STRING THEORY AND GRAVITY

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It is pointed out that string-loop effects may generate matter couplings for the dilaton allowing this scalar partner of the tensorial graviton to stay massless while contributing to macroscopic gravity in a way naturally compatible with existing experimental data. Under a certain assumption of universality of the dilaton coupling functions, the cosmological evolution drives the dilaton towards values where it decouples from matter. At the present cosmological epoch, the coupling to matter of the dilaton should be very small, but non zero. This provides a new motivation for improving the experimental tests of Einstein's Equivalence Principle.

String theory is, at present, the only scheme promising to provide a combined quantum theory of gravity and of the gauge interactions. It is striking that, within string theory, the usual Einsteinian tensor graviton is intimately mixed with a scalar partner: the dilaton. The matter couplings of the dilaton ***a priori*** generate drastic deviations from general relativity, notably violations of Einstein's Equivalence Principle (EP): universality of free fall, constancy of the constants,... This is why it is generally assumed that the dilaton will acquire a (Planck scale) mass due to some yet unknown dynamical mechanism.

An alternative possibility is the following (see [1] for a detailed discussion) : string-loop effects (associated with worldsheets of arbitrary genus in intermediate string states) may naturally reconcile the existence of a massless dilaton with existing experimental data if they exhibit the same kind of universality as the tree level dilaton couplings.

To illustrate this possibility, let us consider the simple case where the effective action for the string massless modes (considered directly in four dimensions) takes the form

$$S = \int d^4x \sqrt{\hat{g}}\, B(\Phi) \left\{ \frac{1}{\alpha'} [\hat{R} + 4\hat{\Box}\Phi - 4(\hat{\nabla}\Phi)^2] - \frac{k}{4}\hat{F}^2 - \overline{\hat{\Psi}}\hat{D}\hat{\Psi} + \ldots \right\} , \tag{1}$$

where the function of the dilaton Φ appearing as a common factor in front is given by a series of the type

$$B(\Phi) = e^{-2\Phi} + c_0 + c_1\, e^{2\Phi} + c_2\, e^{4\Phi} + \ldots \tag{2}$$

The first term on the right-hand side of Eq. (2) is the string tree level contribution (spherical topology for intermediate worldsheets) which is known to couple in a universal multiplicative manner [2] [3] [4]. The further terms represent the string-loop effects: the genus-n string-loop contribution containing a factor $g_s^{2(n-1)}$ where $g_s \equiv \exp(\Phi)$ is the string coupling constant. Apart from the fact that Eq. (2) is a series in powers of g_s^2, little is known about the global behaviour of the dilaton coupling function $B(\Phi)$. For the cosmological attractor mechanism discussed here to apply, two conditions must be fulfilled: (a) the couplings of the dilaton must have the same kind of universality as in the tree level approximation , and (b) the coupling functions of the dilaton must admit a local maximum. In the simple case illustrated in Eq.(1), this universality is guaranteed by the factorization of a common function $B(\Phi)$; string-loop effects, Eq.(2), can then allow this function to admit a local maximum . [More general ways of realizing the needed universality are discussed in [1]].

It is convenient to transform the action (1) by introducing several Φ-dependent rescalings. In particular, one replaces the original "string-frame" metric $\hat{g}_{\mu\nu}$ by a conformally related "Einstein-frame" metric $g_{\mu\nu} \equiv C\, B(\Phi)\hat{g}_{\mu\nu}$, and the original dilaton field Φ by a canonical scalar field φ. The transformed action reads

$$S = \int d^4x \sqrt{g} \left\{ \frac{1}{4q} R - \frac{1}{2q} (\nabla\varphi)^2 - \overline{\Psi} D \Psi - \frac{k}{4} B(\varphi) F^2 + \ldots \right\} , \tag{3}$$

where $q \equiv 4\pi\overline{G} \equiv \frac{1}{4}C\alpha'$ denotes a bare gravitational coupling constant and $B(\varphi) \equiv B[\Phi(\varphi)]$.

The basic clue which allows one to relate string models to the observed low-energy world is the dilaton dependence of the gauge coupling constants apparent in Eq. (3): $g^{-2} = kB(\varphi)$. This dependence implies that the QCD mass scale Λ_{QCD} is given by

$$\Lambda_{QCD}(\varphi) \sim C^{-1/2}B^{-1/2}(\varphi)\exp[-8\pi^2 b_3^{-1} k_3 B(\varphi)]\widehat{\Lambda}_s \ , \tag{4}$$

where b_3 is a (rational) one-loop coefficient associated with the scale dependence of the SU(3) coupling constant, and where $\widehat{\Lambda}_s \simeq 3 \times 10^{17}$GeV [5] is the (string-frame) string unification scale $\propto \alpha'^{-1/2}$. The Einstein-frame mass of hadrons is essentially some pure number times $\Lambda_{QCD}(\varphi)$. More generally, under the assumption of a universal $B(\varphi)$, the masses of all the particles will depend on φ only through the function $B(\varphi)$:

$$m_A(\varphi) = m_A[B(\varphi)] \ . \tag{5}$$

When studying the cosmological evolution of the graviton-dilaton-matter system, one finds that the dilaton vacuum expectation value φ is dynamically driven toward the values φ_m corresponding to a local maximum of $B(\varphi)$, i.e. a local minimum of all the various mass functions $m_A(\varphi)$. [With some important physical differences, this cosmological attractor mechanism is similar to the one discussed in Ref. [6] which concerned metrically-coupled tensor-scalar theories]. The main parameter determining the efficiency of the cosmological relaxation of φ toward φ_m is the curvature κ of the function $\ln B(\varphi)$ near the maximum φ_m:

$$\ln B(\varphi) \simeq \text{ const. } - \frac{1}{2}\kappa(\varphi - \varphi_m)^2 \ . \tag{6}$$

Because of the steep dependence of $m_A(\varphi)$ upon $B(\varphi)$ [illustrated by Eq. (4)], each "mass threshold" during the radiation-dominated era [i.e. each time the cosmic temperature T becomes of order of the mass m_A of some particle] attracts φ towards φ_m by a factor $\sim 1/3$. In the subsequent matter-dominated era, φ is further attracted toward φ_m by a factor proportional to $Z_0^{-3/4}$ where $Z_0 \simeq 1.3 \times 10^4$ is the redshift separating us from the end of the radiation era. Finally, in the approximation where the phases of the ten or so successive relaxation oscillations around φ_m undergone by φ during the cosmological expansion are randomly distributed, one can estimate (when $\kappa \gtrsim 0.5$) that the present value φ_0 of the dilaton differs, in a *rms* sense, from φ_m by

$$(\varphi_0 - \varphi_m)_{rms} \sim 2.75 \times 10^{-9} \times \kappa^{-3}\Omega_{75}^{-3/4}\Delta\varphi \ , \tag{7}$$

where $\Omega_{75} \equiv \rho_0^{\text{matter}}/1.0568 \times 10^{-29}\text{g cm}^{-3}$ and where $\Delta\varphi$ denotes the deviation of φ from φ_m at the beginning of the (classical) radiation era.

The present scenario predicts the existence of many small, but non zero, deviations from general relativity. Indeed, a cosmologically relaxed dilaton field couples to matter around us with a strength (relative to usual gravity)

$$\alpha_A = \left.\frac{\partial \ln m_A(\varphi)}{\partial \varphi}\right|_{\varphi_0} \simeq \beta_A(\varphi_0 - \varphi_m) \tag{8}$$

with $\beta_A \simeq \beta_3 \equiv 40.8\kappa$ for hadronic matter. Therefore, all deviations from Einstein's theory contain a small factor $(\varphi_0 - \varphi_m)^2$ coming from the exchange of a φ excitation. More precisely, the post-Newtonian deviations from general relativity at the present epoch are given by the Eddington parameters

$$1 - \gamma_{\rm Edd} \simeq 2(\beta_3)^2(\varphi_0 - \varphi_m)^2 \,, \tag{9}$$

$$\beta_{\rm Edd} - 1 \simeq \frac{1}{2}(\beta_3)^3(\varphi_0 - \varphi_m)^2 \,, \tag{10}$$

while the residual cosmological variation of the coupling constants is at the level

$$\frac{\dot\alpha}{\alpha} \simeq -\kappa\left[\omega \tan\theta_0 + \frac{3}{4}\right](\varphi_0 - \varphi_m)^2 H_0 \,, \tag{11}$$

$$\frac{\dot G}{G} \simeq -2\beta_3^2\left[\omega \tan\theta_0 + \frac{3}{4}\right](\varphi_0 - \varphi_m)^2 H_0 \,, \tag{12}$$

where $\omega \equiv \left[\frac{3}{2}((\beta_3 - \frac{3}{8})\right]^{1/2}$, and where θ_0 denotes the phase of the matter-era relaxation toward φ_m, while H_0 denotes the present value of Hubble's "constant".

The most sensitive way to look for the existence of a weakly coupled massless dilaton is through tests of the universality of free fall. The interaction potential between particle A and particle B is $-G_{AB} m_A m_B / r_{AB}$ where $G_{AB} = \overline{G}(1 + \alpha_A \alpha_B)$. Therefore two test masses, A and B, will fall in the gravitational field generated by an external mass m_E with accelerations a_A and a_B differing by

$$\left(\frac{\Delta a}{a}\right)_{AB} \equiv 2\,\frac{a_A - a_B}{a_A + a_B} \simeq (\alpha_A - \alpha_B)\alpha_E \,. \tag{13}$$

The difference $\alpha_A - \alpha_B$ introduces a small factor proportional to the ratio $m_{\rm quark}/m_{\rm nucleon}$ or to the fine structure constant α. Finally, one finds an equivalence-principle violation of the form

$$\left(\frac{\Delta a}{a}\right)_{AB} = \kappa^2(\varphi_0 - \varphi_m)^2\left[C_B \Delta\left(\frac{B}{M}\right) + C_D \Delta\left(\frac{D}{M}\right) + C_E \Delta\left(\frac{E}{M}\right)\right]_{AB} \,, \tag{14}$$

where $B \equiv N + Z$ is the baryon number, $D \equiv N - Z$ the neutron excess, $E \equiv Z(Z-1)/(N+Z)^{1/3}$ a Coulomb energy factor and M the mass of a nucleus. The only coefficient in Eq. (14)

which can be reliably estimated is the last one: $C_E \simeq 3.14 \times 10^{-2}$. The largest $\Delta a/a$ will arise in comparing Uranium ($E/M \simeq 5.7$) with Hydrogen (or some other light element). For such a pair Eqs. (7) and (14) yield a violation of the equivalence principle which is well below the present experimental limits

$$\left(\frac{\Delta a}{a}\right)^{max}_{rms} = 1.36 \times 10^{-18} \kappa^{-4} \Omega_{75}^{-3/2} (\Delta\varphi)^2 \ . \tag{15}$$

The results (9)-(12) and (14),(15) provide a new motivation for trying to improve by several orders of magnitude the experimental tests of general relativity, notably the tests of the equivalence principle (universality of free fall, constancy of the constants,...). The scenario summarized here gives an example of a well-motivated theoretical model containing no small parameters and naturally predicting very small deviations from general relativity at the present epoch. In this model, high-precision tests of the equivalence principle can be viewed as low-energy windows on string-scale physics: not only could they discover the dilaton, but, by measuring the ratios C_B/C_E, C_D/C_E in Eq. (14) they would probe some of the presently most obscure aspects of particle physics: Higgs sector and unification of coupling constants.

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