On the Gravitational Structure of Elementary Particles

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Gravitational Bags are spherically symmetric solutions of higher dimensional Kaluza Klein (K-K) theories, where the compact dimensions become very large near the center of the geometry, although they are small elsewhere. K-K excitations become therefore very light when located near the center of this geometry and this appears to affect drastically the naive tower of masses spectrum of K-K theories. In the context of string theories, string excitations can be enclosed by Gravitational Bags, making them not only lighter, but also localized as observed by somebody that does not probe the central regions. Strings however can still have divergent sizes, as quantum mechanics seems to demand, since the extra dimensions blow up at the center of the geometry.

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Normally, when analyzing the physical spectrum of higher dimensional Kaluza Klein theories[1] and of String theories[2], one expands the fields of the theory arround fixed metric and dilaton backgrounds.

One can look for example at the simplest 5-dimensional Kaluza Klein theory, where one dimension is compact, and study the spectrum of matter fields in the compactified background g_{AB}, A,B= 0,1,2,3,5 given by:

$$g_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1) \text{ for } \mu,\nu=0,1,2,3 \qquad (1a)$$

$$g_{55}=(L_5)^2,\ g_{\mu 5}= 0 \text{ , and } 0\le x^5 \le 2\pi, \text{ with 0 and } 2\pi \text{ identified} \qquad (1b)$$

We find that a massless field in 5-dimensions $\psi(x^\mu,x^5)$, periodic in x^5, satisfying $\Box\psi=\left(\eta^{\mu\nu}\partial_\mu\partial_\nu+g^{55}(\partial_5)^2\right)\psi=0$, obeys the mode expansion

$$\psi= \Sigma\, \psi_n(x^\mu)\exp(inx^5),\ n= -\infty, +\infty \qquad (2)$$

where $\psi_n(x^\mu)$ satisfies

$$\left(\eta^{\mu\nu}\partial_\mu\partial_\nu+(m_n)^2\right)\psi_n =0,\ m_n=\left(n/L_5\right) \qquad (3)$$

That is, from a 4-d point of view we get a spectrum of masses instead of a single mass. This spectrum consists of an infinite tower of masses, the spacing between the masses being constant and equal to $1/L_5$.

In the previous analysis, we have assumed that the field ψ is a test field, and therefore the effect it has on the background metric has been neglected. It is precisely the assumption that the back-reaction of the K-K excitations on the metric is small what we wish to examine in more detail.

While it may be natural to consider a background like (1) as a good ground state of the theory, it may be that for a localized Kaluza Klein excitation, as we get closer to the region where the excitation lives, the size of the extra dimension increases so that the K-K excitation becomes lighter. This may be a way of decreasing the energy of these excitations, but this fact is non trivial, because making the size of the extra dimension bigger near the excitation may in principle also cost a lot of energy, since the size of the extra dimension can be regarded as a scalar field and making the size of the extra dimension very big in a small region, while making sure that the size of the extra dimension gets very small (approaching its vacuum value) as we go away from the K–K excitation, implies a big gradient for the scalar field, and this gradient costs energy.

It is easy to see that it is indeed energetically favored to have the size of the extra dimensions become very large near the region where a Kaluza Klein excitation with high quantum numbers lives, although the size of the compact dimensions can approach its vacuum value, which is very small, very quickly as we move away from the region where the excitation lives. This is a consequence of the fact that having a very high and even infinite gradients for the scalar field associated with the size of the extra dimensions, does not cost much energy (in Planck units). In fact, finite energy solutions where the extra dimensions are arbitrarily large at the center of the geometry exist[3],[4].

To illustrate this point, consider the the simplest 5–D, K–K theory, i.e. 5–D Einstein's equations (for that see first two papers in Ref. 3), which for 3–D spherical symmetry has as a solution:

$$ds^2=-A^2dt^2+B^2(dx^2+dy^2+dz^2)+C^2(dx^5)^2 \tag{4a}$$

$$A(r)=\Big((ar-1)/(ar+1)\Big)^{\varepsilon\kappa},\ B=\Big(1/a^2r^2\Big)\{(ar+1)^{\varepsilon(\kappa-1)+1}\}/\{(ar+1)^{\varepsilon(\kappa-1)-1}\},$$

$$C=C_0\Big((ar+1)/(ar-1)\Big)^{\varepsilon},\ \text{where } r=\Big(x^2+y^2+z^2\Big)^{1/2} \tag{4b}$$

and the constraint $\varepsilon^2(\kappa^2-\kappa+1)=1$ must be obeyed. Furthermore, we choose to compactify x^5, so that $0\leq x^5\leq 2\pi$, with 0 and 2π identified.

Notice that for $\varepsilon>0$, the size of the extra dimension, $2\pi C$, explodes at $r=1/a$. We can see that $r=1/a$ corresponds to the center of the geometry, since as we let r approach the value $1/a$ from above, B^2 , i.e. the coefficient of $d\Omega^2=d\theta^2+\sin^2\theta d\phi^2$, if we were to use angular variables, goes to zero.

As we let r approach infinity, A^2 approaches the value $1-(4\varepsilon\kappa)/ar$, which implies that these are finite mass solutions, with mass $M=2\varepsilon\kappa/a$, even though there is an infinite gradient in the size of the extra dimensions at the center of the geometry, which shows that infinite gradients for the size of the extra dimensions may not cost an infinite amount of energy, an essential point for our analysis.

This unexpected phenomenon is due to the existence of the gravitational field, i.e. of the curvature of space−time. In flat space−time, the so called Derrik's theorem [5] prevents us, in dimensions higher than 1+1 from having purely scalar solitons.

In particular, such objects where the extra dimensions become very big at the center of the geometry, can however be quite small for an outside observer [4]. In the cases where this happens, the scalar field associated with the size of the extra dimensions can exist in two phases: massive and massless, the massive phase associated with small perturbations arround a compactified stable vacuum, while the massless phase can be matched to this massive phase through a spherical domain wall. In the inside of this spherical domain wall, where the massless phase lives, the size of the extra dimensions can approach arbitrarily large values, we called these compact (as observed from the outside) objects [4] "Gravitational Bags". These two phases that a Gravitational Bag matches can be interpreted as a perturbative one (inside) and a non perturbative one (outside) since the Kaluza Klein couplings, which are inversely proportional to the size of the extra dimensions are very small inside and much bigger outside. An explicit mechanism, using a generalization of the Freund Rubin mechanism, for having this two phase structure for the scalar field associated with the size of the extra dimensions has been given in Ref. 4, but, if certain conjectures concerning the dilaton potential[6] are proven right, we would have Gravitational Bags using the dilaton field in String theories also.

It is easy to see that (4) is a solution of 5−dimensional Einstein equations everywhere except at the center of the geometry, where we have a singularity. There one must have a source with $T_{55}\neq 0$, and $T_{\mu 5}=0$ i.e., with isotropic motion in the extra dimension. It is therefore consistent to say that it is motion in the extra dimensions what causes the growth of the extra dimensions. Also the analysis of the energy of matter excitations in the WKB approximation proves that the K−K heavy modes become much lighter when considered in the background (4) than when considered in the background (1), which also provides an energy argument of why the extra dimensions must become very large near a K−K heavy mode, it is this enormous energy reduction in the K−K heavy modes what allows Gravitational Bags to have large entropy contents at small energy costs[7]. In the context of String theories, the dilaton field can become very large near a string excitation, and the result of this is that the string excitation becomes much ligther. This is because the mass of a string excitation depends on the local value of the dilaton field [8], like particles in a Brans Dicke theory, therefore string excitations can become very light provided the expectation value of the dilaton field grows very much near the region where the string excitation lives.

We now discuss how Gravitational Bags can play a role in the solution of some fundamental problems of string theory. The first of these puzzles concerns the apparent failure of string theories to generate particles with properties similar to those of ordinary field theories, in particular, as several authors have recently observed[9],[10], strings appear always to have divergent sizes due to quantum fluctuations. Any attempt to construct string wave functions where the r.m.s. size of the string is not divergent, seems always to lead to a state of infinite energy[10]. From these studies, one would apparently conclude that finite energy string excitations do not look like localized particles under any circumstance.

Our suggestion is that one can avoid this pessimistic conclusion, provided string excitations live inside Gravitational Bags. This is because inside a Gravitational Bag, the string has plenty of room to move, since the extra dimensions are of infinite size at the center of the geometry, so strings are certainly allowed to have infinite sizes while confined to the interior of one of these bags. On the other hand, Gravitational Bags do look compact to an observer that does not probe the central region, and also its 4−d projection is certainly compact, so in this case, string excitations would be localizable, at least in a 4−dimensional projected sense, or likewise, for an observer that does not penetrate the domain wall that separates the massless (inside) from massive phases of the scalar field.

This picture appears also self consistent: not only Gravitational Bags help strings to have localized excitations, also strings help Gravitational Bags, since the quantum fluctuations of the string in the center of the bag, where the extra dimensions are infinite, should provide the necessary source at the center of the geometry, which as we mentioned before, must consist of matter that has motion in the extra dimensions. Furthermore, the string excitation in the center of the Gravitational Bag may prove (for the lowest energy excitations) important for the stability of the Gravitational Bags, since the infinite fluctuations in the center of the bag should be something that we cannot get rid of (because of quantum mechanics), and so would be the source (matter with motion in the extra dimensions) that causes the extra dimensions to be very big at the center of the geometry.

Finally, there is yet another puzzle that the model of string excitations living inside a Gravitational Bag appears to solve. This concerns the observation by Casher[11], that although, as Refs. 9 and 10 show, r.m.s. string lengths are infinite, by appropiately defining coordinates corresponding to the center of mass of string bits, these are always inside their 4-dimensional (i.e. in a suitable projection) Schwarzschild radii. One then has the potential of finding several problems with macroscopic causality concerning the prediction of the decay of heavy string excitations. These problems do not show up however if strings live inside Gravitational Bags, because Gravitational Bags are horizon free objects, even if they are much smaller than their Schwarzschild radii (in a projection on to 4- dimensions), and in fact whatever their (projected 4-D) size is.

In a separate paper[12], we will study the issues discussed in this essay in greater detail.

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References:

1. Th.Kaluza, Sitzungsber, Preuss. Akad. Wiss. Berlin, Math. Phys. K1 (1921) 966; O.Klein, Z.Phys. 37(1926) 895.

2. M.B.Green and J.H.Schwarz, Phys. Lett. 149B, 117(1984); E.Witten, ibid. 149B, 351(1984); D.J.Gross, J.Harvey, E.Martinec and R.Rohm, Phys. Rev. Lett. 54, 502 (1985).

3. A.Chodos and S.Detweiler, Gen. Rel. 14, 879 (1982); A.Davidson and D.Owen, Phys. Lett. 155B, 247 (1985); M.Yoshimura, Phys. Rev.D34, 1021 (1986).

4. A.Davidson and E.Guendelman, Phys. Rev. D34, 3264 (1986); A.Davidson and E.Guendelman, Nucl. Phys. B313 , 693 (1989).

5. G.Derrik, J.Math.Phys. 5, 1252 (1964); R.Hobart, Proc. Phys. Soc. Lond. 82, 201 (1963).

6. M.Dine and N.Seiberg, Phys. Lett. 162B, 299 (1985).

7. E.Guendelman and D.Owen, Phys. Lett. 201B, 297 (1988).

8. J.Sherk and J.H.Schwarz, Nucl. Phys. B81, 191 (1974).

9. A.Casher, E.G.Floratos and N.C.Tsamis, Phys. Lett. 199B, 377 (1987).

10. M.Karliner, I.Klebanov and L.Susskind, Int. Journ. Mod. Phys. A 3, 1981 (1988).

11. Casher, Phys. Lett. 195B, 50 (1987), see appendix section.

12. E.I.Guendelman, 'Gravitational Bag effects on Kaluza Klein and String excitations', **LA-UR-89-426**, to appear in the Int. Journ. Mod. Phys. A.