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Essay on Gravitation

submitted to the Gravity Research Foundation

IS THE UNIVERSE EXPANDING ?

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Abstract: It is shown that spherically symmetric static exact solutions of Einstein's Field Equations can reproduce the same cosmological observations as the currently favoured Friedmann-Robertson-Walker universes, if the usual assumptions are made about the local physical laws determining the behaviour of matter, provided that the Universe is inhomogeneous and our galaxy is situated close to one of its centres. Only (i) unverifiable a priori assumptions, (ii) detailed physical and astrophysical arguments, or (iii) observation of the time variation of cosmological quantities, can lead us to conclude that the universe we live in is not such a static space-time.

IS THE UNIVERSE EXPANDING ?

Introduction

The usual deduction that the Universe is expanding is made on the basis of observations of the systematic redshifts of galaxies, which are interpreted as cosmological Doppler shifts, and is supported by the interpretation of the microwave background radiation as the relic radiation from a hot big bang¹. We shall show that these observations can also be explained by a static spherically symmetric universe model with two centres, and our galaxy near one of the centres. In this case the systematic redshifts of the galaxies are interpreted as cosmological gravitational redshifts, while the microwave background radiation originates from a hot gas surrounding a singularity situated at the second centre of the universe.

Cosmography

We proceed as usual by describing the matter and radiation content of space-time as a perfect fluid², and retain the assumption that the universe is spherically symmetric. However we replace the assumption of spatial homogeneity by the assumption that the universe is static. The metric can then be written in the form

$$ds^2 = -e^{2\phi(r)} dt^2 + dr^2 + f^2(r) \{ d\theta^2 + \sin^2\theta d\phi^2 \},$$

where the 4-velocity of matter is $u^a = e^{-\phi} \delta^a_0$ and the central world line C is at $r = 0$; we impose the conditions that $\phi(0) = 0$ and $f(r) = r + O(r^2)$ near $r = 0$. This space-time is spherically symmetric about the world line C. The function $\phi(r)$ (essentially the gravitational potential) determines the redshift z of a source at $r = r_e$ as measured by an observer at C, by $z = e^{-\phi(r_e)} - 1$, and $f(r_e)$ is precisely the observer area distance² of that source as measured by C.

If we are allowed to specify arbitrarily the intrinsic luminosity $L(r)$ and the number density $n(r)$ of galaxies at radius r in this universe, we can reproduce any observed $m-z$ and $N-z$ relations³. Accordingly one can make a cosmographic¹ universe model in this way which reproduces precisely any spherically symmetric observations that are made of the Universe, by supposing that our galaxy is near the centre C of such a space-time.

The General Relativity Field Equations

Given an equation of state the conservation equations determine the radial variation of pressure through the relation $dp/dz = (\mu + p)e^\phi$, where p is the pressure and μ the total energy density of the fluid. The field equations are differential equations for $\phi(r)$ and $f(r)$ which can (in principle) be solved simultaneously with the conservation equations. The solution is determined by 2 constants (or 3 if the Cosmological Constant is taken to be non-zero).

The problem is to find a reasonable equation of state for the matter and radiation dominating the Universe, so that the results one obtains are not totally implausible. There could currently be energy densities of 10^{-20} gm/cc in the form of neutrinos and 10^{-27} gm/cc in the form of gravitational waves without these fields having been detected by direct experiments³. It is therefore possible that the universe is dominated by either gravitational waves or neutrinos⁴, leading to an equation of state $p = \frac{1}{3} \mu$.

Using this assumption one can obtain reasonable solutions of the field equations in which the matter density is a small perturbation of $\mu(r)$, so that the number density of galaxies $n(r)$ does not significantly affect $\mu(r)$; rather $n(r)$ is determined by whatever galaxy formation process takes place in the background space-time, whose curvature is determined by the energy density of gravitational waves and neutrinos.

The Singularity

The existence of the microwave background radiation makes it extremely plausible⁵ that the past null cone from any point p on C refocusses (i.e. $f(r)$ attains a maximum value and then decreases monotonically to zero) at a singularity where $r \rightarrow r_{\max}$, $f(r) \rightarrow 0$, and $\phi(r) \rightarrow -\infty$. If this is so the universe (as well as being spherically symmetric about C) is also spherically symmetric about the singularity S where $r \rightarrow r_{\max}$ and $z \rightarrow \infty$. Space and space-time diagrams of the resulting space-times are given in figure 1.

The Microwave Background Radiation

As well as the matter and dominant radiation in this universe model, blackbody background radiation will be present with a temperature $T = e^{-\phi} \cdot 3 \text{ } ^\circ\text{K}$. As one approaches the singularity this temperature will increase; the matter will be ionised for temperatures greater than $10^4 \text{ } ^\circ\text{K}$, element creation and destruction will take place when $T \approx 10^8 \text{ } ^\circ\text{K}$, and so on. The source for this (and other background) radiation would be the singularity at the centre; the observed microwave background radiation would be the blackbody radiation emitted from the surface of decoupling of matter and radiation at about $3 \cdot 10^3 \text{ } ^\circ\text{K}$.

Non-equilibrium Processes

For this universe model to be viable it is essential that local thermodynamic non-equilibrium processes be able to take place continually. At first sight this presents a problem in a static universe, for one might suppose that in such a universe at most one generation of galaxies could form and die. The key idea is that there could be a continual circulation of matter taking place, with light elements drifting in from near S towards C , being built up to heavier elements in galaxies, then drifting back towards S and being broken down to light elements again by the fireball around the singularity.

The astrophysical situation in the universe would be dominated by this process, for it would determine the galaxy formation conditions and so the number density $n(r)$ and luminosity $L(r)$ of galaxies. There are various possible mechanisms that could cause such a preferential drift of lighter particles 'inwards' (from S towards C) and heavier particles 'outwards', for example the competition between the radiation pressure and the gravitational force acting on the particles.

It is therefore possible to have non-equilibrium processes in such a universe although it is static. An essential role would be played by the singularity S; this would have to be time-asymmetric, emitting radiation (and perhaps matter) in the future direction of time and absorbing radiation (and perhaps matter) arriving there from the past. That is, the master arrow of time for the universe would be built into the singularity.

Correspondence with the FRW Universes

There is a close analogy between these universes and the Friedmann-Robertson-Walker (FRW) universes, the difference being that each variation that was previously ascribed to a time variation is now ascribed to a spatial variation in properties (our observations are made on our past light cone⁶, and one can extend these observations off the light cone in a spatially homogeneous or a static way, see figure 2 ; which of these extensions is more viable depends on detailed astrophysical arguments). For virtually every problem encountered in the one universe there is a corresponding problem in the other. For example the FRW universes have to account for the number counts of radio sources in terms of a time variation of number density or luminosity; the static universe model has to account for the spatial variation in number density or luminosity of these sources.

Problems

The prime questions are whether a more detailed astrophysical examination would vindicate the possibility of a realistic drift mechanism such that light elements drift from S towards C, heavy elements from C towards S, and bound combinations of light and heavy elements (as in stars and galaxies) on average stay put, in a background dominated by gravitational waves or neutrinos; and whether the overall system can be stable. Further research into these questions is required.

Philosophical Questions

Models of the sort described here have not been considered previously because of the assumption - made right at the beginning, in setting up the standard models - of a principle of uniformity (the Cosmological or Copernican principle^{6,7}). This is assumed for a priori reasons, and not tested by observations. However it is precisely this principle that we wish to call into question. The static inhomogeneous model discussed in this paper shows that the usual unambiguous deduction that the universe is expanding is a consequence of an unverified assumption, namely the uniformity assumption.

This assumption is made because it is believed to be unreasonable that we should be near the centre of the Universe¹. This is certainly unreasonable if the implication is taken to be that the universe has been centred on our presence; however there is no need for this implication. Rather one should ask: given a universe model of the type proposed, where would one be likely to find life like that we know on earth? The answer must be, where conditions are favourable for life of this kind; but in the model we are considering, the conditions for life would be most favourable near the centre C where the universe is cool.

Consequently it would be highly probable that if life occurred in such a universe it would occur near the (cool) centre of the universe (this is just the spatial analogue of Carter's statement⁸ that life only occurs at favoured times in the history of FRW universes). Accordingly it is eminently reasonable that we should observe such a static universe from near its centre.

If this point is granted, then these universes have many attractive features. They are, like the steady state universe, unchanging in time; like the Einstein static universe, they are finite in the sense of having only a finite number of galaxies present; they have no particle horizons, obviating the well-known communication difficulties in FRW universes; and the singularity, which in the FRW models is hidden away inaccessibly in the past, is in these universes sitting 'over there' where it can influence, and be influenced by, the universe continually. The question of the concept 'the origin of the universe' might be amenable to more meaningful discussion in this situation.

Comment

It is not claimed that the Universe is actually like this model. What is claimed is that there are no overwhelming arguments that immediately show such a model could not reproduce all the current observations. Such arguments can probably be obtained by closer investigation of the astrophysical aspects of these models; but until they are made explicit, the interpretation of the current evidence as showing the Universe is expanding is based on the assumption of spatial homogeneity, which is made on philosophical rather than observational grounds. The one kind of observation that would rule out a model of

this type immediately is the observation of the time variation of some cosmological quantity (e.g. the Hubble constant). We are of course very far from having such observations available.

Anisotropic possibilities

One point of interest is that in the static model presented here, the blackbody background radiation is exactly isotropic for any observer situated outside the hot fireball. This is quite unexpected; one might have thought at first that one could detect being "off-centre" by anisotropy of this radiation, but this is not so. Accordingly an off-centre observer in such a universe could observe anisotropic Hubble shifts and number counts, but still see precisely isotropic background radiation. This should serve as a cautionary note against using the isotropy of background radiation as evidence that the universe is isotropic about our own position, or that we are at the centre of the universe, should it be spherically symmetric and inhomogeneous.

Developments

As well as further investigations of these static universe, it may be interesting to consider universes which are qualitatively similar in structure (cf. figures 1 and 2), but expanding rather than static.

Acknowledgements

I wish to acknowledge support and hospitality of the Max Planck Institute for Physics and Astrophysics, Munich, and research support from the University of Cape Town. Colleagues at both institutions have helped much in developing these ideas, particularly Judith Perry, Roy Maartens and Stanley Nel.

References

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Figure Captions

Figure 1:

Figure 1(a) Space-time diagram of a static spherically symmetric (SS) universe. This is a section $\{\theta = \text{const}, \phi = \text{const}\}$ of the full space-time. It is a cylinder with a (singular) line S removed

Figure 1(b) Space-section $\{t = \text{const}, \theta = \pi/2\}$ of a SSS universe. It is egg-shell shaped with a (singular) point S removed.

Figure 2:

Figure 2(a) Surfaces of constant density in the past light cone C^- of a point p.

Figure 2(b) Extension of these surfaces into space-time as spacelike surfaces (all the variation is time variation).

Figure 2(c) Extension of these surfaces into space-time as timelike surfaces (all the variation is spacelike).

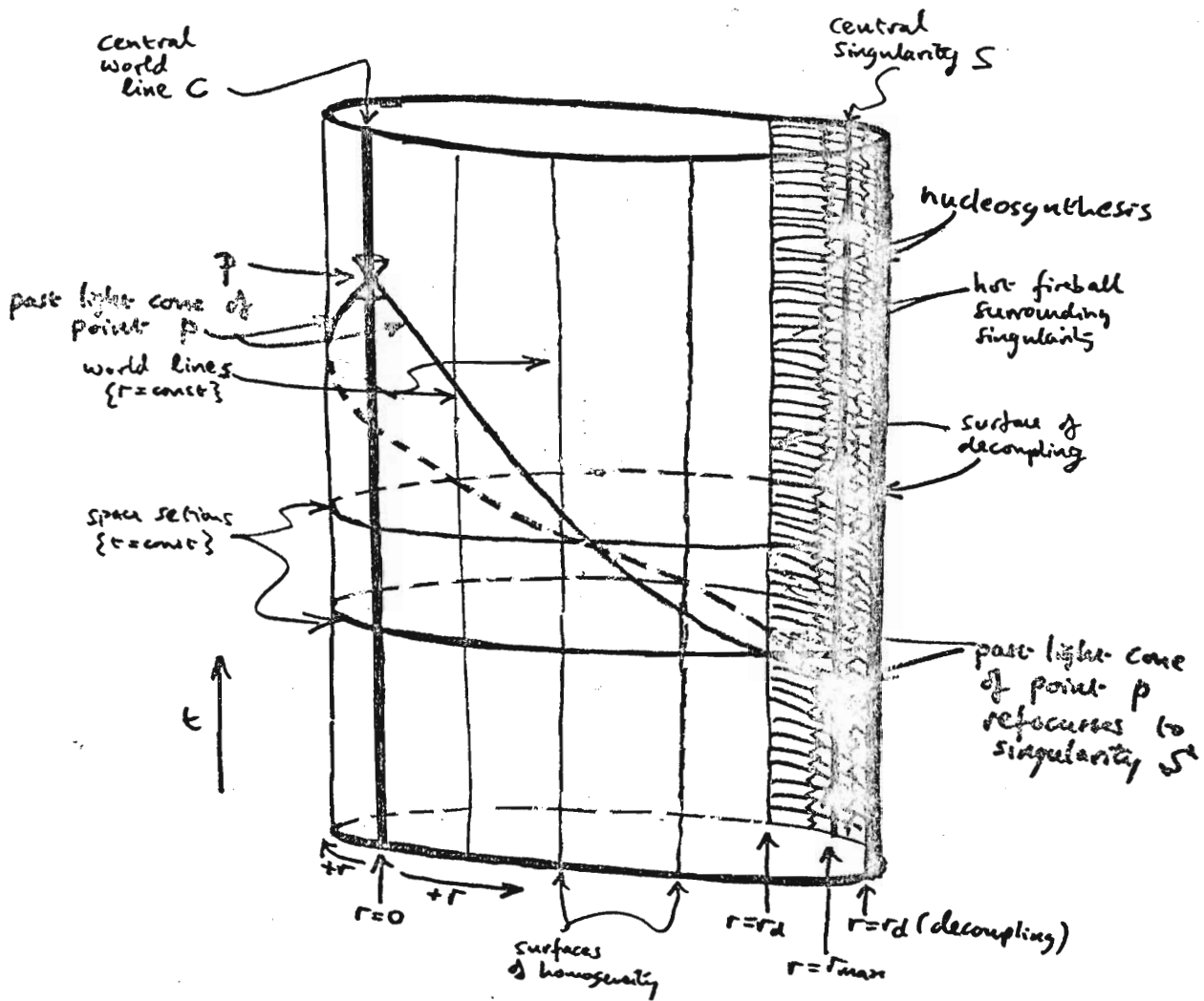


Figure 1(a)

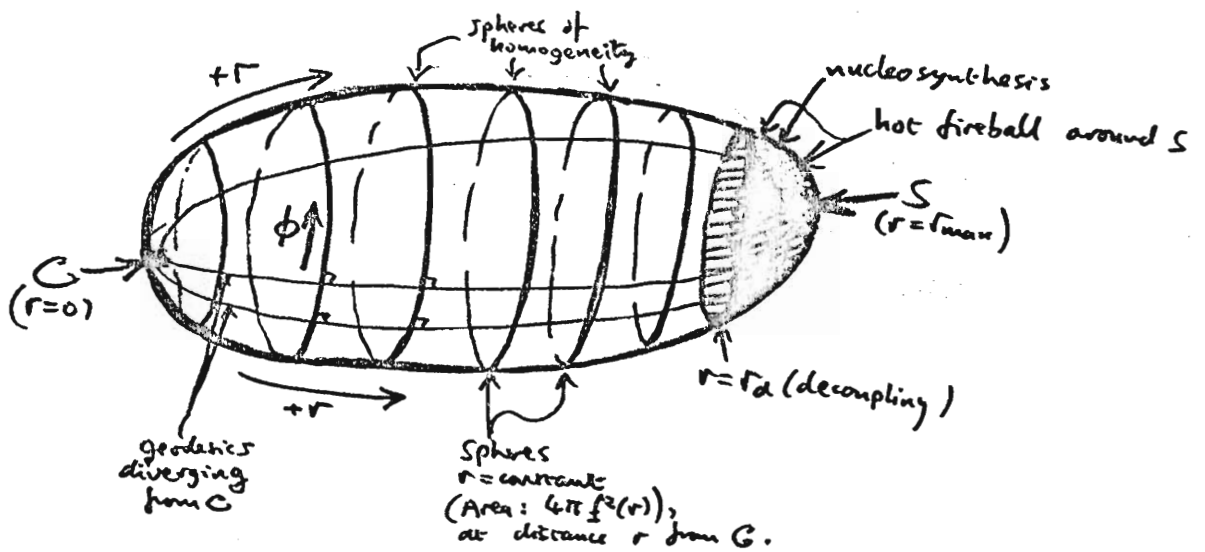


Figure 1(b)

Figure 1

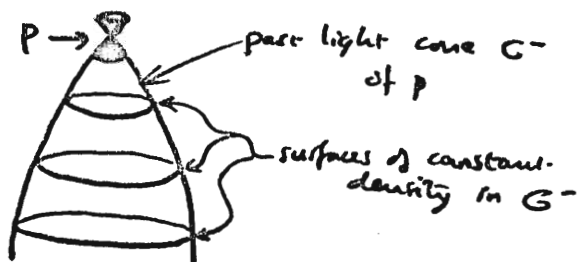


Figure 2(a)

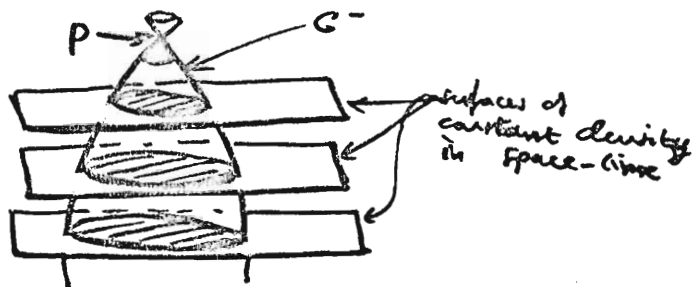


Figure 2(b)

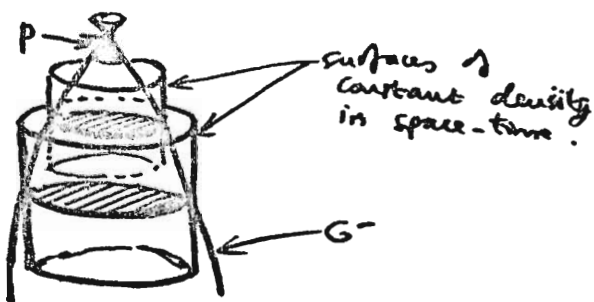


Figure 2(c)

Figure 2

Biographical Sketch

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Between 1964 and 1974 was Fellow of Peterhouse, University Lecturer at Cambridge, visiting Professor at the Universities of Texas, Chicago, Hamburg and Boston, and Fellow of Wolfson College, Cambridge, at various times.

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