MULTI - BARYONS AND VERY MASSIVE STARS

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The Oppenheimer limit on the mass of a cold star can be exceeded by models in which a small core of steeply decreasing density is surrounded by a large envelope of "ordinary" matter. Such models agree with Einstein's theory of gravity if matter under high pressure consists of heavy particles, formed from baryons by many-body forces and here called multi-baryons. A gravitational collapse, forcing us to abandon the law of baryon conservation, is thus avoided.

1. Gravitational Collapse

If a static stellar model consists mainly of neutrons at vanishing temperature, its total mass M_* cannot exceed

(1) $$\overline{M}_* \approx 0.7 \cdot M_\odot \quad ,$$

where $M_\odot$ is the solar mass. The related values of central density and stellar radius are

(2) $$\varepsilon_0 \approx (m_{nucleon})^4 \quad ,$$

(3) $$r_* \approx 10 \text{ km} \quad .$$

[We employ units defined by $\hbar = c = 1$, hence the density has the dimension of the fourth power of a mass.] For an ε_0 below and above (2), r_* goes above and below (3), but such that M_* decreases in both cases.

All this was found [1] by Oppenheimer and coworkers; hence (1) is called Oppenheimer's mass limit. It was confirmed in extensive computations [2] by Wheeler and coworkers. Since many stars have masses above (1), it seems that they must somehow reduce these when cooling off. It was speculated [3] that stars losing so much mass might explain the quasi stellar radio sources, which radiate enormous energies. Therefore, the "gravitational collapse" was recently [4] much discussed, although its connection to observations [5] could not be established.

2. Massive Stellar Models

According to Wheeler [2], the instability of cold stars with masses above (1) even requires abandoning the conservation of baryons. For explaining this conclusion more directly, Wheeler [6] simplified the derivation of Oppenheimer's limit (1). He remarked that a uniform density ε causes a certain curvature of space which limits the occupied volume and thus the total mass M_*. If $\varkappa$ is the gravitational constant, one gets roughly

$$(4) \qquad M_* < (\varkappa^3 \varepsilon)^{-\frac{1}{2}} .$$

With the "nuclear" density (2), this yields again (1), while higher ε make (4) even smaller.

Although these striking arguments [6] presume a homogeneous density, they can qualitatively be trusted for any star in which ε shows a moderate spatial variation. This is true for all the usual models, in which a polynomial

$$(5) \qquad \varepsilon(r) = \varepsilon_0 + \varepsilon_1 r^2 + \varepsilon_2 r^4 + \dots$$

(of low order) approximates ε even at large distances from $r = 0$. Thus the limit (1) and hence the "collapse" of more massive stars seem inescapable.

However, static models of such stars are nevertheless possible, if their density is approximately [7]

$$(6) \qquad \varepsilon(r) = (\ell/r)^2 \, \varepsilon_* + \varepsilon_s .$$

The constant ε_s denotes the density of ordinary solids (≈ 10 g cm^{-3}), ℓ the "elementary" length of gravity ($\approx 10^{-32}$ cm) and ε_* some high density, up to the "gravitational" unit ($\approx 10^{90}\ \varepsilon_s$). Near the center, (6) is so steeply variable, that Wheeler's theorem (4) is certainly not applicable there. But just this steep decrease makes the superdense core of our model so small that almost the total stellar mass M_* is contained in a large envelope of densities about ε_s. For Wheeler's[6] theorem (4), the whole star can therefore be considered as having $\varepsilon \approx \varepsilon_s$. For its maximal mass $\overline{M}_*$, this rough estimate gives

$$(7) \qquad \overline{M}_* < (\varkappa^3 \varepsilon_s)^{-\frac{1}{2}} \approx 10^6 \cdot M_\odot ,$$

clearly exceeding Oppenheimer's limit (1).

3. The Multi-Baryons

A density variation as steep as (6) contradicts Einstein's gravitational theory, if one assumes as usual[1] that cold matter at any density above (2) consists of simple baryons. In this case, the equation of state at vanishing temperature and high pressure p becomes

$$(8) \qquad \varepsilon = 3p + m^2 \sqrt{p} \qquad \text{with} \qquad m \approx m_{\text{nucleon}} ,$$

making (6) impossible. There is, however, no reason for the stability of single baryons under high pressure. Since a sufficient pressure favors neutrons above their decay products, why cannot much higher pressures produce even heavier particles?

At "ordinary" pressures, such massive states may be highly unstable; and the rates of their formation (cross sections) may be very small. This is irrelevant for our static problem, but would make it difficult to detect those particles directly. One might compare them with many metastable states in chemistry, which are better accessible since the needed pressures (law of mass action) are less than "nuclear" ones.

The unstable states concerned here decay under lower pressure into many baryons; therefore let us call them multi-baryons. Compared with atomic nuclei of corresponding baryon numbers, they must be spatially much smaller because the density even of the whole stellar core highly exceeds that of "nuclear" matter. None of those multi-baryons can be stable, otherwise they might occur in ordinary matter. Some will be electrically charged, but the positive and negative ones must occur in equal densities, because no compensating electrons are present under high pressures. Possibly, the "resonances" observed in the scattering of baryons on baryons [8] already indicate the simplest of our multi-baryons.

4. Many-body Forces

Forces strong enough for producing multi-baryons can be expected from the cyclic exchange of quanta among several particles. In ordinary matter, those "many-body forces" are unimportant because simultaneous close encounters of several particles happen seldom. But they will exceed the usual "two-particle forces" at high densities because they superpose non-linearly. If we assume the average multi-baryon to contain as many single baryons as one finds within the range of that many-body force which is strongest at the density ε , its mass becomes

(9) $$m = \text{const} \cdot m_*^{-3} \cdot \varepsilon \quad ,$$

where m_* is the mass of the exchanged quanta. If this m_* varies like m (the force quanta are the multi-baryons themselves), (9) yields

(10) $$m \approx \gamma \cdot \sqrt[4]{\varepsilon} \quad , \qquad \gamma \approx 1 \quad .$$

Here, the constant γ is such that m becomes the nucleon mass if ε equals the "nuclear" density (2).

Inserting (10) in (8) yields $\varepsilon = \text{const} \cdot p$. Accepting this for all high pressures including that of a neutron gas, one has merely to add the density ε_s of the solid state, which persist at vanishing pressure. Thus assuming

(11) $$\varepsilon = \alpha \cdot p + \varepsilon_s \, , \qquad \alpha = \text{const} > 3 \quad ,$$

one finds [7] that Einstein's equations are solved in good approximation by (6); and also the new limitation (7) of the total mass results again.

5. Difficulties and Refinements

Actually, one should use at low pressures a more detailed [2] equation of state, which requires modifying (6) and hence (7) by numerical computations. When these were performed by Misner and Zapolsky [9], the old Oppenheimer limit (1) was found. But this happened because (11) was used only at densities well above the "nuclear" ones. Just in the decisive region of these densities themselves, those authors [9] assumed the usual neutron gas. Moreover, it is certainly difficult to integrate numerically over densities varying by 10^{90} .

Another difficulty is that Einstein's equations, if solved by (6) down to $r = 0$, imply a definite relation between ε_* and the constant α of (11), thus allowing only one stellar model for a given equation of state. Always regarding this conclusion [9] as unphysical, we formerly stated [7] that the center of the star, where (6) is mathematically singular (though integrable), cannot be treated macroscopically. This is especially true because (10), the average mass of the multi-baryons, grows toward $r = 0$ so strongly that only a few are in a central region of some gravitational units in diameter. This region should be treated by a quantum theory which does not yet exist for such a situation (heavy particles in very strong, continual interaction and under enormous gravity).

Therefore, we better consider another model, which can be treated entirely by macro-physics, though only numerically. Very near the center, we assume the density behavior (5), allowing an arbitrary central density ε_0. If this is chosen high enough, while (11) is the equation of state, the solution starting as (5) will soon go over into (6). As discussed above, this yields a small core of steeply decreasing density and a vast envelope of "ordinary" matter, which contributes almost the total mass M_*. Computations for models of this type are in progress.

6. Quantum Statistics

Being suggested by the forces creating multi-baryons, (10) is also distinguished by simplicity because it does not involve any constant (except $c = \hbar = 1$). Every other relation $m = m(p)$ must, in order to yield (2), at least contain the nucleon mass and the "nuclear" density. In spite of this, (10) might only be a limiting case of possible relations for the average mass of a multi-baryon (a stronger variation leads to difficulties). Actually, this question and that for the equation of state must be treated anew by statistics. It was certainly inconsistent merely to plug (10) into (8), which holds for particles of uniform mass.

From the quantum statistics[10] of relativistic particles, one easily derives[11] the correct formalism for a gas in which none of the particle numbers, but only the total baryon number is conserved (besides energy and momentum). This formalism involves the masses m_i of the particles with any baryon number b and their multiplicities g_i. If one assumes simple power laws for the dependence among m_i, g_i and b, an approximate evaluation shows a weaker increase of m with ε than in (10); but we find[11] again an equation of state like (11). Only this, especially the absence of terms such as the last one of (8), is important for the possibility of very massive stars.

All these details of models with vanishing temperature may find adequate treatment, if one performs the required computations. Before any comparison with observations one should, however, establish models with different temperatures. Thus one must also consider the generation and transformation of those energies, which flow through the star and finally appear as its radiation (from radio waves up to γ - rays).

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