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ROTATING GRAVITATIONAL SENSORS*

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ABSTRACT

We have been investigating a novel class of gravitation sensor which utilizes the rotational properties of tensors in order to separate the effects of forces from the effects of the gradients of the forces. These sensors are theoretically capable of distinguishing the gravitational effects of a nearby mass from the inertial effects of acceleration and rotation. The basic concept is that tensors of n^{th} rank, when examined in the rotating reference frame of a sensor, will be found to produce time-varying signals that are at n times the rotational frequency of the sensor. Work has been started on a research model of a gravitational mass sensor. The program objective is a sensor that will detect the presence of a small, nearby moving mass through gravitational interactions. The ultimate objective of our effort on gravitational mass sensors is the development of a small, lightweight, rugged sensor to be used on lunar orbiters to measure the mass distribution of the moon and on deep space probes to measure the mass of the asteroids.

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A. Introduction

In order to measure the mass of an object at a distance,[1-6] when both the object and the sensor are in free fall, and in order to determine the attitude of a spacecraft in orbit around the earth without using external referents,[7-18] it is necessary to develop force measuring instruments that will allow us to distinguish between the effects of gravitational forces and inertial reaction forces. At first glance it might be assumed that Einstein's principle of equivalence might preclude such a differentiation, since basically it states that there is no way to distinguish between a gravitational field and an accelerated reference frame. However, the principle of equivalence is valid only for uniform gravitational fields or an infinitesimal region of the reference frame.[19]

The separation of the gravitational effects from the inertial effects is accomplished by using the physical fact that the various force fields differ in their gradient or tensor characteristics and the mathematical fact that a tensor of n^{th} rank, when examined in the rotating reference frame of a sensor, will be found to produce time-varying signals that are n times the rotational frequency of the sensor.

B. Characteristics of Gravitational and Inertial Fields

1. Inertial Fields

When a nongravitational force F acts on a vehicle with mass m, it causes a linear acceleration. The linear acceleration of the vehicle creates in the frame of reference of the vehicle a uniform inertial field which has purely vector properties and no spatial gradients:

$$a_f = \frac{F}{m} . \tag{1}$$

If the vehicle using the sensor is rotating at a rate Ω, the rotation sets up a cylindrically symmetric inertial field which increases with increasing distance r from the axis of rotation.

$$a_r = \Omega^2 r \quad . \tag{2}$$

This field has a uniform gradient in the plane of rotation

$$\Gamma_r \approx \Omega^2 \tag{3}$$

but no higher order gradients.

2. Gravitational Fields

When a gravitational field of a mass M at a distance R acts on a vehicle, it sets up a spherically symmetric acceleration field

$$a_g \approx -\frac{GM}{R^2} \quad , \tag{4}$$

which has not only a first order gradient

$$\Gamma_g \approx -\frac{GM}{R^3} \tag{5}$$

but also an unlimited number of higher order gradients

$$T_{ab\,\ldots\,n} \approx -\frac{GM}{R^n} \quad . \tag{6}$$

3. Differentiation of Gravitational and Inertial Fields

As is shown in the previous sections, gravitational and inertial effects have different tensor characteristics. The inertial field created by acceleration is a uniform vector field and has no gradients, while the inertial field created by rotation has a uniform cylindrically symmetric tensor gradient but none of higher order. The gravitational field created by a mass is highly nonuniform, with essentially no limit to the number of higher order gradients. These differences make it theoretically possible to measure independently gravitation, rotation, and acceleration effects; to do so, some form of differential force sensor with tensor response characteristics must be used.

The differential force sensors discussed in the literature[7-18] usually consist of spaced pairs of low level accelerometers with opposed outputs. However, a very good accelerometer is capable of a linearity of only one part in 10^5, and the outputs of two accelerometers cannot be matched to even this degree of accuracy. Thus, it has not been possible to make differential force sensors whose outputs could be combined to cancel out the acceleration terms in order to obtain the rotation and gravitation terms.

The most promising technique is a dynamic one.[1-6, 13, 17] By rotation of specially designed differential force sensors, the static spatial variations of the fields can be transformed into temporal variations in the sensor. Because of the rotational properties of tensors, the various inertial and gravitational effects come out at different frequencies.[3,4]

The basic concept is that forces are vectors (tensors of first rank), the gradients of forces are tensors of second rank, and higher order gradients are higher rank tensors. In general, the components of a tensor of n^{th} rank, when examined in the rotating reference frame of a sensor, will be found to have time-varying coefficients that are at n times the rotational frequency of the sensor.[3]

C. Rotating Gravitational Gradient Sensors

The basic dynamic gravitational sensor configuration being studied at Hughes[1-6] consists of a mass-spring system with one or more vibrational modes. The system is rotated at some subharmonic of the vibrational mode. If a nonuniform gravitational field is present, the differential forces on the sensor resulting from the gradients of the gravitational field will excite the vibrational modes of the sensor structure. In the schematic of Fig. 1, the gradient of the gravitational field excites vibrations at twice the rotation frequency of the sensor. Similar devices have been proposed by Diesel,[13, 17] Kalmus,[20] and Fitzgerald,[21, 22] although only the device proposed by Diesel was designed to measure the gradient of the force rather than the gravitational force itself.

Because theoretical analysis has shown that there would be limitations on the use of radial type mass sensor design and because design problems of a radial type support structure were quite complex, it was decided to try a cantilever type sensor design (see Fig. 2) for our preliminary studies of sensor support and drive mechanisms. The gradient of the gravitational field causes differential torques on the arms. As the sensor rotates, the direction of the applied differential torque varies at a frequency which is twice the rotation frequency of the sensor, and a phase which is related to the direction to the exciting mass.

The general electronic circuit for the mass sensor is shown in Fig. 3. The parallel connection of four strain transducers (Gulton SC-2) serves two purposes: It reduces the source impedance seen by the preamplifier by 4, and it cancels output of vibrational modes other than the desired one. The output of the strain transducers (1 to 100 μV at 190 cps) is amplified by a low noise preamplifier to a level of 1.5 to 150 mV. The

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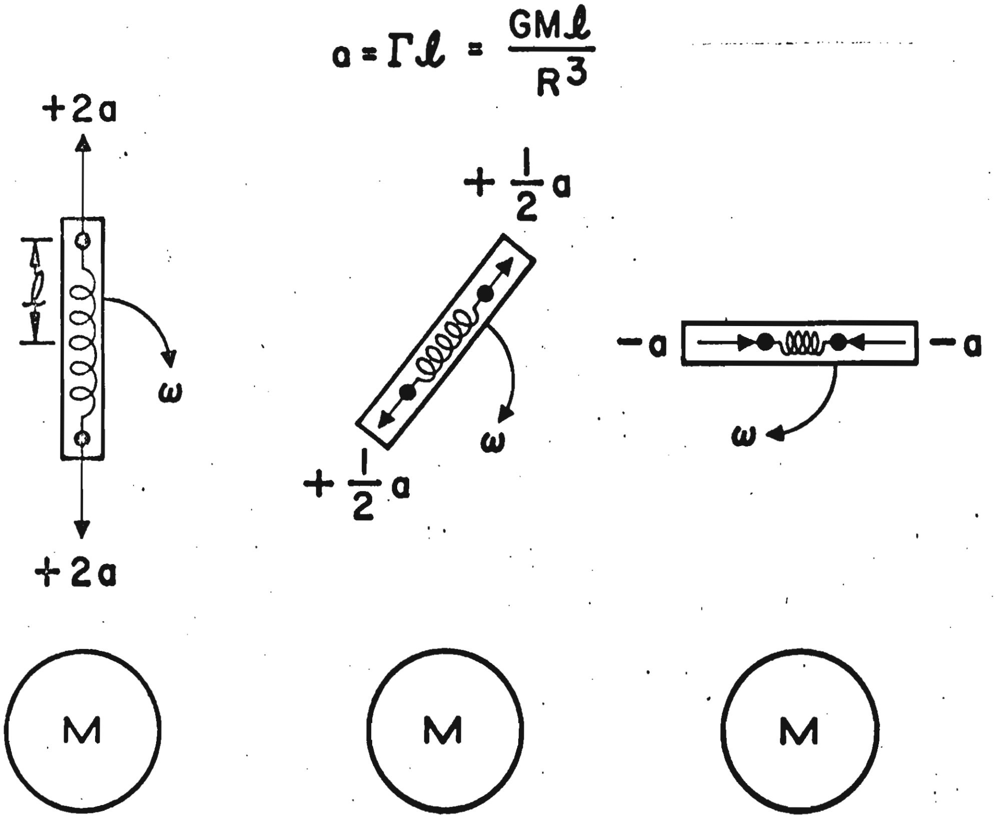

Fig. 1. Response of rotating gradient sensor to gravity gradients.

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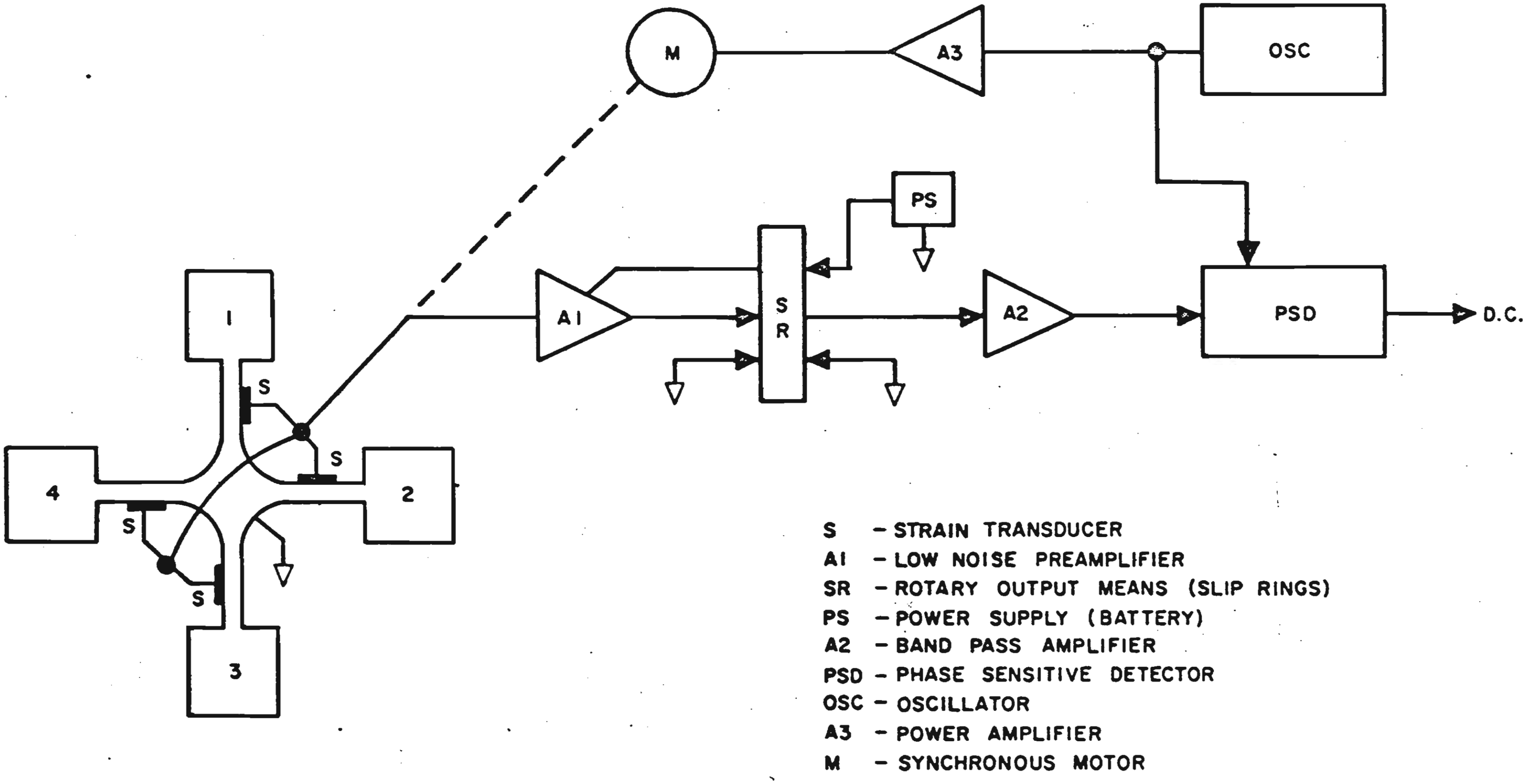
Fig. 2. Five-inch diameter cruciform gravitational mass sensor



S - STRAIN TRANSDUCER
A1 - LOW NOISE PREAMPLIFIER
SR - ROTARY OUTPUT MEANS (SLIP RINGS)
PS - POWER SUPPLY (BATTERY)
A2 - BAND PASS AMPLIFIER
PSD - PHASE SENSITIVE DETECTOR
OSC - OSCILLATOR
A3 - POWER AMPLIFIER
M - SYNCHRONOUS MOTOR

Fig. 3. General mass sensor electronics.

preamplifier is mounted in the rotating sensor vacuum chamber. Power for the preamplifier and the preamplifier output are retrieved from the sensor via slip rings.

The preamplifier output in the frequency region desired is next amplified to a level of approximately 1 V by a band-pass amplifier such as the GR 1232-A and fed to a lock-in amplifier (or phase sensitive detector) such as the Princeton Applied Research JB-4.

The sensor is rotated at a synchronous speed (6000 rpm) by an oscillator-amplifier-motor system. The oscillator also supplies the phase reference for the lock-in amplifier.

The output of the amplifier is a dc level proportional to the amplitude of the sensor output at the frequency and phase selected by the oscillator and lock-in amplifier.

D. Discussion of Noise

1. Thermal Noise

The fundamental sensitivity of any sensor is determined by the thermal noise limitation. In practice, this limit can never be reached, but many systems can approach it very closely. This is especially true of low frequency devices, since the electronics available in this region has been highly developed and will contribute only a few degrees of extra equivalent noise temperature to the physical temperature of the sensor.

Because this basic limit is dependent upon energy considerations, its calculation depends only upon very general parameters of the sensor, such as its temperature, mass, effective length, and time of integration. The results can then be applied to all sensors, regardless of their detailed design. The basic formula states that the signal-to-noise ratio is given by the ratio of the signal energy stored in the sensor to the thermal energy (kT) present in the sensor.

$$\frac{S}{N} = \frac{m\,\omega^2 \xi_o^2}{2\,kT} \tag{7}$$

where we assume that the amplitude of the spring extension due to the signal forces is given by[1]

$$\xi = \xi_o \sin \omega t \approx \frac{GM\,\ell\,\tau}{R^3\,\omega} \sin \omega t \quad . \tag{8}$$

So the minimum gradient that can be measured for a thermally limited sensor is:

$$\Gamma = \frac{GM}{R^3} = \frac{(S/N)^{1/2}}{\ell\,\tau} \left(\frac{2\,kT}{m}\right)^{1/2} \tag{9}$$

where S/N is the desired signal-to-noise ratio; T, m, and ℓ are the temperature, mass, and length, respectively, of the sensor; M is the mass of the object at distance R; and τ is the integration time.

The major problem to be faced in the design, construction, and operation of the proposed gravitational mass sensors is the identification and elimination of external and internal sources of electronic and mechanical noise so that the sensor is limited only by thermal fluctuations.

2. Mechanical Noise

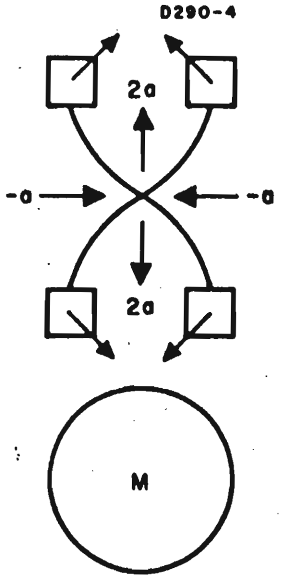
Since the proposed sensors consist of mechanically resonant mass-spring systems, they are potentially susceptible to mechanical noise with frequency components at the frequency of resonance of the sensor. The various types of mechanical noise that could presumably be present are acoustic noise coupling to the sensor through the air, vibrational noise from the bearings or drive coupling through the sensor support, and inertial noise due to poor balance in the sensor support or accelerations and rotations of the vehicle containing the sensor.

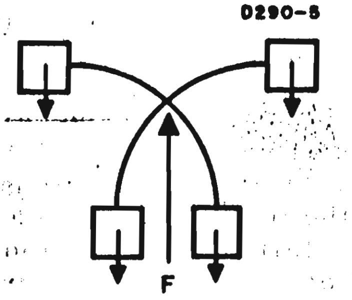
In theory, these sources of mechanical noise (except the inertial noise due to certain types of rotation of the using vehicle) should not affect the operation of the sensor, since they do not excite the proper mode of oscillation. This is most easily seen by reference to Fig. 4 which shows the response of a cruciform sensor. This applies to all sensors since in general the mechanical noises are forces, forces are vector quantities or first rank tensors, and the sensors are designed to sense only second rank tensors.

In practice, however, these mechanical noises do couple into the sensor and out through the electronics because of asymmetries or nonlinearities in the various mechanical and electrical components. Thus, the mechanical noise problem is a second order one. However, the gravitational interaction which we are seeking is very weak, and a second order sensor coupling to mechanical noise can easily hide the gravitational signal if the noise sources are not carefuly eliminated.

To eliminate the acoustic coupling, the sensor will be operated in a vacuum chamber. Previous experience has shown that a pressure of a few microns is sufficient for decoupling any acoustic noise.

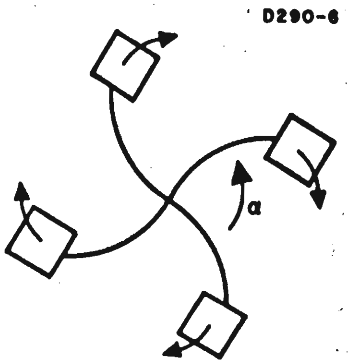
The vibrational noise from the bearings and drive and the inertial noise due to unbalance of the rotating systems, which couple to the sensor through the sensor support, are major problem areas that are being investigated in the experimental portion of the program. The investigation to eliminate this type of noise has just begun, and will be our major concern during the next year. Various different bearings and drives are being purchased, and their noise characteristics will be determined by test. Various sensor supports will be investigated in an effort to find one which transmits the low frequency torques and forces needed for

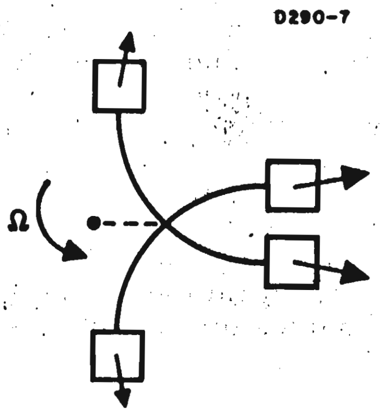




(b).
Response to linear acceleration.

(a).
Response to gravitational force gradient.





(c).
Response to angular acceleration.

(d).
Response to asymmetric rotation.

Fig. 4. Response of cruciform sensor to various gravitational and inertial sources.

rotation and support, but does not transmit the high frequency torques and vibrations that will excite the sensor resonance.

E. Applications

The initial applications of a gravitational mass sensor will be to measure the masses of the asteroids and the variations in the gravitational field of the moon. However, since the devices respond in different ways to inertial and gravitational forces, they could also be used as sensors for active attitude control as proposed in Refs. 7-18.

1. Asteroid Mass Measurement

The investigation of the asteroid belt will be primarily directed toward determining the origin of the asteroids. An important parameter in this determination is a measurement of their density. Measurement of the volume can be obtained from photographs during a flyby, and the development of a sensor to measure the mass during a flyby will relieve the necessity for matching orbits with an asteroid in order to determine its mass.

Some estimates of the maximum range obtainable for a room temperature sensor of effective mass 100 g and effective length of 1 ft on a space probe with a relative velocity of 10 km/sec are summarized in Table I.

TABLE I

Asteroid Measurement Range for Small Flyby Sensor

Number of Asteroids	Radius, km	Mass, kg	Measurement Range, km	Integration Time $\tau = 2R/v$ sec (v = 10 km/sec)
6	140	5×10^{19}	15000	3000 (~ 1 hour)
25	70	6×10^{18}	5000	1000
80	44	1.5×10^{18}	2500	500
200	28	4×10^{17}	1300	260
500	18	1×10^{17}	660	130
1250	11	2.5×10^{16}	330	66 (~ 1 min)

2. Gravity Survey of Moon

A detailed gravity survey of the moon will tell us a great deal about its past history and internal structure. A surface survey with gravimeters is obviously time consuming, and a survey using the variations in the orbital parameters of a lunar satellite will smooth out localized features. The proposed gravitational force gradient sensors will read out the gravity difference directly, and thus the geodetic information will be available in real time. The gravitational force gradient of the moon is 1.7×10^{-6} sec^{-2}, which is about the same as that of the earth. (For measurements near the surface of a body, the gradient is proportional to the density.) The magnitude of the variations in this gradient due to the higher harmonics of the mass distribution of the moon are also expected to be as large or larger than those of the earth since the moon's lower gravity allows larger mass imbalances to exist. The largest effects will occur for density variations on the moon's surface that have an effective radius equal to or greater than the altitude of the lunar orbiter. If we assume an altitude of 50 km, objects with characteristic dimensions of 50 km will create gravitational gradients of the order of 10^{-7} to 10^{-8} $\sec^{-2}$. This is a variation of a few percent in the basic gradient of the moon and should be easily measured by a sensor in a lunar orbiter, since the thermal noise limit is over 30 dB down.

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