# DEMISE OF THE COSMIC CENSOR?

Viqar Husain

Department of Physics and Astonomy
University of British Columbia
6224 Agricultural Road
Vancouver, BC V6T 1Z1 Canada

**********************************************************************

FIRST AWARD-WINNING ESSAY

Gravity Research Foundation
PO Box 81389
Wellesley Hills, MA 02181-0004
USA

1998

# Demise of the Cosmic Censor?

Viqar Husain*

*Department of Physics and Astronomy,*

*University of British Columbia,*

*6224 Agricultural Road*

*Vancouver, BC V6T1Z1, Canada*

## Abstract

Initial data for general relativity may be such that there are trapped surfaces on a spatial initial data surface. Penrose has proposed that if the cosmic censorship hypothesis is true, the ADM mass $M$ of asymptotically flat initial data and the area $A$ of the outermost apparent horizon surface should satisfy the inequality $A \leq 16\pi M^2$. Initial data which does not satisfy this inequality may be viewed as providing a counterexample of the cosmic censorship conjecture. We describe initial data that appears to violate this inequality.

*Email: husain@physics.ubc.ca

A well known open problem in classical general relativity is the cosmic censorship conjecture. In general terms, this conjecture states that if spacetime singularities arise in a "physical" context, they are always shrouded by event horizons [1,2].

The "physicality" of the context usually takes the form of finite energy-momentum. This restricts consideration to spacetimes which are asymptotically flat at spatial and null infinity, because these boundary conditions allow unambiguous definition of the energy and momentum of the gravitational field. An additional physical condition is that matter should be "resonable." This is taken to mean that matter should satisfy at least the weak, and preferably the dominant energy condition. The last condition requires positive energy density and timelike or lightlike energy fluxes.

An essentially equivalent way of stating the asymptotic conditions is to impose restrictions on the initial value data for spacetime: The initial data on a spatial surface, consisting of the gravitational variables $(q_{ab}, \tilde{\pi}^{ab})$ and matter variables $(\phi, \tilde{P}_\phi)$ should be asymptotically flat. For example, a spherically symmetric shell of electromagnetic matter with flat interior and Schwarzschild exterior would be considered physical. The cosmic censorship conjecture then states that under evolution, such initial data cannot give rise to a naked singularity.

Stated this way, it appears that full time evolution of initial data is required to determine if the cosmic censorship hypothesis holds in any given example. This would make it rather difficult to test cosmic censorship. Luckily however, there is a simpler test due to Penrose, which uses initial data alone [3]. This test is based on a reasonable physical picture.

It is known that trapped surfaces form in regions of sufficiently strong gravitational field. Therefore, as initially diffuse matter collapses under gravity, there is a possibility that trapped surfaces will form as the collapse proceeds. If this happens, then on a sufficiently late time spatial surface, there will be a boundary that separates the trapped region from the normal region. This is the apparent horizon surface. As matter continues to collapse, the area of this boundary grows until a portion, or all of the matter has collapsed. In the long time static or stationary limit, the area of the apparent horizon surface tends to the area of the event horizon. This physical picture suggests Penrose's inequality

$$A(S) \leq A_0 \leq 16\pi M^2, \tag{1}$$

on any spatial slice of the spacetime containing trapped regions; $A(S)$ is the area of the apparent horizon surface, $A_0$ is the area of a surface that bounds $S$, and $M$ is the ADM mass of the initial data.

This inequality has successfully passed many tests [4]. The most general of these is a theorem [5] that rules out data which includes the condition $\tilde{\pi}^{ab} = 0$, known as time-symmetric data. However, there is now an example of naked singularity formation from time-symmetric initial data for the spherically symmetric Einstein-scalar field system [6]. This initial data is finely tuned and appears to be a set of measure zero. In particular, it appears to evade at least the intuition behind the initial data test.

In this essay, we give a class of time-symmetric and asymptotically flat initial data for gravity coupled to a scalar field. We show that there are ranges of parameters in this data for which Penrose's inequality appears to be violated. This suggests that it may constitute a counterexample of the cosmic censorship conjecture.

The initial value constraints for scalar field coupling are

$$\frac{1}{\sqrt{q}}\left(q_{ad}q_{bc} - \frac{1}{2}\,q_{ab}q_{cd}\right)\tilde{\pi}^{ab}\tilde{\pi}^{cd} - \sqrt{q}R + \frac{1}{\sqrt{q}}\tilde{P}^2 + \sqrt{q}q^{ab}\partial_a\phi\partial_b\phi = 0, \tag{2}$$

$$\partial_b\tilde{\pi}^{ab} + \tilde{P}q^{ab}\partial_b\phi = 0, \tag{3}$$

where $(\phi, \tilde{P})$ are the scalar field phase space variables, and $D_a$ is the covaraint derivative satisfying $D_a q_{bc} = 0$.

Consider three-space to be $R^3$. An ansatz leading to a solution of these constraints in spherical symmetry is

$$\tilde{P} = 0, \qquad \tilde{\pi}^{ab} = 0, \qquad q_{ab} = \psi^4(r)\delta_{ab}, \tag{4}$$

where $\delta_{ab} = \mathrm{diag}(1, r^2, r^2\sin^2\theta)$ is the flat metric. The initial scalar field $\phi$ is arbitrary at this stage. This ansatz solves the spatial diffeomorphism constraint. The Hamiltonian constraint becomes

$$\nabla^2\psi + \frac{1}{8}\left(\partial_r\phi\right)^2\psi = 0. \tag{5}$$

Now, for $\phi(r) = \sqrt{2}\, C/r$ where $C$ is a constant, the solution of the Hamiltonian constraint is

$$\psi(r) = A\cos\left(\frac{C}{2r}\right) + B\sin\left(\frac{C}{2r}\right), \tag{6}$$

where $A$ and $B$ are integration constants. This solution gives an asymptotically flat spatial metric. For large $r$, $A$ can be identified as a constant conformal factor, and so can be set to unity. The ADM mass $M$ is determined by both the "geometric" parameter $B$ and the "matter" parameter $C$ via $M = BC$. The next order term $(1/r^2)$ is determined entirely by $C$. The spatial metric has a curvature singularity at $R(r) \equiv r\psi^2(r) = 0$. This two parameter solution is the one we will use to test Penrose's inequality.

In spherical symmetry the apparent horizon equation is $g^{\alpha\beta}\partial_\alpha R\partial_\beta R = 0$, where $g_{\alpha\beta}$ is the spacetime metric. Furthermore, the inequality (1) reduces to the simple form

$$R_{AH} \leq 2M, \tag{7}$$

where $R_{AH}$ denotes the radius of the apparent horizon sphere. For the above time-symmetric ansatz, the apparent horizon equation is

$$\psi + 2r\partial_r\psi = 0. \tag{8}$$

This equation is easy to solve numerically for $\psi$ given by (6). There are an infinite number of positive real solutions because of the sinusoidal behaviour. Thus, there are an infinite number of apparent horizons! For the initial data test, we are interested in the horizon with largest $r$.

By probing the two parameter space $(B, C)$, we find that there appear to be regions where Penrose's inequality is violated. One such region is at and near the point $(0.01, 2.00)$. At this point $2M - R_{AH} = -0.286$. Larger violations of the inequality also occur. For example, at the point $(0.01, 10.0)$, $2M - R_{AH} = -1.43$.

Why is this happening? The scalar field amplitude $C$ plays two roles in this solution: it determines the period of oscillations in $\psi$, and partly determines the ADM mass $M(= BC)$. The first can have a significant affect on the (local) apparent horizons, as may be seen by varying $C$ for fixed $B$. Now, Penrose's inequality may be viewed as providing a "correlation" between local ($R_{AH}$) and global ($M$) quantities. The solution presented here appears to indicate that local oscillations affecting apparent horizon size can become "uncorrelated" with a global quantity such as $M$, which measures an average.

What does this result imply for the cosmic censorship conjecture? In a recent review Wald states [4]: " ... Failure of this inequality in any example would be nearly fatal to cosmic censorship, as only a few small loopholes would remain – such as the possible 'unsuitability' of ... matter, the possibly 'non-generic' nature of the example, and the (very remote) possibility that the black hole does not become asymptotically stationary."

The matter used in the example above is manifestly physical in the sense of energy conditions. Furthermore, the solution forms a significant set which does not appear to be non-generic. Thus, barring the last "very remote" possibility, which can apparently only be tested by full evolution of initial data, this result suggests that the cosmic censorship conjecture may be false.

A numerical evolution could perhaps be used to further test the result. It would require setting up non-singular data. This may be done for the data given here by patching flat space in the inner region from $r = 0$ to some $r = a$. The outcome would undoubtedly be interesting.

*Acknowledgements:* It is a pleasure to thank Ted Jacobson and Bill Unruh for discussions. This work was supported by the Natural Science and Engineering Research Council of Canada.

*Note added:* A few weeks after this essay was submitted, it was pointed out to me by Ted Jacobson that the apparent horizon in the above calculation lies in a region which is disconnected from the asymptotically flat region; ie. the conformal factor $\psi$ goes to zero at a point outside the horizon. Whether this point is a spacetime curvature singularity is not clear, because the scalar field is singular only at $r = 0$. Based on this observation, the answer to the title question, for the specific initial data considered in this essay, is "No."

## REFERENCES


[1] R. Penrose, Revistas del Nuovo Cimento **1**, 252 (1969).

[2] R. Penrose, in *General Relativity, an Einstein Centennary Survey,* ed. S. W. Hawking and W. Israel, Cambridge University Press (Cambridge 1979).

[3] R. Penrose, Ann. N. Y. Acad. Sci. **224**, 125 (1973).

[4] A recent review is given in R. M. Wald, "Gravitational Collapse and Cosmic Censorship," gr-qc/9710068.

[5] P. S. Jang and R. M. Wald, J. Math. Phys. *18*, 41 (1977).

[6] M. Choptuik, Phys. Rev. Lett. **70**, 9 (1993).