INFLATION -
AN ALTERNATIVE TO THE SINGULAR BIG BANG

Dalia S. Goldwirth* **and Tsvi Piran†**

Racah Institute for Physics

The Hebrew University, Jerusalem 91904, ISRAEL

Abstract

Domains larger than the horizon in which $\Phi >$ (a few) $\times M_{pl}$ are required for the onset of inflation. Two different, equally plausible, arguments lead us to opposite conclusions about the feasibility of the existence of such regions. It seems that inflation does not free us completely from the need for special initial conditions. However, Linde [1] has pointed out that inflation can be eternal. He stresses the fact that inflation will never cease, but this also means that it did not necessarily have a beginning. We argue that this is the simplest solution to the initial value problem and that inflation might not only solve the problems of the Big Bang model, it might also provide us with an alternative that will replace it altogether.

* **Electronic mail: Dalia@hujivms**
† **Electronic mail: Tsvi@hujivms**

The discovery of the microwave background radiation and the amazing success of cosmological nucleosynthesis consolidated our belief in the hot Big Bang theory. According to the classical version of this theory, the Universe began, some twenty billion years ago, with a singularity which gave rise to a hot Friedmann Universe which has been expanding and cooling ever since. It is generally accepted that the initial classical singularity should be superseded by a Quantum era (taking place at $t < t_{pl}$) during which the rules of Quantum Gravity or some more fundamental theory replace those of Classical General Relativity.

The standard big bang model faces two well known problems - the horizon problem and the flatness problem. Both problems can be considered not to be problems if we accept the idea of very special initial conditions. We can simply declare that there is no problem - God created a Friedmann Universe with extremely small initial curvature and acausal homogeneity. Dicke and Peebles [2] have pointed out that to a physicist this should seem quite unnatural. It would be much nicer if a physical mechanism, rather than ad hoc initial conditions, will lead to the observed Universe. Such a mechanism is inflation [3].

For inflation we need a slowly varying scalar field ($\dot{\Phi}/\Phi < \dot{R}/R$), whose potential dominates the energy density of the Universe:

$$\rho_{total} \approx \rho_\Phi = \frac{\dot{\Phi}^2}{2} + \frac{(\nabla\Phi)^2}{2R^2} + V(\Phi) \approx V(\Phi) \quad .$$

The potential acts as an effective cosmological constant giving rise to a de Sitter phase - commonly called inflation. During this phase the scale factor of the Universe, R, increases exponentially, making the curvature term negligibly small and the horizon exponentially large.

There are two main variants of inflation: "chaotic inflation" [4], with $V(\Phi) = \lambda\Phi^n$, and "new inflation" [5]. In the sequel we discuss only "chaotic inflation" since the problems that we raise are only exasperated in the "new inflation" scenario. For simplicity we use an $n = 2$ potential, for which $\lambda = m^2/2$ (m is the mass of the scalar field). None of our arguments depends on this specific choice.

Inflation solves both the horizon and the flatness problems - but does the inflationary paradigm live to its promise to free cosmology from the worry about initial conditions? We know that inflation takes place when initially $\Phi_i >$ (a few) $\times M_{pl}$ and when the potential dominates the energy density $\rho_{total} \approx \rho_\Phi \approx V$. In general the kinetic term $\dot{\Phi}^2/2$ and the gradient term $(\nabla\Phi)^2/2R^2$ also contribute to ρ_Φ. The central question is: what happens under initial conditions when these other terms are comparable or larger than the potential term?

The effect of the kinetic term has been discussed quite extensively [6]. By now it is well known that inflation will develop even when the kinetic term dominates the energy density provided that initially $\Phi_i >$ (a few) $\times M_{pl}$. In this case Φ decreases logarithmicaly with t whereas $\dot{\Phi} \propto 1/t$. The kinetic term disappears rapidly and inflation begins.

Until recently the onset of inflation was tested only under homogeneous initial conditions and it was not clear what are the possible consequences of a gradient term. However, to justify its claim to fame, inflation should be able to emerge from truly generic inhomogeneous initial conditions. To explore the full effect of initial inhomogeneities we must turn to numerical calculations [7]. A numerical solution of a spherically inhomogenous Universe with rapid variations of the scalar field is shown in Fig. 1a. We see that inflation occurs even in the presence of large gradients, provided that they are superimposed on a large average scalar field ($\bar{\Phi} >$ (a few) $\times M_{pl}$).

The situation becomes more complicated and more interesting when we consider a different inhomogeneous configuration in which at some point, say the origin, $\bar{\Phi} \gg$ (a few) $\times M_{pl}$ while in other regions it is not so large [8]. Figs. 1b and 1c display the evolution of two almost similar Universes that differ in the width, $R\Delta$, of the "effectively homogeneous" region over which Φ is above some critical value. When this region in large the Universe inflates (Fig. 1b) it does not inflate when this region is narrower (Fig.1c). Generally the question whether inflation commences or not depends on the ratio between

$R\Delta$ and the horizon size H^{-1}. Fig. 2 displays the expansion at the origin as a function of $R\Delta/H^{-1}$. We see that inflation does not begin unless the scalar field is higher than (a few) $\times M_{pl}$ across several (at least 2) horizons.

Clearly, inflation solves the horizon problem by many orders of magnitude and the initial conditions for inflation are much more general than those required for a Friedmann Universe. But there still remains a problem with initial conditions: *Is it reasonable to expect that regions of several horizons over which the average scalar field will have a large value, appropriate for inflation, will exist in the pre-inflationary era?*

Suppose that at the end of the quantum era (when $R \approx M_{pl}^{-1}$ and $\rho \approx M_{pl}^4$) the energy of the scalar field is distributed equally between the kinetic, the gradient and the potential terms:

$$\frac{\dot{\Phi}^2}{2} \approx \frac{\delta\Phi^2}{2R^2\Delta^2} \approx \frac{m^2\Phi^2}{2} \approx M_{pl}^4 \quad .$$

The scalar field varies with a typical wavelength $R\Delta \approx M_{pl}^{-1}$ and amplitude $\delta\Phi \approx M_{pl}$. $\delta\Phi$ is much smaller than the average value of the scalar field, $\bar{\Phi}$. To see this recall that the quantum fluctuation constraint [9] $\delta\Phi/\Phi \approx H/2\pi \ll 10^{-4}$ limits the coupling constant of the scalar field: $m \ll M_{pl}$. The scalar field must have a very large average value, $\bar{\Phi} \approx M_{pl}^2/m \gg M_{pl}$ in order that the potential term will be in equipartition with the kinetic and gradient terms in spite of its small coupling constant. Since $\Phi \gg \delta\Phi$, large regions with $\bar{\Phi} \gg$ (a few) $\times M_{pl}$ will exist.

Our conclusion can be drastically different if we assume that the scalar field Φ emerges from the quantum era in a thermal equilibrium (with $T \approx M_{pl}$) (it has been argued [10] that a weakly coupled scalar field does not have enough time to termalize during the quantum era, but other workers [11] assert that the scalar field is in a thermal equilibrium during the whole quantum phase). In this case:

$$\frac{\dot{\Phi}^2}{2} \approx \frac{\delta\Phi^2}{2R^2\Delta^2} \approx M_{pl}^4 \quad ,$$

and $\delta\Phi \approx M_{pl}$. But the potential energy is, in this case, much lower than the kinetic energy:

$$\frac{m^2\bar{\Phi}^2}{2} \approx \frac{m^2}{T^2} M_{pl}^4 \quad ,$$

and $\bar{\Phi} \approx M_{pl}$. $\bar{\Phi} \approx \delta\Phi$ for a thermal field at $T \gg m$ and we do not expect to find the required large regions with $\bar{\Phi} \gg$ (a few) $\times M_{pl}$.

Two different, but plausible, arguments have led us to opposite conclusions. We are faced once more with the question of initial conditions. It seems that we must understand better the conditions at the end of the pre-inflationary era, in order to know whether inflation did take place in our Universe. We can turn to quantum gravity and hope that quantum processes favor configurations in which domains larger than the horizon with $\bar{\Phi} >$ (a few) $\times M_{pl}$ appear at the end of the quantum era. At this stage practically nothing is known about inhomogeneous quantum gravity. Lacking a clear theory the statement - "quantum gravity will provides the necessary initial conditions for inflation" seems to be only an "idea for an idea" which, unfortunately, cannot be pursued further today.

The uncertainty about the initial conditions turns our attention to an attractive alternative - perhaps there were none? Linde [1] has recently pointed out that *eternal inflation* can take place under relatively simple conditions. Consider an inflating Universe. Classically, Φ decreases, in one e-folding time H^{-1}, by:

$$d\Phi_{classical} \approx \frac{M_{pl}^2}{\Phi} \quad .$$

At the same time Φ undergoes quantum fluctuations of the order

$$d\Phi_{quantum} \approx \frac{m\Phi}{M_{pl}} \quad .$$

Evidently, if Φ is large enough, (more specifically if $\Phi > M_{pl}\sqrt{M_{pl}/m}$), these quantum fluctuations are larger than the classical change. The combined effect is a stochastic random walk with an average decrease in Φ. In some regions Φ will decrease but in others it will increase by $d\Phi_{quantum} - d\Phi_{classical}$. The latter regions will expand faster than the former and will contain a larger volume of the Universe. This process repeats itself again and again every e-folding time. It is stochastic and it is impossible to predict where Φ will increase but the overall effect is clear - some regions of the Universe will inflate for ever. Inflation can be eternal.

Once Φ drops somewhere below $M_{pl}\sqrt{M_{pl}/m}$ the classical motion takes over, Φ decreases and this region eventually exits inflation and emerge as Friedmann Universes like the one in which we live. It seems that eternal inflation provides us with a grand steady state cosmology, in which quantum fluctuations overcome the classical evolution to maintain a stochastic kernel of an inflating Meta-Universe that keeps forever forming domains which exit inflation, one of which is our Universe.

The most remarkable feature of eternal inflation is that it can be eternal on both "ends". It does not halt, but there is no need to turn it on either. One can, out of conceptual inertia, assume that the eternally inflating Universe had a pre-inflationary epoch (with an initial singularity or a quantum era) and immediately be faced with the problem that we have just encountered: how did it start? However, this is not necessary. One can just as well assume that there was no beginning (and that there will be no ultimate end) and that we live in a Universe which is a minuscule part of a steady state eternally inflating Meta-Universe.

This seems like a drastic proposal - but for the time being, and most likely for a very long time in the future, there does not seem to be a single observational clue - or even an idea for one - which will enable us to distinguish between a Universe which began with a bang (and has undergone an inflationary phase later) and one which is a part of an ever inflating Meta-Universe. The fact that there is no present observational distinction between these two options is not necessarily a virtue: it be nice to be able to test this radical proposal. But this also means that eternal inflation without a beginning cannot be ruled out right away. At least for the time being it should be taken as seriously as the (by now more conventional) initial singularity or initial quantum era proposal. Since the assumption of no initial conditions seems to be the simplest one, Okham's razor will tell us to prefer it and to conclude that we live in a tiny part of a steady state inflating Meta-Universe that has existed and will exist forever.

Acknowledgment

It is a pleasure to acknowledge helpful discussions with A. D. Linde. This research was supported by a grant from the US-Israel Binational Science Foundation to the Hebrew University.

REFERENCES

1. A.D. Linde, Phys. Lett. **175B**, 395 (1986). A.D. Linde, "Inflation and Quantum Cosmology", Acad. Press, Boston, (1990). A.D. Linde, "Particle Physics and Inflationary Cosmology", Gordon and Breach, N.Y., (1990).

2. R.H. Dicke and P.J.E. Peebles, in General Relativity: An Einstein Centenary Survey, eds. S. Hawking and W. Israel. Cambridge: Cambridge University Press.

3. R. Brout, F. Englert and E. Gunzing, in Gravitational Award First Prize Essay, (1978); Gen. Relativ. Gravit. **10**, 1 (1979); A.H. Guth, Phys. Rev. **D23**, 347 (1981).

4. A.D. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

5. A.D. Linde, Phys. Lett. **129B**, 177 (1983).

6. V.A. Belinsky,L.P. Grishchuk,I.M. Khalatnikov and Ya.B. Zel'dovich, Phys. Lett. **155B**, 232 (1985); A.D. Linde, Phys. Lett. **162B**, 281 (1985); T. Piran, and R.M. Williams, Phys. Lett. **163B**, 331 (1985); T. Piran, Phys. Lett. **181B**, 238 (1986); V.A. Belinsky, H. Ishihara, I.M. Khalatnikov, and H. Sato, Prog. Theo. Phys. **79**, 676 (1988).

7. D.S. Goldwirth and T. Piran, Phys. Rev. **D40**, 3263 (1989).

8. D.S. Goldwirth and T. Piran, Phys. Rev. Lett. in press (1990).

9. V.F. Mukhanov and G.V. Chibisov, **JETP** Lett. **33**, 523 (1981); S.W. Hawking, Phys. Lett. **115B**, 339 (1982); A.A. Starobinsky, Phys. Lett. **117B**, 175 (1982); A.H. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); J. Bardeen, P.J. Steinhardt and M. Turner, Phys. Rev. **D28**, 679 (1983); R. Brandenberger, Rev. Mod. Phys. **57**, 1 (1985); V.F. Mukhanov, **JEPT** Lett. **41**, 493 (1985); V.F. Mukhanov, L.A. Kofman and D.Yu. Pogosyan, Phys. Lett. **193B**,427 (1987).

10. A.D. Linde, Phys. Lett. **132B**, 317 (1983); A.D. Linde, Phys. Lett. **162B**, 281 (1985); G.F. Mazenko, W.G. Unruh, and R.M. Wald, Phys. Rev. **D31**, 273 (1985).

11. R. Brout, G. Horwitz and D. Weil, Phys. Lett. **B192**, 318 (1987).

FIGURE CAPTIONS

Fig. 1: The scale factor, R (left) and the scalar field, Φ (right) as a function of the radial coordinate χ for deferent times. In all cases the solid line describes the initial data.

1a: Large gradients on top of a large $\bar{\Phi}$ - the fluctuations in the scalar field decay and inflation starts.

1b: Gaussian with $R\Delta/H^{-1} = 4$, inflation begins at the origin but not at the exterior region.

1c: Gaussian with $R\Delta/H^{-1} = .87$ inflation does not appear anywhere.

Fig. 2: The scale factor at the origin at the end of the computation as a function of the the proper width of the initial gaussian relative to the horizon size for three different families of initial data and different potentials.

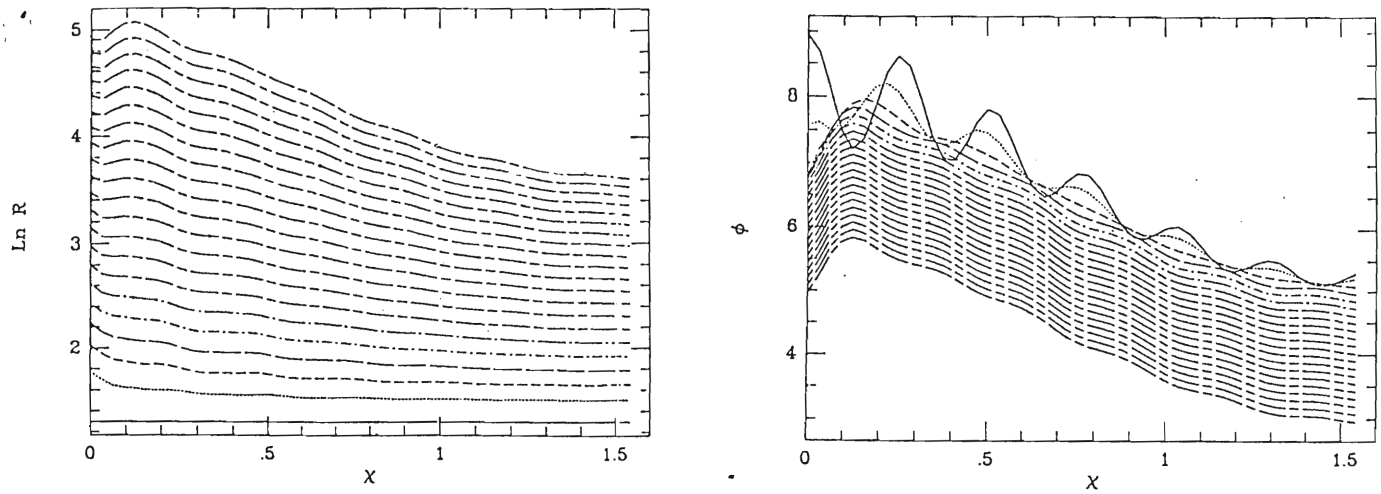

Fig. 1.a

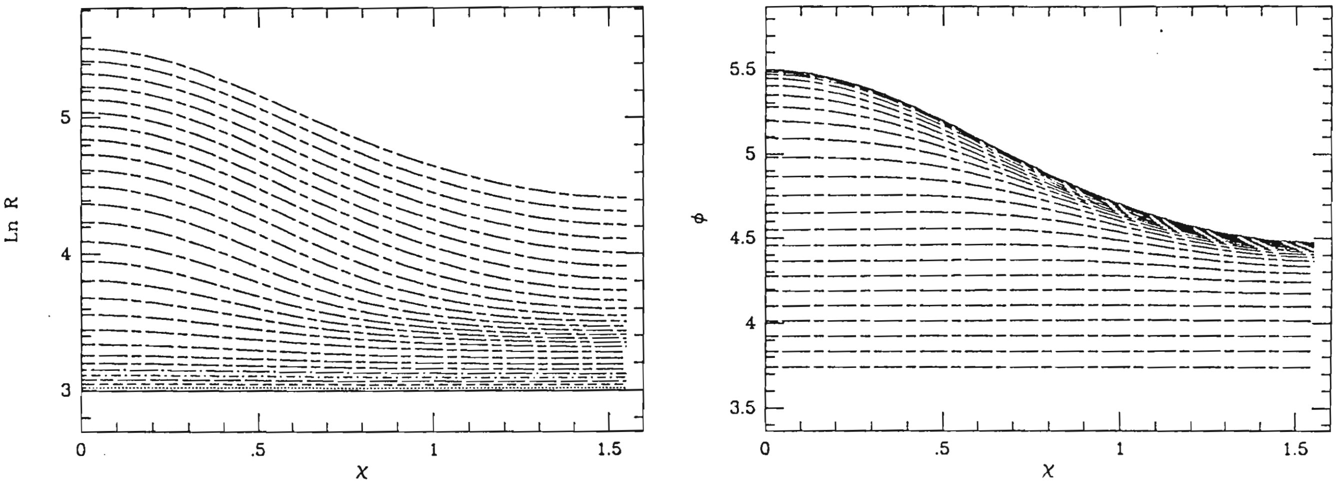

Fig. 1.b

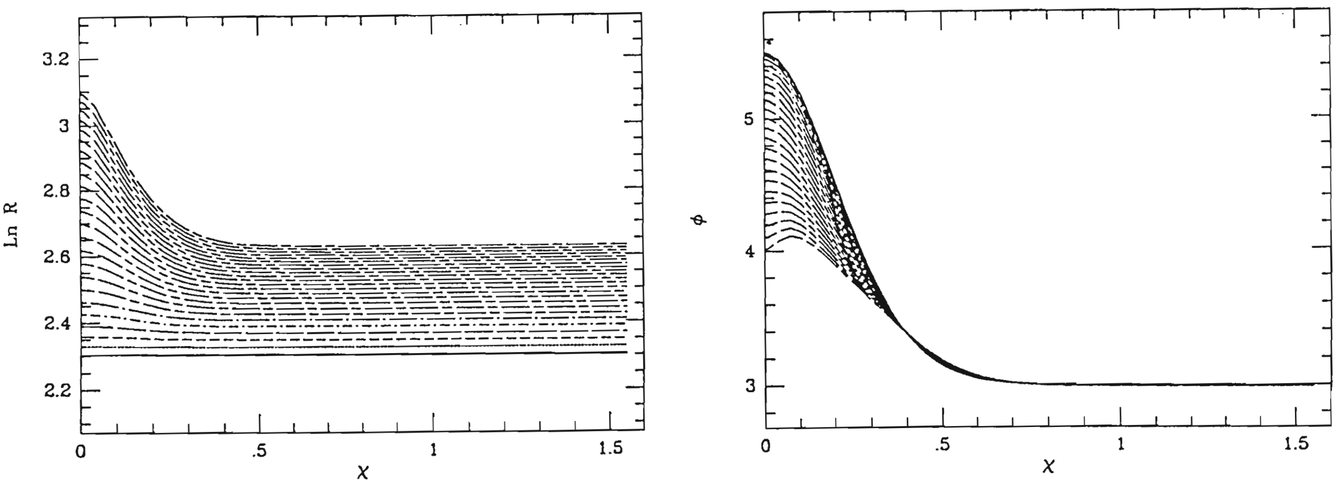

Fig. 1.c

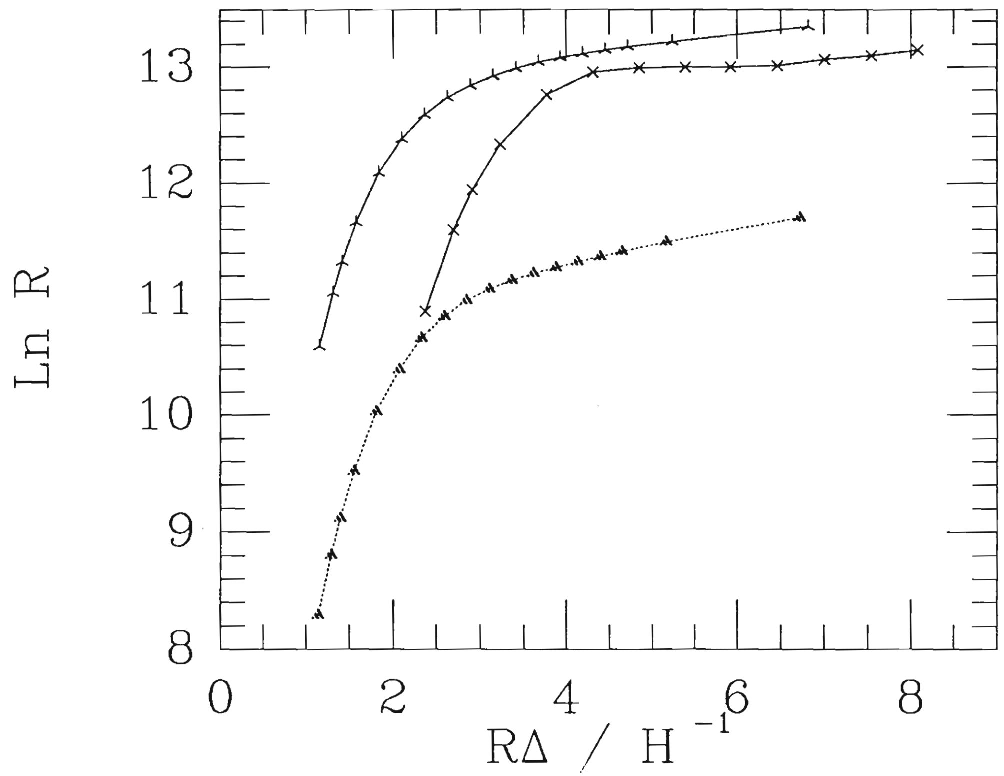

Fig. 2