21

TUFTS UNIVERSITY

Department of Physics
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Mr. George M. Rideout
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Dear Mr. Rideout:

I am enclosing three copies of an essay entitled "Does $\Omega < 1$ Imply that the Universe will Expand Forever", which I am submitting for the 1986 competition. This essay was written especially for the Gravity Research Foundation competition.

Sincerely,

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Enclosures

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Does $\Omega < 1$ Imply that the Universe will Expand Forever?

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SUMMARY

The issue of whether the present observational evidence that the mean mass density of the universe is less than the critical density (i.e. $\Omega<1$) implies an infinite future expansion of the universe is discussed. Although in convential cosmological scenarios, $\Omega<1$ necessarily leads to a universe that will grow infinitely old, this conclusion can be avoided in ways which are reasonably natural. One of these is to assume the existence of a small negative cosmological constant. Another way is to postulate the existence of unstable fields with a long time scale for the onset of the instability. An example is a scalar field with a negative squared mass and $|m| \lesssim 10^{-32}$ eV. Other examples include fields for which the instability is generated by quantum corrections in curved spacetime. All of these are capable of halting the expansion of an open universe and forcing it to recollapse into a "big crunch".

The issue of whether the universe is open or closed has been the subject of intense investigation and debate since the development of relativistic cosmology in the 1930's. Some cosmologists have expressed a strong philosophical preference for a closed universe. This is presumably on the grounds that the possibility of recollapse and recycling of the universe appears to be a less bleak prospect than does unceasing expansion with its resulting "heat death", the eventual stoppage of all organized activity as entropy increases. At the present time, observational evidence appears to favor an open universe. The amount of luminous matter in the universe clearly amounts to much less than the critical density. The flat rotation curves of galaxies and the application of the virial theorem to clusters of galaxies reveal the presence of large amounts of dark matter.[1] Nonetheless, the best current estimates of Ω fall well below one, [2,3] typically of the order of $\Omega \sim 0.2$.

The question which will be addressed in this essay is the following: if we accept that Ω is indeed less than unity, are we forced to accept that the universe is doomed to expand unceasingly. The answer is no; either a negative cosmological constant or an unstable field can force the recollapse of an open universe. One often speaks as though a closed (k=1) universe is synonymous with eventual recollapse and an open (k=-1) universe with indefinite expansion. This is only true, however, with certain restrictions on the form of the energy momentum tensor of matter. If the metric is of the form

$$ds^2 = dt^2 - a^2(t)\, d\sigma^2 \qquad (1)$$

where $d\sigma^2$ is the 3- space metric, then the equation which governs the dynamics of the universe is

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho \qquad (2)$$

Here ρ is the total energy density, including any contribution from the cosmological constant. If $\rho > o$, then the $k = -1$ universe can have no turning point and $a(t)$ increases monotonically. However, if $\rho < o$, then the possibility of a turning point and recollapse to a final singularity exists.

A negative cosmological constant will eventually cause ρ to become negative and stop the expansion of an open universe. If

$$\rho = \rho_{matter} + \Lambda \qquad (3)$$

where $\Lambda < 0$ and $\rho_{matter} \to 0$ as $a \to \infty$ then recollapse occurs. In order that the universe have reached its present age, it is necessary that $|\Lambda| < 10^{-30} gm/cm^3$. Beyond this, there are no observational constraints upon Λ. The effects of a negative Λ upon the dynamics of the universe have long been known.[4] However, the advent of modern theories with spontaneous symmetry breaking has made the possibility of a nonzero value of Λ seem more natural. Such theories lead naturally to cosmological constant terms in the energy momentum tensor which change value during a phase transition. Although one can adjust the value of the effective cosmological constant to be zero in one phase, it cannot vanish in all phases. The typical magnitudes of the effective value of Λ in theories of the fundamental interactions is enormously larger than the upper bound on $|\Lambda|$ quoted above; in grand unified theories, for example, $|\Lambda| \simeq 10^{80}$ gm/cm^3. The question of why Λ has such a small value at the present time is still unanswered. Whatever the final resolution of this puzzle may be, the possibility that Λ is small but nonzero cannot be excluded. Furthermore, a negative value of Λ seems to be as plausible as does a positive value.

A second possibility for avoiding an endless expansion is the existence of unstable fields.[5] An example of such a field is a scalar field with a "tachyonic" mass, whose equation of motion is

$$\Box \phi - m^2 \phi = 0 \tag{4}$$

It must first be emphasized that the existence of such a field in nature does not imply the existence of particles which travel faster than light. The classical field theory of this field is perfectly causal, with the characteristics of (1) being the light cone regardless of the sign of m^2. The quantum theory of an unstable field can be defined, but one cannot associate a particle interpretation with the unstable modes. For example, Guth and Pi[6] have recently given a discussion of this topic which shows that the evolution of the expectation value of ϕ in a quantum treatment is essentially equivalent to the classical evolution at late times. Thus for our purposes, we may regard ϕ as a classical field.

Let us now understand why this field can lead to negative energy densities and hence stop the expansion of the universe. The energy density of ϕ field is

$$\rho_\phi = 1/2\ (\dot{\phi}^2 - m^2\phi^2) \tag{5}$$

The most rapidly growing solution of (4) is the spatially homogeneous one. In flat spacetime this is $\phi = \phi_o e^{mt}$, for which $\rho_\phi = 0$; the energy density does not increase as ϕ increases. In a Robertson-Walker spacetime the behavior is quite different. The spatially homogeneous mode satisfies (for all values of k) the equation

$$\ddot{\phi} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} - m^2\phi = 0 \tag{6}$$

Over a time interval small compared to the age of the universe, we may assume that $H = \dot{a}/a$ is constant. Then the growing solution is

$$\phi \sim \exp\left\{\left[\tfrac{1}{2}\,(H^2+4m^2)^{1/2} - H\right]t\right\} \tag{7}$$

The energy density of this solution is

$$\rho_\phi \sim -H(H^2+4m^2)^{1/2} \exp\left[(H^2+4m^2)^{1/2} - H\right]t \tag{8}$$

In an expanding universe ($H > 0$), ρ_ϕ is negative and in a contracting universe ($H < 0$), it is positive.

Suppose that we start with an expanding, matter dominated universe, so the total energy density is of the form

$$\rho = \rho_{\text{matter}} + \rho_\phi \tag{9}$$

If m is very small, ρ_ϕ will be negligible for a very long time. Eventually it will dominate ρ_{matter} and force ρ to become negative. The expansion will then stop and the universe begins to recollapse. At this point, ρ_ϕ changes sign and then becomes large and positive, accelerating the collapse into the final singularity. This behavior is illustrated in Fig. 1, which shows the results of a numerical integration of the dynamical equations (2) and (6) with ρ of the form of (9). For the expansion phase of the universe to last at least 10^{10} years, we must have that $m = \lesssim 10^{-67}$gm $= 10^{-32}$eV.

The tachyonic mass scalar field is one of several unstable fields that could cause an open universe to recollapse. Other examples include scalar fields with a $\lambda\phi^3$ or $\lambda\phi^4$ ($\lambda<0$) self-interaction. It is only necessary that λ be sufficiently small to allow the universe to reach its present age. An especially intriguing possibility is that the instability could be generated by quantum corrections rather than be postulated at the classical level.[7,8] This phenomenon can be illustrated by the following model: Let ϕ be a massless field which is coupled to a quantized scalar field ψ by an interaction of the form $g\phi^2\psi^2$. If the ψ field is in its vacuum state, the dynamics of the ψ field is described to lowest order in g by

$$\Box \phi + g \langle\psi^2\rangle\phi = 0 \tag{10}$$

Here $\langle\psi^2\rangle$ is the vacuum expectation value. Although classically ψ^2 is positive definite, its renormalized quantum expectation value can be negative. It then has the effect of a dynamically generated tachyonic mass for the ϕ field. In a curved spacetime characterized by a length scale ℓ, the magnitude of $\langle\psi^2\rangle$ is of the order of ℓ^{-2}. Thus typical values of $\langle\psi^2\rangle$ that would be produced by the large scale curvature of our present universe are of the order of 10^{-64} eV^2. It is possible in a Robertson-Walker universe to have $\langle\psi^2\rangle$ change sign from positive to negative as the universe expands.[9] If this transition occurs at an age of 10^{10} years or more, this mechanism could allow the universe to reach its present age but still force recollapse in the future.

If the unstable field couples only gravitationally, it will produce no other observable effects than modification of the cosmic dynamics. A field with a long time scale for the onset of instability cannot be ruled out as unreasonable. On the contrary, the possibility of such a field allows one to contemplate avoiding an unceasing expansion of the universe even if $\Omega < 1$.

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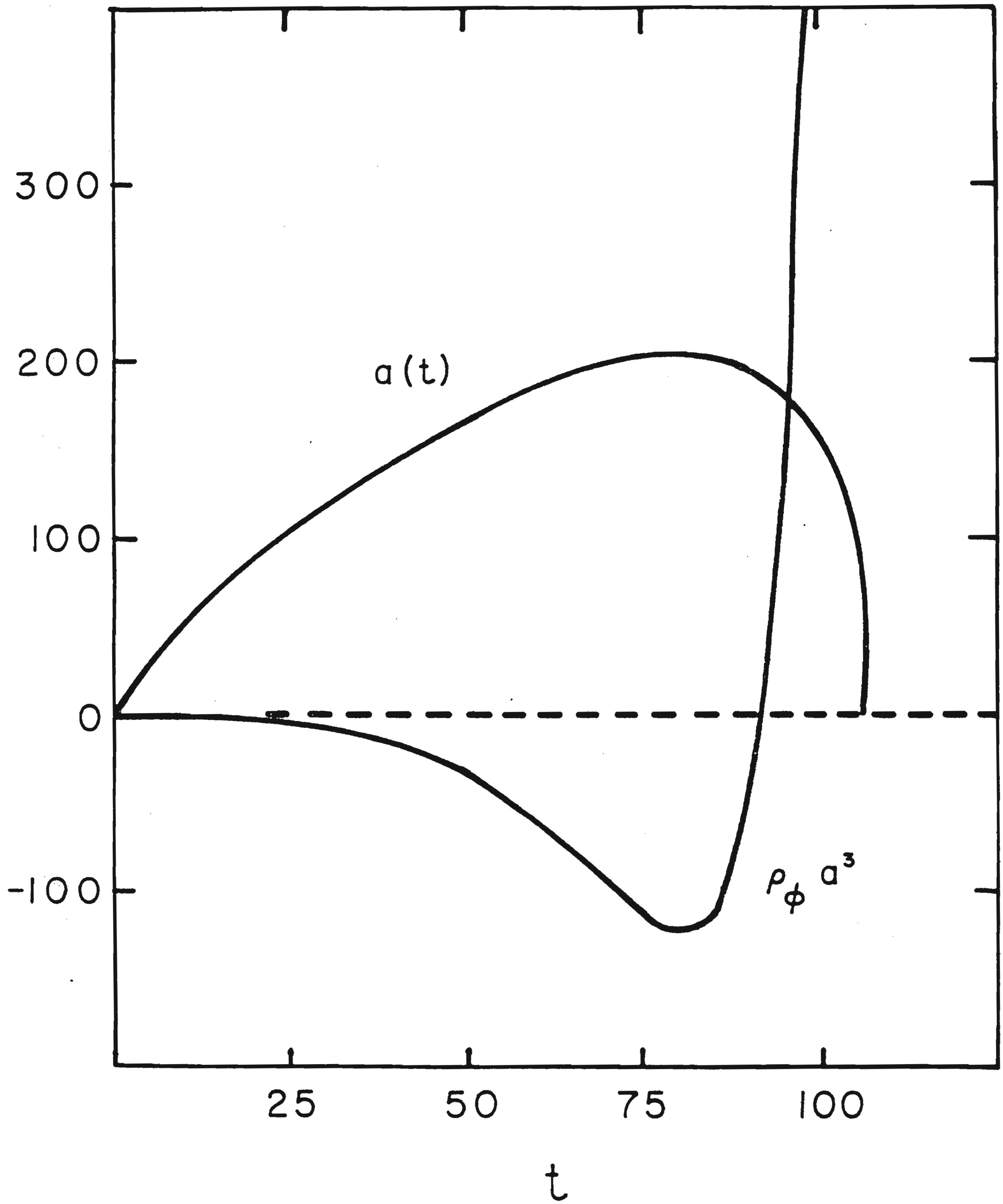
FIGURE CAPTION

1. The results of solving the coupled Einstein-Klein-Gordon equations for an open universe are illustrated. The energy density is assumed to be that of ordinary matter plus an unstable scalar field: $\rho = A/a^3 + \rho_\phi$, A = constant. The scale factor a(t) and the scalar field energy density ρ_ϕ are plotted in arbitrary units, showing that the universe recollapses.


300
200
100
0
-100
a(t)
$\rho_\phi a^3$
25
50
75
100
t