

ASTROPHYSICS RESEARCH DIVISION

University of Science & Technology of China  
Hefei, Anhui, The People's Republic of China

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Dear Dr. G.M.Rideout:

Enclosed please find an essay on gravitation by us.  
I would like to send it for the 1985 Awards for Essays on  
Gravitation.

Yours sincerely

*Fang L. Zhi*

Li Zhi FANG

MAR 9 1985  
*Letter sent  
LH*

Is the Periodicity in the Distribution of Quasar  
Redshifts an Evidence of Multiply Connected Universe ?

Li Zhi FANG and Humitaka SATO\*

Astrophysics Research Division  
University of Science and Technology of China  
Hefei, Anhui, CHINA

\* Research Institute for Fundamental Physics  
Kyoto University, Kyoto 606, JAPAN

Abstract

The periodicity in the distribution of quasar redshifts is interpreted assuming that the cosmological space is a topologically compactified manifold like three dimensional torus. The size of such compact space is estimated as the order of 600 Mpc. <sup>present</sup>

In the standard cosmology, it is assumed that the volume of the three-dimensional space is infinite for  $k=0, -1$  and the space form <sup>of</sup>  $k=1$  is uniquely sphere  $S^3$ ,  $k$  being the sign of the constant curvature. However, the cosmological principle and the Einstein equation do not, in fact, determine the topology of the space-time as a whole. The space is called as simply connected if the geodesics between two points is unique and as multiply connected otherwise. The multiply connected manifold  $\tilde{M}$  with the same local

properties with the simply connected manifold  $M$  is also available for the universe model. Such manifold  $\tilde{M}$  can be constructed by identifying the point  $P$  with the other point  $P'$  in  $M$ . For instance, the multiply connected topology for flat ( $k=0$ ) universe  $ds^2 = -c^2 dt^2 + R^2(t) [dx^2 + dy^2 + dz^2]$  is obtained by identifying  $(x, y, z, t)$  with  $(x + \ell a_x, y + m a_y, z + n a_z, t)$  for all integer  $\ell, m, n$ . The spatial section obtained in this manner is so called three-dimensional torus  $T^3$ . The above mentioned identification is one of the eighteen topologically different types of identification in the case of flat space. <sup>(1)</sup> Each identification constitutes a group  $\Gamma$  of isometrics of  $M$  which has not a fixed points, and  $\tilde{M}$  is the quotient space  $M/\Gamma$ .

On the other hand, the observational constraints on the size of the compactified universe have also been examined by Sokolov and Shvartsman <sup>(2)</sup> and Gott. <sup>(3)</sup> The main result of their arguments is to show that the compactified universe like  $T^3$  does not contradict with observations of the distributions of galaxies and clusters of galaxies, if the size at present  $R_H = R(t_0) [a_x^2 + a_y^2 + a_z^2]^{1/2} / 2$  is larger than about  $200 h_0^{-1}$  Mpc, where  $h_0 = H_0 / 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . However, they did not suggest any positive evidence for that.

In this letter, we point out a possible positive observational evidence for such multiply connected topology. According to this interpretation about the distribution of quasars, most of the quasars might be "ghosts" of the original image. Here, we shall call the image of the closed source as the original image and that of the sources located in the equivalent points in  $M$  as the ghosts. This kind of ghost, which Sokolov and Shvartsman called "ghost of the first kind", is different from "ghost of the second

kind" which might arise in the positive curvature universe. (2)

Recently, it has been demonstrated from the statistical analysis of the observational data that the distribution of the emission line redshifts of quasars have a periodic feature with respect to the argument  $x \equiv F(z, q_0)$  defined by (4)

$$F(z, q_0) = \int_0^z \frac{dz}{(1+z)(1+2q_0z)^{1/2}} \quad (1)$$

where  $q_0$  denotes the deceleration parameter. In the distribution of the redshifts there exists a set of peaks at  $z_n$  given by

$$F(z_n, q_0) = An + B, \quad (2)$$

where  $n$  is zero or positive integer and  $A$  and  $B$  are constants.

This above periodicity might be interpreted as the existence of a large scale periodic perturbation in the density distribution of cosmic matter. According to this interpretation,  $A$  is related to the "wave-length"  $\lambda_0$  of such periodic perturbation as (4)

$$A = H_0 \lambda_0 / c, \quad (3)$$

the subscript 0 denoting the present value. From the statistical analysis,  $\lambda_0$  has been estimated as

$$\lambda_0 \sim 600 h_0^{-1} \text{ Mpc}. \quad (4)$$

The constant  $B$  in (2) is related to our position relative to the

perturbation waves. However, it is very difficult to explain why B is not random depending on directions. If we were located in a preferred position near center of the large scale spherical wave perturbation, the above relation would be explained. But such interpretation would be not acceptable from the point of view of the cosmological principle.

The observational data seems to teach us that the "whole-sky-coherent" periodicity does not exist for the small scales up to about  $200 h_0^{-1}$  Mpc but does exist for the larger scales above  $600 h_0^{-1}$  Mpc. Namely, the small scale inhomogeneities do not show the correlation between the different directions.

For large scale, however, there exists some correlation even in different directions. If we insist on the simply connected universe, it will be very difficult to explain this whole-sky-coherent correlation. But such type of correlation results naturally if we assume the multiply connected universe. The horizon problem associated with the isotropy of the microwave background radiation might be also resolved by the multiply connected universe. (5)

Let us consider the flat universe with topology  $T^3$  for a simple example. The redshifts of the multiple images of the source located at  $x_s$  in the x direction is given from

$$x_s + na_x = \int_{t_n}^{t_0} \frac{cdt}{R(t)} \quad \text{and} \quad z_n + 1 = \frac{R(t_0)}{R(t_n)} \quad (5)$$

as

$$z_n + 1 = \left(1 - \frac{R(t_0)x_s}{2(c/H_0)} - \frac{R(t_0)a_x}{2(c/H_0)} n\right)^{-2} \quad (6)$$

From (2) for  $q_0=1/2$ ,  $A=R(t_0)a_x/(c/H_0)$  and  $B=R(t_0)x_s/(c/H_0)$ .

Considering the  $R(t_0)a_x/2 \sim 600$  Mpc and  $R(t_0)x_s < a_x/2$ , the quasars with  $z < 3$  are the original image ( $n=0$ ) or the ghosts of  $n=1$  or  $n=2$ .

In the case of the whole-sky correlation, however, the situation is more complicated. For example, in the case of  $a_x=a_y=a_z=a$ , the units of space periodicities are  $Ra$ ,  $\sqrt{2}Ra$ ,  $\sqrt{3}Ra$ ,  $\dots$ , depending on the observed directions. Then, the superposition of the two periodicities of  $Ra$  and  $\sqrt{2}Ra$  produces an approximate periodicity of  $\sim 0.5Ra$ . In the actual situation, the observed periodicity may be the superposition of a few fundamental space periodicity.

In spite of the above ambiguity, we can predict such a general tendency that the "wave-length" of periodicity for the quasars in the given direction should be larger than the "wave-length" estimated from the whole-sky correlation. This result might be used to interpret the following observational evidence: the "wave-length" of the whole-sky data is about a half of the "wave-length" estimated from the quasars listed by Savage and Bolton,<sup>(6)</sup> which are the quasars in the two given direction of the south Galactic Polar region of  $02^h00^m$ ,  $-50^000'$  and  $22^h04^m$ ,  $-18^055'$ .

Finally, we want to point out that the multiply connected universe might also contribute to solve the problem of association between quasars and galaxies. Since 1971, after the first finding of bright galaxies associated with 3CQSO, many statistical analyses have been done to check a significance level of association between quasars and galaxies. Some analyses show a positive correlation.<sup>(7)</sup> In some direction of the multiply connected universe, we can see the original image and its ghost images in the same direction.

If we see some clump where the formation of quasar and galaxy were active, the association in appearance between the small redshift galaxies of the original image and the large redshift quasars of the ghost images will result naturally. Since quasar is thought to be a short lived phenomena lasting only a few million years, we will see only one image of the quasar if  $R_H \sim 600$  Mpc and  $Z < 3$ . According to this interpretation, the associated regions may provide us a good sample for studying the evolutionary relationship between galaxy and quasar; the remnants of the quasars should belong to the same clump of the associated galaxies.

Anyhow the further statistical analyses of the quasar redshifts will be important to find the "positive" evidence of the compactified universe or to improve the lower limit on its size.

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