A NEW TEST FOR GRAVITATIONAL BINDING OF GALAXY CLUSTERS

Donald Goldsmith and Joseph Silk

Department of Astronomy
University of California
Berkeley, California

SUMMARY

The discrepancy between the gravitational binding mass and the observed mass of clusters of galaxies poses a formidable problem for attempts to apply gravitation theory over cosmological distances. We describe a new test to determine whether the Coma cluster of galaxies forms a gravitationally bound system. Recent analyses have shown that the "missing mass" required to bind the cluster should be present in the form of diffuse, hot gas. We have calculated the emission from this gas, and show that new ultraviolet observations of Lyman alpha radiation can finally determine the parameters of the elusive intergalactic gas. Such observations are now within our technological capabilities.

A NEW TEST FOR GRAVITATIONAL BINDING OF GALAXY CLUSTERS

Within the hierarchy of structures that astronomers have established in the visible universe, clusters of of galaxies occupy the foremost position. Some question still remains as to whether these clusters, which contain anywhere from a few dozen to many thousand member galaxies, are themselves grouped into "super-clusters." However, there is no doubt that the clusters themselves are coherent entities, since measurements of the line-of-sight velocities of different galaxies within the same cluster have revealed that the cluster members are receding from us at almost the same velocity. From Hubble's law for the expansion of the universe, we may conclude that this similarity in the velocities of recession implies that the galaxies do lie at almost the same point in space.

One of the best-known examples of a nearby compact cluster of galaxies is found in the constellation of Coma Berenices, about twenty degrees from the bright star Arcturus. This cluster spreads over a few degrees on the sky and contains almost a thousand known members. The average recession velocity in the cluster, denoted by V_{av}, is 6850 kilometers per second and implies that the cluster is some 270 million light years distant.[1] Thus the cluster actually occupies an enormous volume, since its true linear diameter must be about 15 million light years.

The measurement of the recession velocities (denoted by V) of individual members of the Coma cluster of galaxies has produced a disquieting result for gravitation theory. If we form the deviation of

---

[1] We shall assume that the Hubble parameter is 75 km $\sec^{-1}$ Mpc^{-1}; this value is uncertain by a factor of about 30%.

the recession velocity from the mean recession velocity, $V - V_{av}$, and then compute the square root of the average value of $(V - V_{av})^2$, we obtain the velocity dispersion, a large velocity, which is characteristic of the velocity spread among cluster members. The process of squaring the deviation, averaging, and then taking the square root, allows both positive and negative values of $(V - V_{av})$ to contribute to the average dispersion without cancellation. We are interested in both positive and negative deviations from V_{av} without regard to the sign.

According to an important result from the theory of dynamics called the virial theorem, the velocity dispersion, i.e., $\langle (V - V_{av})^2 \rangle^{\frac{1}{2}}$, is related to the mass M and radius R of a galaxy cluster by the relation

$$\langle (V - V_{av})^2 \rangle^{\frac{1}{2}} = \left(\frac{GM}{R}\right)^{\frac{1}{2}} \qquad (1)$$

provided that the cluster is in overall equilibrium. That is, the virial theorem states that for a system which is neither expanding nor contracting significantly, the average kinetic energy of a test particle of unit mass, which is half of the square of the left-hand side of equation (1), is equal to half the absolute value of the potential energy of the particle at the cluster boundary, which would be -GM/R.[2] For the Coma cluster, astronomers have found that $\langle (V - V_{av})^2 \rangle^{\frac{1}{2}}$ amounts to 1800 kilometers per second for the line-of-sight velocities. We multiply this number by a factor of $\sqrt{3}$ to obtain the three-dimensional velocity dispersion, and then find that the mass of the cluster should be some fifty thousand times the mass of our own Milky Way galaxy.

---

[2] Here G is the universal coefficient of gravitation, equal to 6.7×10^{-8} in the cgs system of units which we are using.

Since less than 1000 galaxies (many of them smaller than the Milky Way) have been seen in the Coma cluster, we must adopt one of two conclusions: either the Coma cluster does not have equilibrium between its kinetic and its gravitational energy (in which event the virial theorem would not be applicable), or there is an unseen component of mass within the cluster which contributes at least an order of magnitude more mass than observed for the galaxies within the cluster. The first alternative seems unlikely, because if the cluster is not held together by the mutual gravitational interaction of its constituent parts, it will soon disintegrate. If the average velocity is 3×10^8 cm $\sec^{-1}$, the cluster will fly apart in about a billion years. But the Coma cluster galaxies must be of comparable age to our own galaxy, which is some ten billion years old. Moreover, astronomers have found that galaxy formation appears to have ended fairly soon after the start of the expansion phase of the universe, some thirteen billion years ago, and hence they are understandably reluctant to believe that the Coma cluster was formed at a cosmologically recent time.

The preferable alternative is the second possibility, that of unseen mass with the cluster. A natural form for this mass would be an intergalactic medium of gas between the galaxies, perhaps the relic of the original epoch when galaxies condensed out of the diffuse gas. In this essay, we wish to describe a new test for establishing the existence of this intergalactic matter within the Coma cluster. Verification of the existence of this hitherto hidden mass is a crucial hurdle that must be surmounted before the theory of gravitation can be said to be established over distance scales of up to some hundreds of millions of light years.

Now a diffuse intergalactic medium would probably be formed almost entirely of hydrogen, with about one atom in ten being a helium atom, as one finds for almost all of the components of the universe that astronomers have investigated. However, radio observations of galaxy clusters have shown that the hydrogen could not be atomic, because neutral hydrogen atoms emit easily detectable radio waves.[3] The amount of radio emission observed at the earth limits the possible mass of hydrogen atoms within the cluster to an amount that is one hundred times less than the total mass that the virial theorem requires.

Now the temperature of the ionized gas cannot exceed one million degrees, otherwise excessive X-radiation would already have been observed. How then can one verify the existence of an intergalactic gas that is ionized, but not sufficiently hot to emit large amounts of X-rays?

We wish to point out that the best way to observe an ionized intergalactic medium would be to detect the emission in the Lyman alpha line that would follow the occasional recombinations (of electrons onto protons) which re-form hydrogen atoms for a short while before their subsequent re-ionization. Lyman alpha radiation occurs when an electron jumps from the second lowest to the lowest energy level in hydrogen atoms, and this transition is the strongest of those allowed for the atoms.

Let us estimate the Lyman alpha flux expected from the hidden mass in the Coma cluster, if in diffuse form. The total number of recombinations onto hydrogen ions within an ionized medium of mass M would be, using

---

[3] This emission is due to a fine structure spin-flip transition, and occurs at a rest wavelength of 21 cm.

$M = 10^{49}$ gm,

$$\frac{M}{m}\alpha \simeq 8 \times 10^{59} \left(\frac{10^4}{T}\right)^{0.75} \quad \mathrm{cm}^3\ \mathrm{sec}^{-1} \tag{2}$$

Here m is the average mass per particle in the gas (taken to be a factor 1.1 times the mass of a hydrogen atom), and α(cm^3 sec^{-1})is the hydrogen recombination coefficient.[4] Each recombination will eventually produce a Lyman alpha emission, because other transitions yield photons that are degraded in energy to that of a Lyman alpha transition before they can escape from the cluster. Now because of the finite size of the Coma cluster, we observe the flux of Lyman alpha photons spread over about ten square degrees (or two percent of a steradian), and in "detector" units, the flux would be

$$F(\mathrm{Ly}\text{-}\alpha) = 2 \times 10^4 \left(\frac{10^4}{T}\right)^{0.75} \quad \text{photons cm}^{-2}\ \text{sec}^{-1}\ \text{sterad}^{-1}$$

or, since each Lyman alpha photon has an energy of 1.6×10^{-11} erg,

$$F(\mathrm{Ly}\text{-}\alpha) = 3 \times 10^{-7} \left(\frac{10^4}{T}\right)^{0.75} \quad \text{erg cm}^{-2}\ \text{sec}^{-1}\ \text{sterad}^{-1} \quad .$$

If the intergalactic gas were not diffused uniformly throughout the Coma cluster, but instead occupied a fraction 1/C of the volume, the Lyman alpha emission would be enhanced by a factor C, since the recombinations per unit volume vary as the <u>square</u> of the density. Thus

---

[4] α is approximately given by

$$\alpha \simeq 2 \times 10^{-13} \left(\frac{10^4}{T}\right)^{0.75} \quad \mathrm{cm}^3\ \mathrm{sec}^{-1}$$

where T is the temperature in degrees Kelvin.

even if the intergalactic gas occupies a volume smaller than the actual volume of the Coma cluster by a factor C because of the clumping, there will nevertheless occur C times more recombinations within the same amount of material.

In fact, the missing mass in the Coma cluster must be clumped, otherwise it would necessarily acquire a velocity dispersion similar to that of the galaxies, or it would collect at the center of the cluster. X-ray observations rule out the first possibility (since the gas temperature would have to exceed 10^8 °K), and the observed distribution of galaxy velocities eliminates the second alternative (since it follows that missing mass cannot be any more centrally condensed than the galaxies themselves). Hence we obtain the final result that the total Lyman alpha emission should be

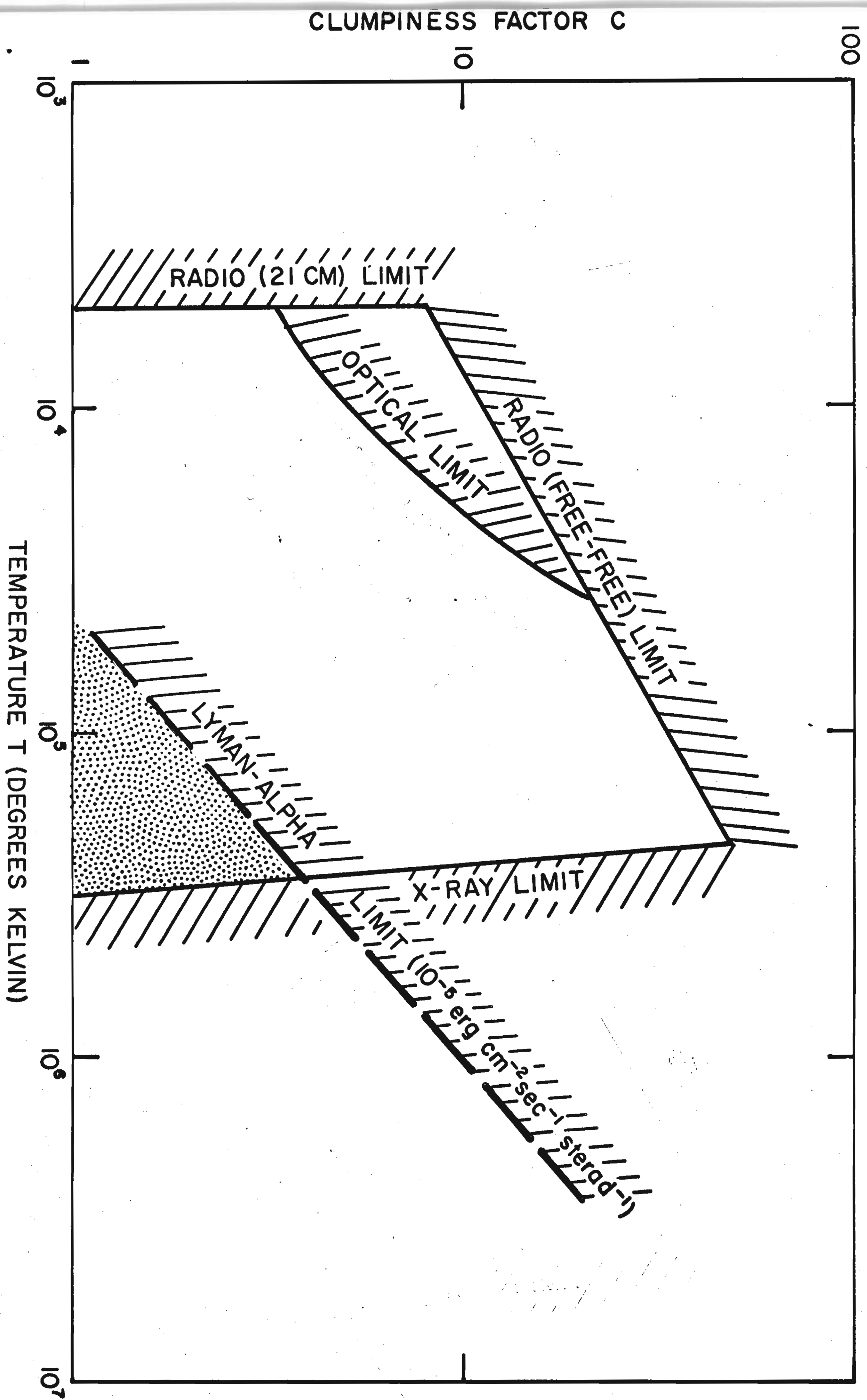
$$F(\text{Ly-}\alpha) = (3 \times 10^{-7})\ C\ \left(\frac{10^4}{T}\right)^{0.75} \quad \text{erg cm}^{-2}\ \text{sec}^{-1}\ \text{sterad}^{-1}$$

Can we observe such a flux? The Lyman alpha line falls in the ultraviolet spectral region, at a wavelength of 1218 Angstroms. The velocity redshift from the cluster's motion changes this wavelength to 1248 Angstroms, conveniently removed from local interference arising from absorption in the interstellar medium of our own galaxy. But we must go above the atmosphere--by rocket or satellite--to observe this wavelength. The Soviet Venus spacecraft have apparently achieved a detection threshold of 10^{-5} erg cm^{-2} sec^{-1} sterad^{-1} in this wavelength region. This is already sufficient to set interesting limits (Figure 1). If United States technology could surpass this level by a factor of ten we could hope to uniquely find the Lyman alpha line from an ionized medium dense enough to stabilize the Coma cluster. The reason that ultraviolet observations provide such a powerful tool is that optical and radio measurements have already been extended to their ultimate limits. Only ultra-violet observations have not yet been fully exploited. As is apparent from Figure 1, it is the combination of new Lyman alpha measurements with existing X-ray data that can enable a new test of the gravitational binding of the Coma cluster to be performed.

Thus it appears likely that Lyman alpha experiments can soon establish whether or not an ionized intergalactic medium <u>does</u> exist in the Coma cluster with enough mass to meet the requirements of the virial theorem and end the uncertainties posed by the large velocity dispersion in the cluster. Of course, the missing mass could have taken another form--burnt-out galaxies or even super-dense "black holes." In any case, it is the <u>gravitational</u> balance of the cluster which must be accounted for, since the electromagnetic, strong, and weak interactions that govern small assemblies of matter are completely overwhelmed in large bodies by the additive power of gravitational forces, which never cancel but always attract, ever seeking an equilibrium between motion and cohesion.

FIGURE CAPTION

We show various observational limits on the intergalactic gas in the Coma cluster, assuming the virial mass to be present in diffuse form. Limits on the clumpiness factor C (i.e., the inverse of the fractional volume occupied by the intergalactic gas clouds) are shown as a function of the kinetic temperature T. Existing observational limits are denoted as "optical" (H-beta emission), "radio" (21-centimeter and free-free emission), and "X-ray" (emission observed down to a threshold energy of 0.25 keV). The new type of measurement that we propose is labelled as "Lyman-alpha." We have assumed a Lyman-alpha sensitivity equal to that obtained by experiments on the Soviet Venus space probes. Only the shaded area corresponds to allowable parameters for the gas in the Coma cluster. A factor-of-ten improvement in the Lyman-alpha sensitivity would finally solve the mystery of the "missing matter."



CLUMPINESS FACTOR C
1
10
100
TEMPERATURE T (DEGREES KELVIN)
10^3
10^4
10^5
10^6
10^7
RADIO (21 CM) LIMIT
OPTICAL LIMIT
RADIO (FREE-FREE) LIMIT
LYMAN-ALPHA
X-RAY LIMIT
LIMIT (10^{-8} erg cm^{-2} sec^{-1} sterad^{-1})

BIOGRAPHY

Donald Goldsmith was born in Washington, D.C., on February 24, 1943. He received a B.A. degree from Harvard College in 1963, and the Ph.D. degree from the University of California, Berkeley, in 1969. After a year at the Institute for Plasma Research of Stanford University, Dr. Goldsmith became a research associate in the astronomy department of the University of California, Berkeley, in June 1970. Dr. Goldsmith's primary research interests have been the thermal and ionization balance of the interstellar medium and of a possible intergalactic medium. He has also worked on the thermal balance of the solar corona, and on the photometry of quasi-stellar objects.

Joseph Silk was born in London, England, on December 3, 1942. He received a B.A. from Clare College, Cambridge University, in 1963 and the Ph.D. degree from Harvard University in 1968. After spending 1968-1969 as a visiting fellow at the Institute for Theoretical Astronomy, Cambridge, England, and the following academic year as a research associate of the Princeton University Observatory, Dr. Silk joined the astronomy department at the University of California, Berkeley, in the fall of 1970 as an Assistant Professor. Most of Dr. Silk's research has dealt with the high-energy problems of astrophysics, especially with the generation and production of X-rays and of gamma-rays. He has also worked on cosmological problems involving the growth of inhomogeneities in the early universe.