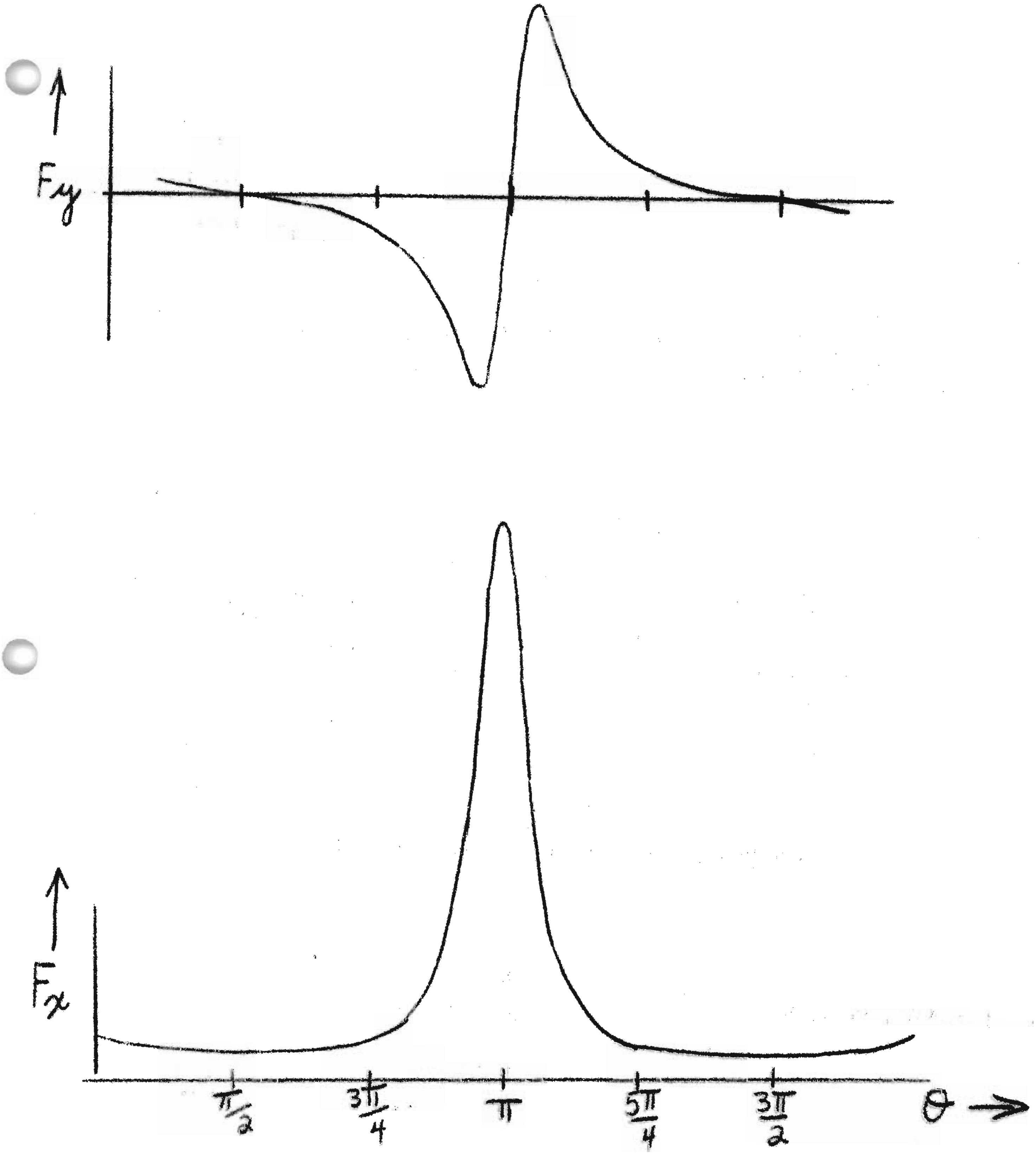
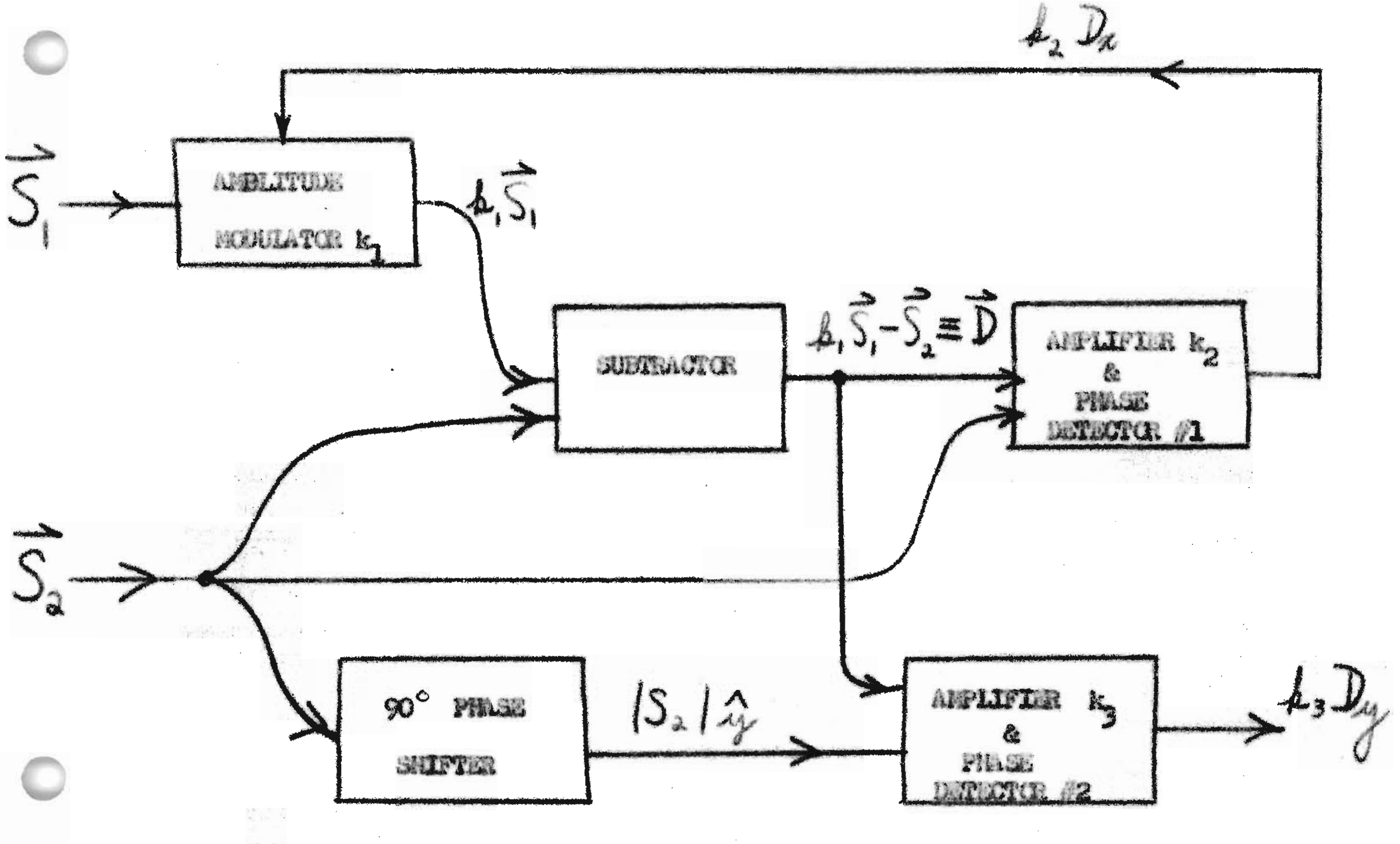
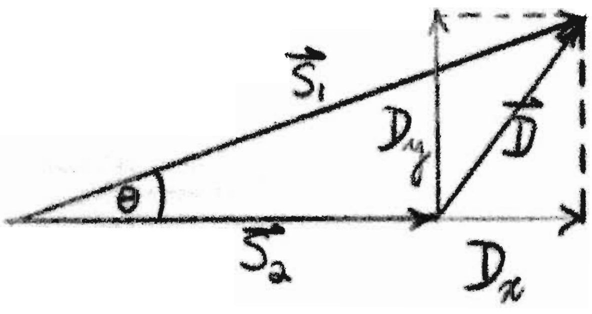


Fig. 4 Form of x and y forces as a function of transmitter angle θ.



Signals S_1 and S_2 represented as vectors.



$\theta = \dfrac{D_y}{|S_2|}$ very nearly, for small θ.

Fig. 5 Above, block diagram of phase-measuring method. Below, vector representation of signals.

D_x; $\frac{D_y}{|S|}$ is the phase angle sought.

The subtraction is performed by a shielded transformer, as in Figure 6; ordinarily, differences of 10^{-10} volt between $\vec{S_1}$ and $\vec{S_2}$ are readily detectible in the subsequent circuits.

Synchronous frequency conversion:

Referring to Figure 2, one sees that some improvement in signal-to-noise ratio would appear at a frequency above 50 cycles/second. There is a useful method of avoiding the excess noise at low frequency without raising the transmitter frequency. That is to replace the D.C. signal on the fixed plates of the receivers R by an audio frequency signal of say 5 KC. The input signal to the electrometer tube then consists of a pair of side bands of frequencies 5 KC ± 50 cps, with no accompanying carrier. The modulation process is of the form:

$$\sin \omega_1 t \ \sin \omega_2 t = \frac{1}{2} \cos(\omega_1 t - \omega_2 t) - \frac{1}{2} \cos(\omega_1 t + \omega_2 t).$$

The phase is preserved, so that measuring the phase relation between a pair of similar side bands is equivalent to a phase measurement at the frequency W_1 provided that W_2 comes to both receivers from a common source. Frequency conversion of this nature is incorporated in the circuit of Figure 5 where desireable.

The upper and lower sidebands in this example are sufficiently well separated (100 cps) so that either may be selected with a filter. (Special forms of phase detectors may use both sidebands.)

The electrometer circuit input impedance must be kept very high to avoid loading the high-Q mechanical resonator; reactive loading will

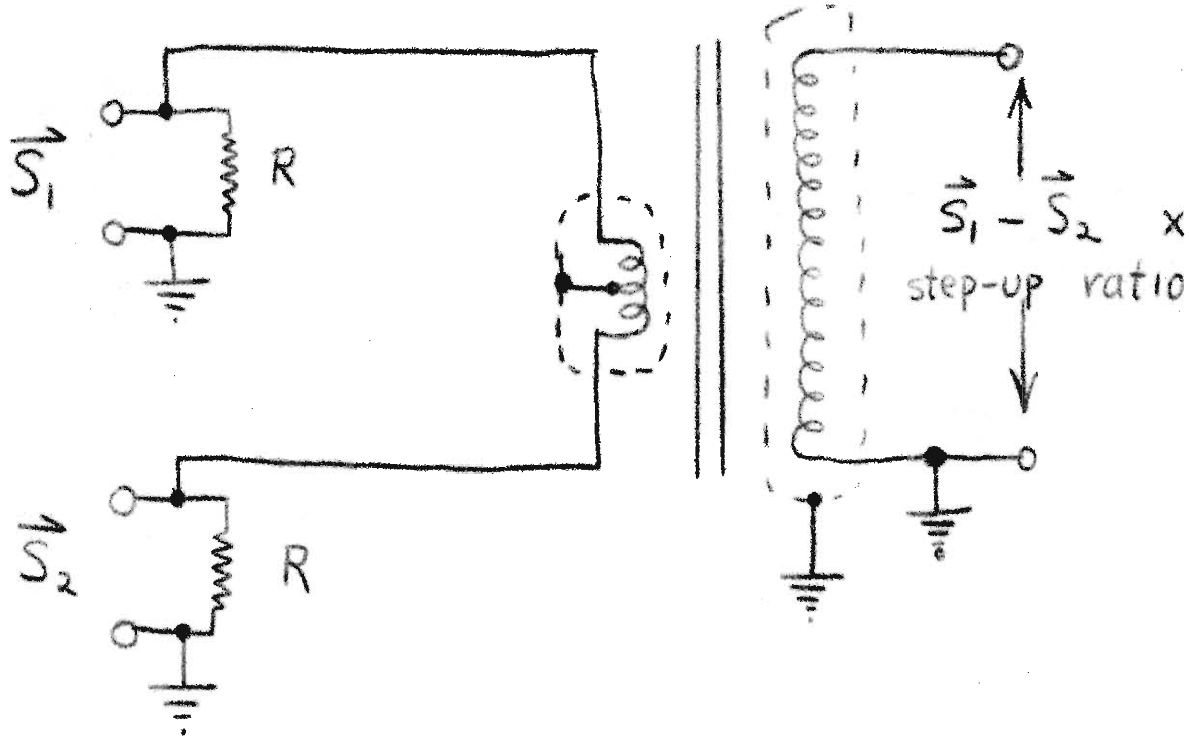


Fig. 6 Electrostatically shielded transformer for measuring very small difference signals.

shift the resonant frequency and hence the phase; resistive loading will reduce the Q.(6) The input impedance, using appropriate feedback, can be kept suitably high in the frequency range chosen.

SUMMARY

Signals S_1 and S_2 may be expected to have a phase relation which depends on the distance X in Figure 3. It is the phase difference as a function of X which is to be measured. The receivers R_1 and R_2 do not operate in the wave zone, but in the induction zone, much less than a wavelength away. The phase detector must thus be capable of measuring a very small phase difference. The time difference may be expected to be of the order of 1×10^{-9} sec. if the receivers are 30 cm. apart and the interaction travels with the velocity of light. This time difference may be measured with an estimated error of 1×10^{-11} second with the circuit of Section III. The receivers are self-contained, and can be located as desired to plot the propagation of fields. Interesting variations of the experiment are possible in which the space X is filled with some material. The measurement of the velocity in free space, however, is of first importance; it must be regarded as a milestone in the experimental progress of the science of gravity.

APPENDIX - SECTION IV

Calibration and testing procedures

Transmitter Frequency Stability

The transmitter is inherently stable over short periods of time. Over longer periods, it will be locked to the frequency of the receivers. It could in fact be locked to an atomic clock, but it is more reasonable to use a receiver as the standard since only relative frequency drift is of great importance.

Transmitter Spurious Radiation

Stray electric and magnetic fields from the transmitter can be looked for with sensitive instrumentation and suitable shields designed and tested. The vibration coupled from the transmitter through its suspension can be monitored continuously by a pickup device like the receiver, but directly attached to the suspension rather than isolated. A magnetic suspension for the transmitter would allow it to choose its own spin axis with the restrictions that the vertical force of gravity be just overcome, and a couple be applied to cause precession of the correct magnitude to follow the earth's rotation. ($I\,ww' \approx 1$ in-lb).

Magnetic Suspension:

"Magnetic suspension" refers to support of an object against gravity by magnetic forces. Position detectors electronically control the field. The instantaneous equilibrium is not required to be stable;

if the system responds in 1 microsecond, the object may fall for part of this time and then be picked up. The resulting displacement, however, is insignificant:

$$S = \frac{1}{2} g t^2 \approx \frac{1}{2} \times 10^3 \times 10^{-12} = .05\,\text{Å} \; .$$

The restoring forces may be quite non-Hookian if desired, and such a suspension may be given excellent properties as a mechanical filter.

Receiver Response:

The receivers can be tested by exciting them with electromagnetic (light or microwave) radiation chopped at the desired frequency. A radiation pressure of $\sim$ 5 microdynes (5×10^{-11} Newton) during the "on" part of the cycle should be exerted on the mass M_2; the required power beamed toward M_2 is $\approx$ $5 \times 10^{-11}\, c$ $=$ 15 milliwatts. The Q-curves of the receivers can be plotted in this way, and the frequencies of resonance adjusted to be equal. Aging effects on the resonant frequency can be followed over a period of time (1 part in 10^9 / day is anticipated) on a group of several receivers, and a pair having similar drift characteristics be selected for the propagation velocity experiment.

The receiver properties are fundamental to the experiment, and must be measured carefully. A way to select a pair of similar receivers is to excite them from a common source of radiation, and compare the relative phase angle with the device of Section XII. If a significant phase difference appears when the source-to-receiver distances are equal, a relative adjustment of receiver resonant frequency is required.

Near the resonant frequency, there will be a phase shift $\Delta\phi$; in radians,

$$\Delta\phi \approx \frac{Q\pi}{2} \frac{\Delta f}{f_0}$$

where f_0 is the resonant frequency and Δf is the difference between the driving frequency and f_0. The adjustment is, of course, delicate; this is the price we pay for dealing with a small force signal in the first place.

Receiver Shielding:

The shielding of the receiver for spurious fields can be tested by placing it in much stronger fields than the measured spurious fields of the transmitter, and noting its response. Because of the high Q and the coherent detection scheme, only frequencies in a very narrow band near the operating frequency will cause significant disturbance.

Testing the phase measuring device:

An audio frequency oscillator supplies test signals to simulate S_1 and S_2. ~~By feeding the oscillator signal through different lengths of coaxial cable to the inputs for S_1 and S_2.~~ By feeding the oscillator signal through different lengths of coaxial cable to the inputs for S_1 and S_2, which contain resistive terminations, a pair of signals having a known phase relation is obtained. Separate noise generators inject noise voltages into the two signals; the noise generators must be separate so the noise is not coherent.

This test has been performed with a simple version of the device. The experimental result is that time differences of 1×10^{-11} sec. between audio frequency signals can be detected in the presence of noise.

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Biographical sketch of Quentin A. Kerns, author of "A Proposed Laboratory Measurement of the Velocity of Propagation of Gravity."

Quentin Kerns is group leader of the Counting Research Group at the University of California Radiation Laboratory. He has been associated with the electronics of particle accelerators since 1942, when he joined the Radiation Laboratory. He recalls participating in the design of regulated high-voltage D.C. sources for the isotope-separating Calutrons. After a time at Berkeley, the Army beckoned, and he spent two years with the Special Engineer Detachment at Oak Ridge, Tennessee, developing among other circuitry some analogue computers to aid Calutron ion source research. He rejoined the U. C. Radiation Laboratory in 1946, working in the experimental electronics group and attending school part time, receiving his B.A. in Physics from the University of California in 1951. Along with the Counting Research Group effort toward long-range improvements in the fast electronics of nuclear physics particle counting, he is attending graduate school part time, with an interest in the field of solid-state physics. He resides in Orinda, California, is married and has four children: Susan, 8, Robert, 6, Jimmy, 2½, and Jonathan, 8 months.

Kerns Essay -

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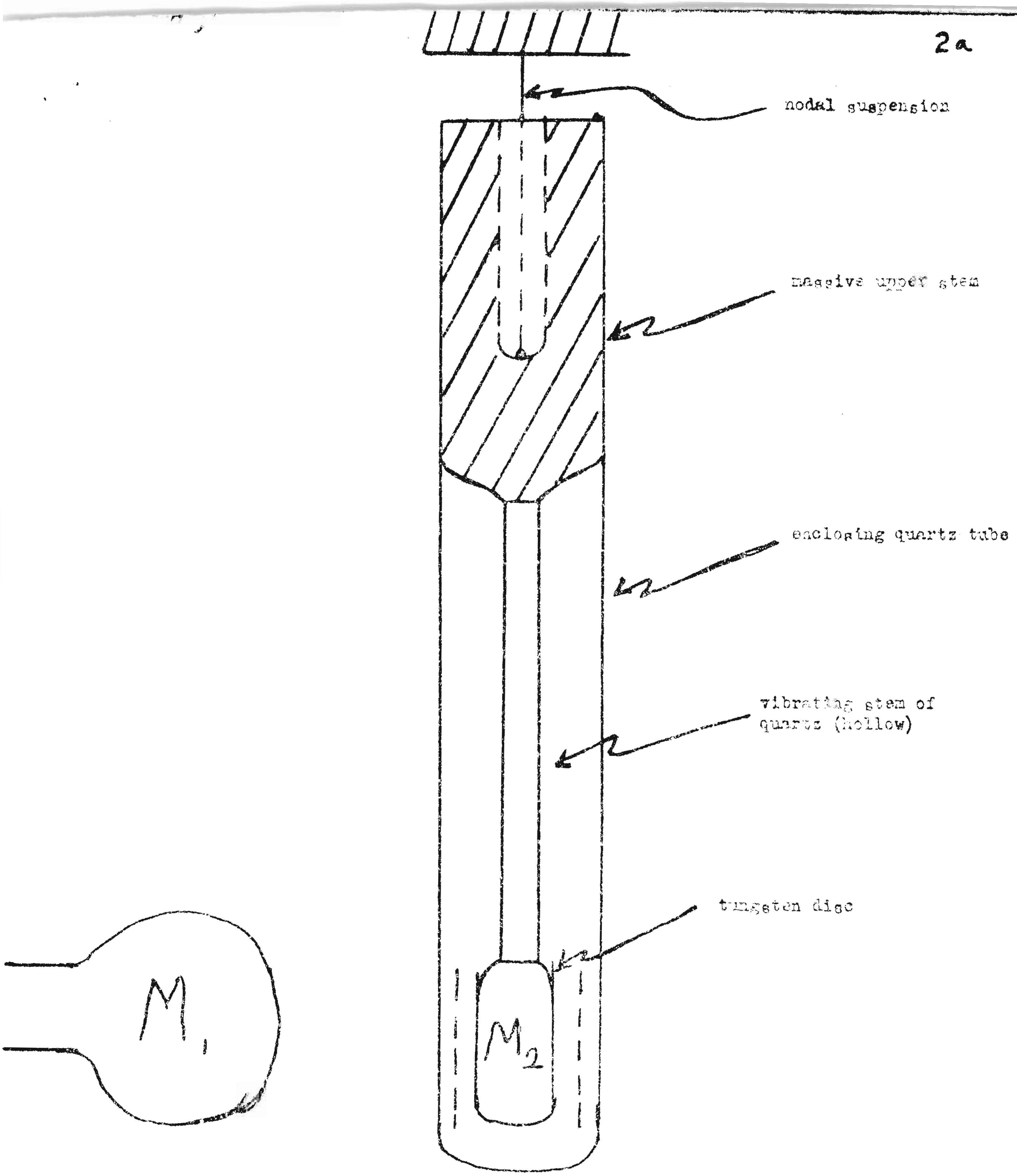

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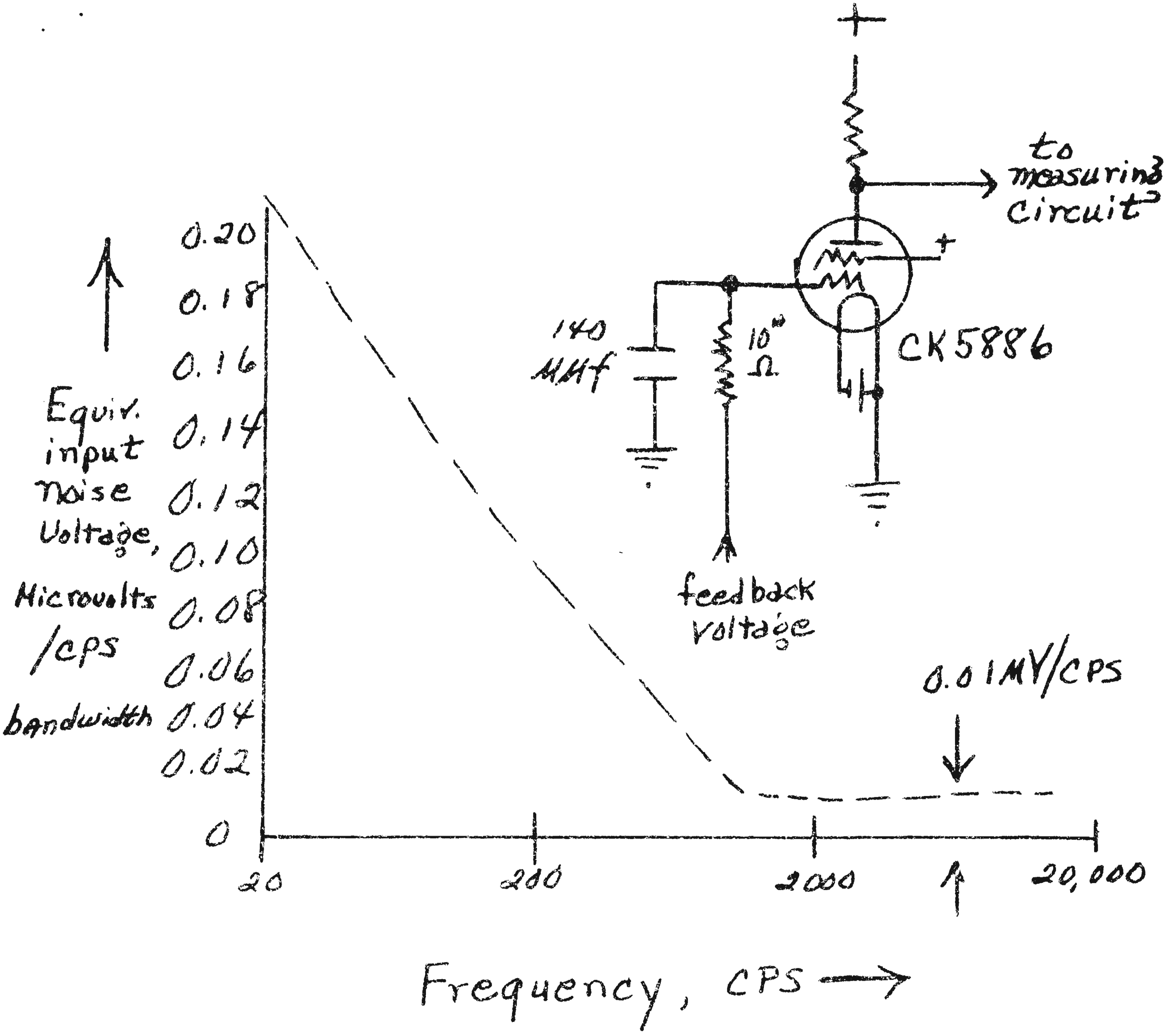

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5a

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PERIPHERAL SPEED $= \omega\rho$	$\omega\rho \simeq$ Mach 1/2

Table 1. Dipole transmitter properties

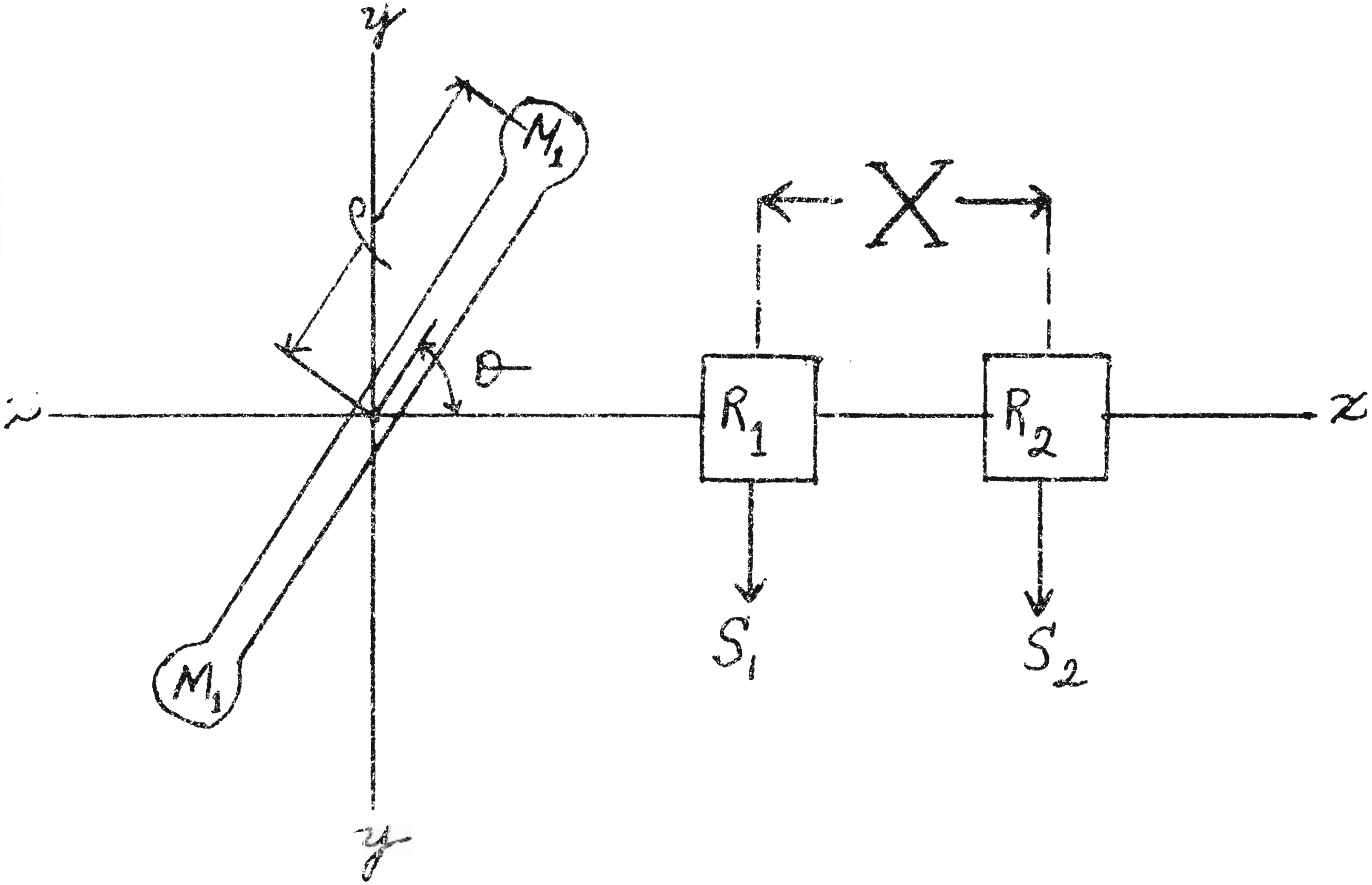


Fig. 3 Proposed experimental layout.

SECTION III

Method for measuring ultra-small phase angles.

Figure 5 is a block diagram.

$\overrightarrow{S_1}$ and $\overrightarrow{S_2}$ are sine wave signals of the same frequency but slightly different in phase and amplitude. The difference $\overrightarrow{D}$ has components D_x and D_y in phase and out of phase with respect to $\overrightarrow{S_2}$. By amplitude modulation of one of the signals, say $\overrightarrow{S_1}$, (with no accompanying phase shift), the signals can be made to differ only in phase. A subtraction will then give a signal whose amplitude is directly proportional to phase difference. In general, the amplitudes will vary with time. In order not to saturate the amplifier K_3 with the amplitude difference signal, its presence is noted as an error signal by the phase detector #1, and the amplitude of S_1 is continuously adjusted so that D_x is small. D_y can be determined in the presence of a reasonable or zero amount of D_x: $\frac{D_y}{|S|}$ is the phase angle sought.

The subtraction is performed by a shielded transformer, as in Figure 6; ordinarily, differences of 10^{-10} volt between $\overrightarrow{S_1}$ and $\overrightarrow{S_2}$ are readily detectible in the subsequent circuits.

Synchronous frequency conversion:

Referring to Figure 2, one sees that some improvement in signal-to-noise ratio would appear at a frequency above 50 cycles/second. There is a useful method of avoiding the excess noise at low frequency without raising the transmitter frequency. That is to replace the D. C. signal on the fixed plates of the receivers R by an audio frequency signal of say 5 KC. The input signal to the electrometer tube then consists of a pair of side bands of frequencies 5 KC $\pm$ 50 cps, with no accompanying carrier. The modulation process is of the form:

$$\text{Sin}\ \omega_1 t \cdot \text{Sin}\ \omega_2 t = \tfrac{1}{2}\,\text{Cos}(\omega_1 t - \omega_2 t) - \tfrac{1}{2}\cos(\omega_1 t + \omega_2 t).$$

The phase is preserved, so that measuring the phase relation between a pair of similar side bands is equivalent to a phase measurement at the frequency W_1 provided that W_2 comes to both receivers from a common source. Frequency conversion of this nature is incorporated in the circuit of Figure 5 where desireable.

The upper and lower sidebands in this example are sufficiently well separated (100cps) so that either may be selected with a filter. (Special forms of phase detectors may use both sidebands.)

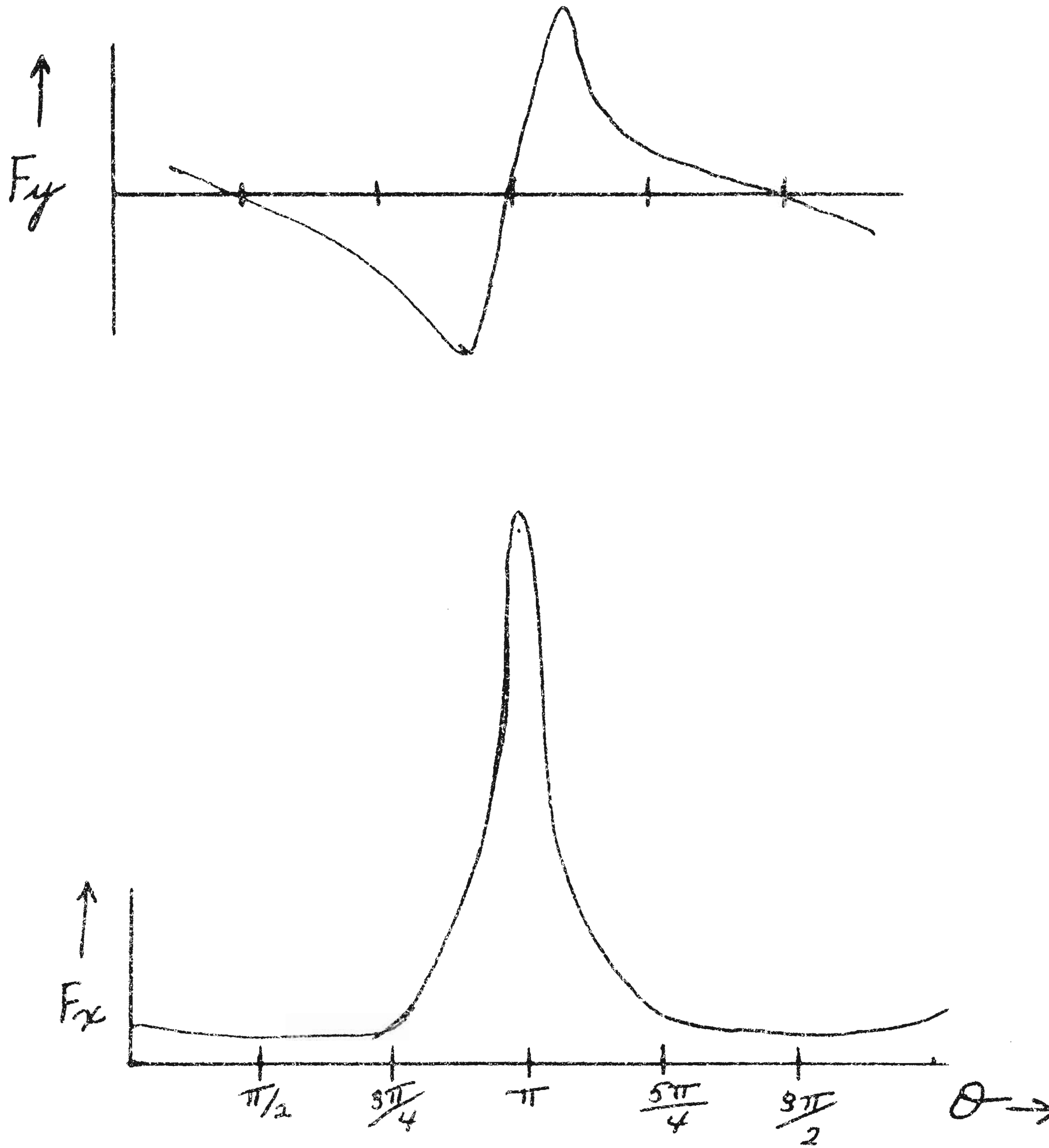
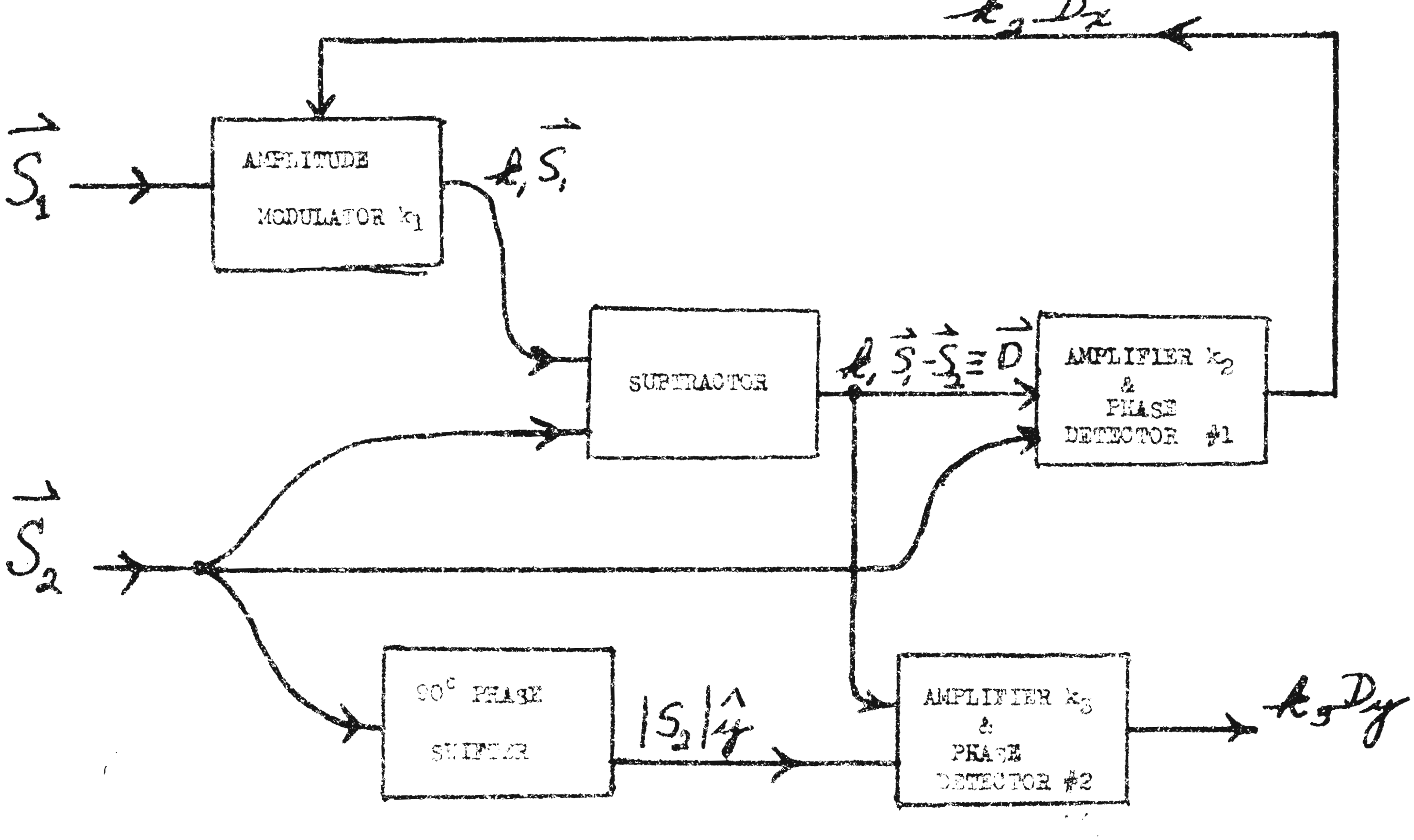
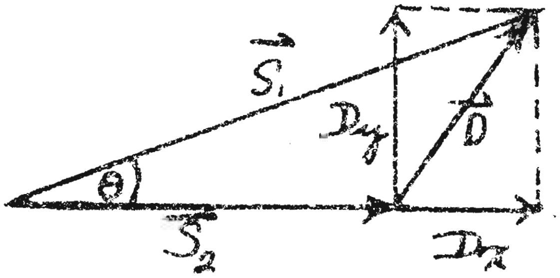


Fig. 4 Form of x and y forces as a function of transmitter angle θ.



Signals S_1 and S_2 represented as vectors.



$$\theta = \frac{D_y}{|S_2|}$$

very nearly, for small .

Fig. 5 Above, block diagram of phase-measuring method. Below, vector representation of signals.

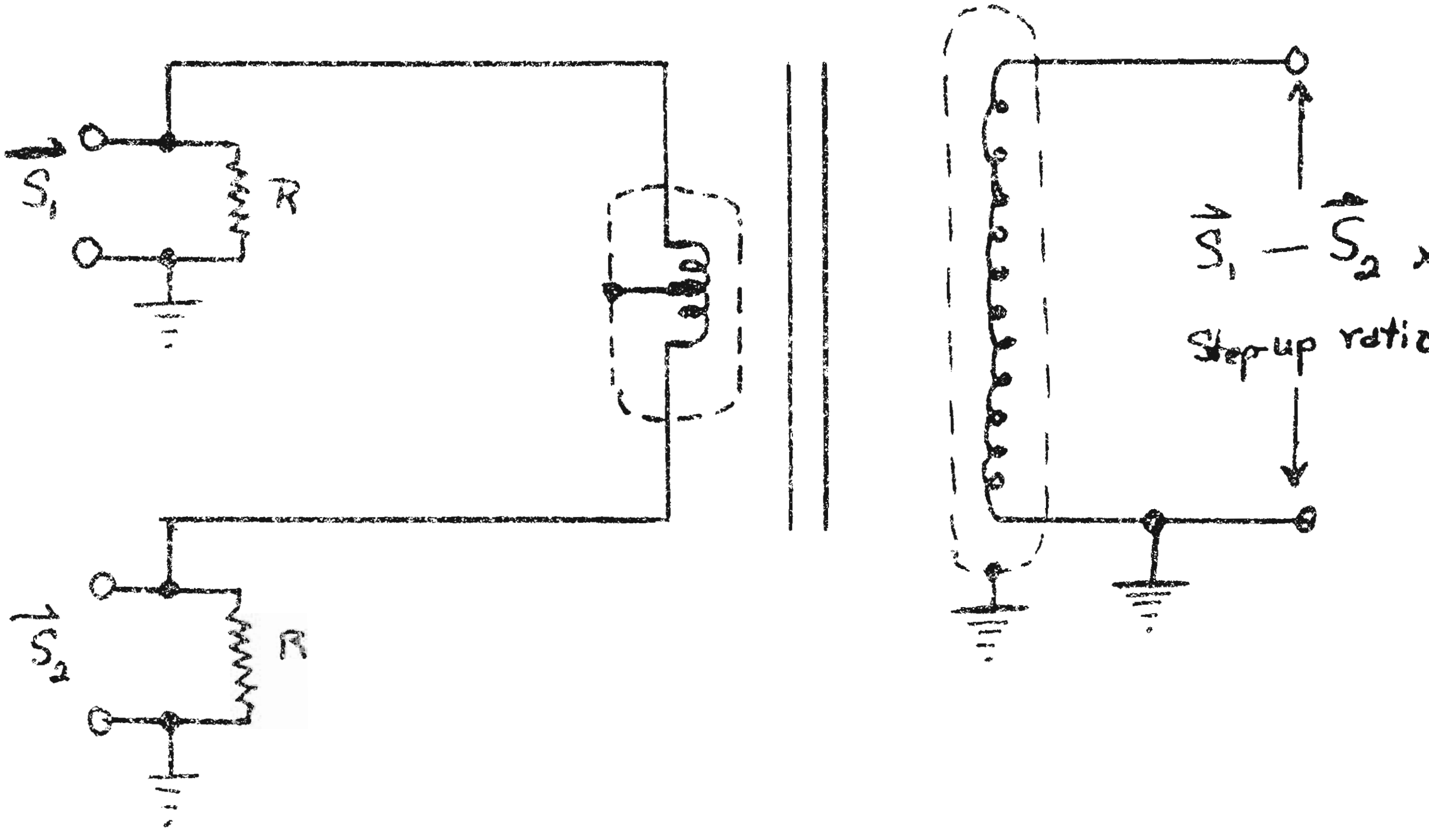


Fig. 6 Electrostatically shielded transformer for measuring very small difference signals.

The electrometer circuit input impedance must be kept very high to avoid loading the high-Q mechanical resonator; reactive loading will shift the resonant frequency and hence the phase; resistive loading will reduce the Q. (6) The input impedance, using appropriate feedback, can be kept suitably high in the frequency range chosen.

SUMMARY

Signals S_1 and S_2 may be expected to have a phase relation which depends on the distance X in Figure 3. It is the phase difference as a function of X which is to be measured The receivers R_1 and R_2 do not operate in the wave zone, but in the induction zone, much less than a wavelength away. The phase detector must thus be capable of measuring a very small phase difference. The time difference may be expected to be of the order of 1×10^{-9} sec. if the receivers are 30 cm. apart and the interaction travels with the velocity of light. This time difference may be measured with an estimated error of 1×10^{-11} second with the circuit of section III. The receivers are self-contained, and can be located as desired to plot the propagation of fields. Interesting variations of the experiment are possible in which the space X is filled with some material. The measurement of the velocit in free space, however, is of first importance; it must be regarded as a milestone in the experimental progress of the science of gravity.

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APPENDIX - SECTION IV

Calibration and testing procedures

Transmitter Frequency Stability

The transmitter is inherently stable over short periods of time. Over longer periods, it will be locked to the frequency of the receivers. It could in fact be locked to an atomic clock, but it is more reasonable to use a receiver as the standard since only relative frequency drift is of great importance.

Transmitter Spurious Radiation

Stray electric and magnetic fields from the transmitter can be looked for with sensitive instrumentation and suitable shields designed and tested. The vibration coupled from the transmitter through its suspension can be monitored continuously by a pickup device like the receiver, but directly attached to the suspension rather than isolated. A magnetic suspension for the transmitter would allow it to choose its own spin axis with the restrictions that the vertical force of gravity be just overcome, and a couple be applied to cause precession of the correct magnitude to follow the earth's rotation. (I ww $\approx$/in-lb).

Magnetic Suspension:

"Magnetic suspension" refers to support of an object against gravity by magnetic forces. Position detectors electronically control the field. The instantaneous equilibrium is not required to be stable; if the system responds in 1 microsecond, the object may fall for part of this time and then be picked up. The resulting displacement, however, is insignificant: $S = \frac{1}{2} g t^2 \approx \frac{1}{2} \times 10^3 \times 10^{-12} = .05\ \AA$.

The restoring forces may be quite non-Hookian if desired, and such a suspension may be given excellent properties as a mechanical filter.

Receiver Response:

The receivers can be tested by exciting them with electromagnetic (light or microwave) radiation chopped at the desired frequency. A radiation pressure of $\sim$ 5 microdynes (5 x 10^{-11} Newton) during the "on" part of the cycle should be exerted on the mass M_2; the required power beamed toward M_2 is $\approx 5 \times 10^{-11}\, c \;=\; 15$ milliwatts. The Q-curves of the receivers can be plotted in this way, and the frequencies of resonance adjusted to be equal. Aging effects on the resonant frequency can be followed over a period of time (1 part in 10^9 / day is anticipated) on a group of several receivers, and a pair having similar drift characteristics be selected for the propagation velocity experiment.

The receiver properties are fundamental to the experiment, and must be measured carefully. A way to select a pair of similar receivers is to excite them from a common source of radiation, and compare the relative phase angle with the device of section III. If a significant phase difference appears when the source-to-receiver distances are equal, a relative adjustment of receiver resonant frequency is required. Near the resonant frequency, there will be a phase shift $\Delta\phi$: in radians,

$$\Delta\phi \approx \frac{Q\pi}{2} \quad \frac{\Delta f}{f_0}$$

where f_0 is the resonant frequency and Δf is the difference between the driving frequency and f_0 . The adjustment is, of course, delicate; this is the price we pay for dealing with a small force signal in the first place.

Receiver Shielding:

The shielding of the receiver for spurious fields can be tested by placing it in much stronger fields than the measured spurious fields of the transmitter, and noting its response. Because of the high Q and the coherent detection scheme, only frequencies in a very narrow band near the operating frequency will cause significant disturbance.

Testing the phase measuring device:

An audio frequency oscillator supplies test signals to simulate S_1 and S_2. By feeding the oscillator signal through different lengths of coaxial cable to the inputs for S_1 and S_2 which contain resistive terminations, a pair of signals having a known phase relation is obtained. Separate noise generators inject noise voltages into the two signals; the noise generators must be separate so the noise is not coherent.

This test has been performed with a simple version of the device. The experimental result is that time differences of 1 x 10^{-11} sec. between audio frequency signals can be detected in the presence of noise.