HOW FAST CAN A BLACK HOLE EAT ?

by

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Summary : We construct stationary spherically symmetric solutions of the equations for accretion of large mass flows onto a black hole, including the interaction of matter and radiation due to Thomson scattering in diffusion approximation. We discuss the relevance of these solutions for a decision of the following question: Does the limitation of the luminosity (Eddington limit) also imply an upper bound to the possible rate of mass flow? The question remains open until all instabilities have been studied. At the moment we still tend to a negative answer.

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A black hole seems to embody the idea of gravity : Nothing will be released, everything is attracted and can be devoured.
A closer look, however, shows matter to behave rather refractorily in front of the unsatiable gullet. It raises all its other properties against gravity to prevent free fall, and in this fervid fight the battle field may shine red-hot or even in X- and Gamma-rays.

Will the escaping radiation throttle the mass flow so efficiently as to limit the rate at which the black hole can eat ?

There are two classes of scenarios for this spectacle : Disc accretion, if angular momentum is a dominant feature, and a more or less spherical pattern, if it is not. [(1 - 2)] In an accretion disc, the inward mass flow is governed by the rate at which angular momentum can be transported outward. This transport seems to be unlimited in principle (though quite limited in practice).
However, the suspicion has been expressed [(3)], that the radiation from the inner parts of the disk will blow away most of the arriving matter and eject it along the rotation axis, if the flow exceeds a critical value. This value was expected to be a small multiple of L_E / c^2 , where L_E is the Eddington luminosity.
This luminosity exerts the same force on the Thomson cross section of an electron in a spherically diverging radiation field, as a proton experiences in the gravitational field of the black hole.
This balance condition gives the value $L_E = 1.2 \cdot 10^{38}\ M/M_\odot$ ergs/sec for a black hole of mass M .

Clearly, in a spherical mass flow, the Eddington limit confines the luminosity : More radiation would overcompensate the gravitational attraction and prevent any stationary flow. With deviations from spherical symmetry the picture becomes more complicated: matter might be swallowed from some directions while radiation might escape and blow away matter in other directions. The Eddington limit might then not only confine the order of magnitude of the luminosity, but also of the mass flow. In the case of the disc this suspicion, if corroborated, could perhaps supply us with a very interesting mechanism for the ejection of relatively stable jets. In the spherical scenario, flickering of the escaping light and irregular ejection of matter would also lead to interesting features of the corresponding objects.

We have set out to study these questions, starting from stationary spherically symmetric accretion flows.

To save space, let us measure all quantities in units of the velocity of light, the Schwarzschild radius $r_s = 2GM/c^2 = 3 \cdot 10^5 \cdot M/M_\odot$ cm, and the Eddington luminosity. Then, e.g., the unit of density is $L_E/r_s^2c^3 = 5.3 \cdot 10^{-5}\ M_\odot/M\ \mathrm{g/cm^3}$, the unit of the rate of mass flow is $L_E/c^2 = 1.4 \cdot 10^{17}\ M/M_\odot\ \mathrm{g/sec} = 2.2 \cdot 10^{-9}\ M$ per year.

If the black hole sits or moves in a medium of density ρ_a, the accretion flow is roughly determined [4-6] by the "accretion velocity" v_a (thermal velocity in the medium, or relative velocity between the hole and the medium, if this is bigger). The "accretion radius" r_a which characterizes the size of the drainage area, is given by the distance at which the free-fall velocity equals v_a.

If we take into account the radiation drag from the luminosity λ, produced in the accretion itself, the attraction force is reduced by a factor $(1-\lambda)$ and we have $r_a \approx (1-\lambda) / v_a^2$. The mass rate will be given by

$$\mu \approx (1-\lambda)\, \varrho_a\, v_a^{-3}$$

(up to numerical factors depending on the detailed picture).

For many years it seems to have been public opinion that spherically symmetric accretion onto a black hole cannot produce much radiation, because matter would fall rather uninhibited.(7-12) Only recently it has been realized that the presence of magnetic fields or turbulence must change this picture drastically(13-15), because such fields cannot fall freely below a certain point. If they are frozen into the matter, they build up more and more pressure against the flow; if they are dissipated they deposit heat and thus radiation density, which will also work against the flow. Up to now, nobody seems to have looked at the consequences of radiation flux diffusion through a dense accretion flow into a black hole. But this can be done quite easily in a reasonable approximation : Consider an accretion flow with velocity v and density ϱ onto a non-rotating black hole, and a radiation flux F with energy density U and pressure $U/3$ which dominate all kinds of internal energy and pressure associated with the matter (like thermal energy and magnetic fields). Forget all spectral details and let matter and radiation interact only by Thomson scattering. Let us also neglect the modifications of geometry near the Schwarzschild radius. (The latter neglection might easily be avoided, but from the experiences with a full relativistic description of accretion flows [8,16,17] we expect little changes : In this respect a static black hole seems very much to behave like a Newtonian singularity!) Finally, we neglect the complications of radiation transport in the transition shell between optically thin and thick regions, and replace this zone by the surface of optical depth $\tau = 2$. Outside we assume free fall (with gravity reduced by $1-\lambda$ as before), inside we use the diffusion approximation. The results will not be sensitive to the choice of the transition surface - which shows that the simplification was reasonable.

We use our adapted units, define $E = (v^2 - 1/r)/2$, and write a prime for differentiation with respect to r. Then the equations for stationary flow read :

$$4\pi r^2 \rho v = \mu \qquad \text{(conservation of matter)}$$

$$F = -U'/6\pi\rho - \frac{4}{3} Uv \qquad \text{(diffusion and convection term!)}$$

$$\rho E' = -U'/3 \qquad \text{(pressure balance)}$$

$$L \equiv 4\pi r^2 F = \mu E + \lambda \qquad \text{(integrated energy balance).}$$

Apart from a missing factor 4/3 in the convection term, these equations have also been used in [18] for accretion onto a neutron star. The basic difference is, of course, that the neutron star fixes the outside luminosity λ, once the mass flow μ is given, because the matter stops, and the full difference of potential energy must appear as luminosity at "infinity".

The black hole does not define such a relation between μ and λ. Hence, for given μ, one can construct solutions of the equations for any luminosity within the range $o < \lambda < 1$. Having assumed values for μ and λ , we determine the boundary conditions at the surface $r = r_o$ where $\tau = 2$ from the reduced free-fall law, and put $U_o = \lambda/4\pi r_o^2$. Then we integrate the equations to smaller radii, using a Runge-Kutta scheme. Results are shown in the figure for a mass rate $\mu = 100$ and various values of the luminosity λ. Besides the flow velocity v there are also shown lines of constant radiation temperature and lines with $r\tau' = \text{const}$, (which allow to calculate the optical depth along each curve), as well as lines with $L/\lambda = \text{const}$, representing the surfaces from below which a certain fraction of the luminosity emerges. The line $L = 0$ corresponds to balance between convection and diffusion; From the definition of L one finds that it lies near $r = \mu/2$. We see that for dense accretion flows (large μ) the "point of no return" for the photons has moved outward from the Schwarzschild radius $r = 1$ to $r = \mu/2$.

For low luminosity the flow is at first supersonic with respect to our sound velocity $\approx (U/3)^{1/2}$. A sort of shock has to be passed, but it doesn't disturb the numerical integration because it is widened by radiation diffusion. (On the other hand, this prevents a simple treatment by junction conditions.) In the solutions with $\lambda \lesssim 0.5$, where a shock appears, it is hidden below the surface with $L = 0$.

For a different mass rate μ , the velocity curves and the lines $L/\lambda = \text{const}$ are simply shifted along the free-fall line (apart from small influences of the outside boundary condition), such that $L = 0$ lies near $r = \mu/2$. (Temperature and optical depth change in a different way!) At $r = r_o$, where $\tau = 2$ (i.e. $r \cdot \tau' = 1$ in reduced free fall), the temperature is

$$T_o \approx 3 \cdot 10^7 \cdot \mu^{-1} \cdot \left(8\lambda(1-\lambda)^2 M_\odot/M \right)^{1/4}$$

The matter in the optically thick region will share the radiation temperature; one finds that the neglection of thermal energy was justified in our solutions.

A clear consequence of this picture is : A large accretion rate excludes high luminosities in X- or Gamma-rays. They can only originate in "critical" or "subcritical" accretion, i.e. with $\mu \leq 1$. But are there perhaps ultraviolet or infrared sources near the Eddington limit which remain to be discovered ? Or does large mass flow never occur in nature ? Or are our solutions unstable ?

Let us speculate a little :

If there is a sort of global spherically symmetric instability (perhaps enhanced by relativistic effects) it would probably lead to collapse. In our picture this must mean: as near to free fall as possible. But as soon as we introduce the dissipation of magnetic fields, luminosity $\lambda = o$ and free fall are excluded. Some higher luminosity is required. (A frozen-in field would enforce a similar velocity law as radiation does at small r - but usually already further outside!) Such an instability might help to select the "true" model from the family of solutions.

The dependence of the accretion radius on λ, both through the reduction in gravity and through heating of the gas, may or may not lead to a stable self regulation.

The deep zone with frozen-in radiation becomes asymptotically adiabatic with $\gamma \rightarrow 4/3$. It seems just to be stable against convection. A formation of channels or bubbles should be difficult due to the high pressure in this region. On the other hand, in the outer shells of our picture, diffusion dominates, and this also seems to have stabilizing rather than destabilizing influence.

Clearly, such hand-waving cannot replace a proper analysis of all sorts of instabilities. We shall try hard, but it may be too difficult or cumbersome for us.

At the moment it seems quite possible that our solutions, or at least some of them, are stable against break-up into threads or lumps of matter and channels or bubbles of radiation. In this case one might find that even a disk can feed a black hole at an arbitrary rate. If only enough matter is brought down to small radii, it might spread itself rather spherically and squash all attempts of the radiation to break through with high temperature. An immense accretion flow would then be detectable only by the soft and cool exhalations from its outskirts. The true battle field, where extremely hot radiation just succumbs to the weight of huge layers of accreted material, would remain hidden.

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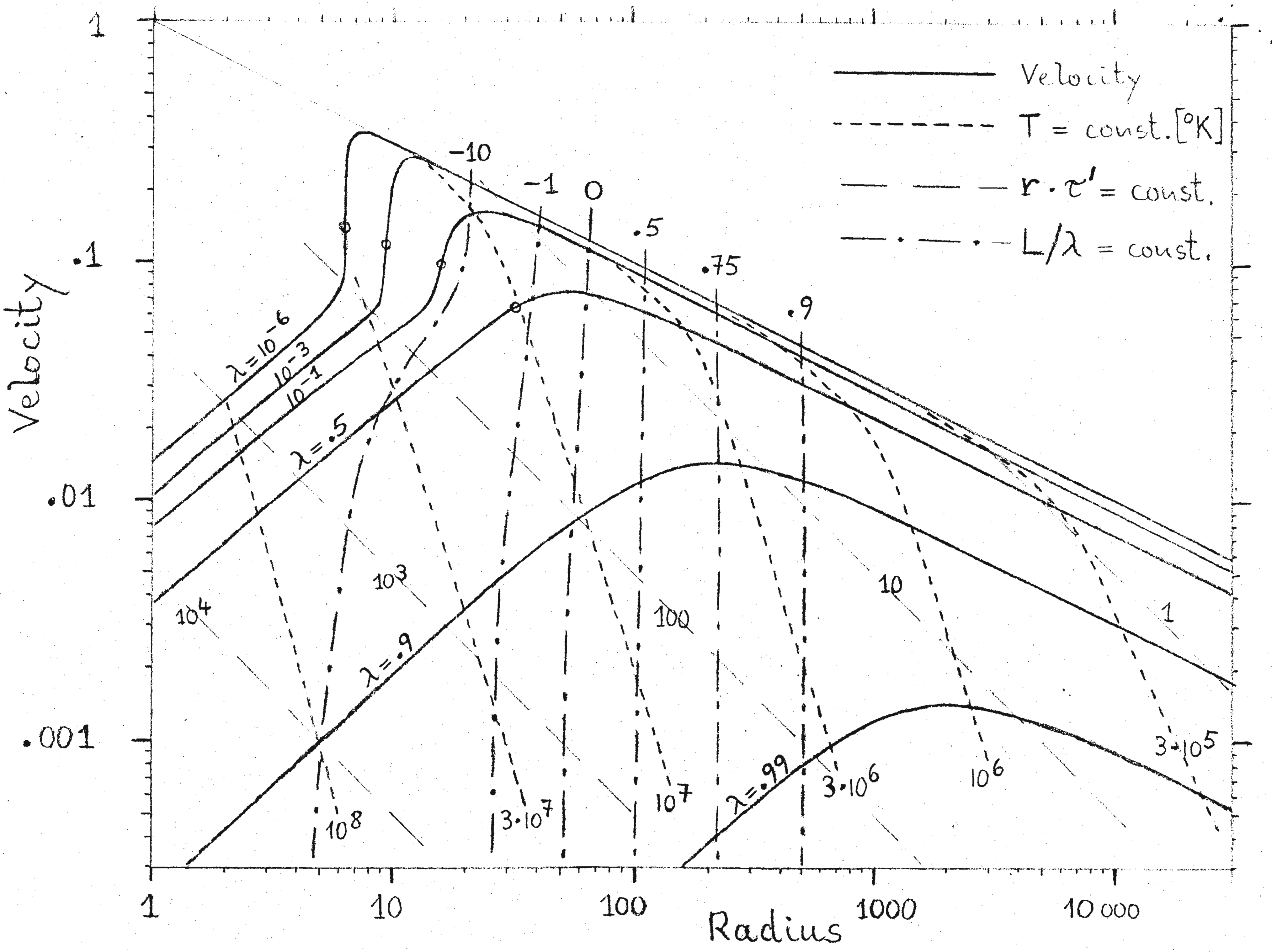

Velocity
T = const. [°K]
r · τ' = const.
L/λ = const.
Velocity
1
.1
.01
.001
Radius
1
10
100
1000
10 000
λ = 10^-6
10^-3
10^-1
λ = .5
λ = .9
λ = .99
-10
-1
0
.5
.75
.9
10^4
10^3
100
10
1
10^8
3·10^7
10^7
3·10^6
10^6
3·10^5

Biographical Sketches.

Peter Kafka was born in 1933 in Berlin, Germany. He studied Physics in Erlangen and Berlin from 1952 to 1957, lived as a casual labourer from 1957 to 1962, and obtained a diplom-degree in Physics at the University of Munich in 1965. He worked there as an assistant for a while, and then joined the Max Planck Institute for Physics and Astrophysics, where he has been working since 1965 in cosmology, gravitational radiation and other problems of relativistic astrophysics. He also took part in the 1972 competition of the Gravity Research Foundation and won the second award for his essay "Are Weber's Pulses Illegal ?".

Peter Mészâros was born in 1943 in Budapest, Hungary. He studied Physics in Buenos Aires from 1962 to 1967 and then did research in Radioastronomy for a year, before going to Berkeley, where he obtained a Ph.D. in Astronomy in 1972. After a year of research on the physics of the interstellar medium at Princeton, cosmology and black hole astrophysics occupied him for two years spent at the Institute of Astronomy in Cambridge, England, and these topics continue to claim his attention at the Max Planck Institute for Physics and Astrophysics, which he joined in 1975.

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