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Dear Sir,

Enclosed is a submission to your essay contest by Louis Crane, of the University of Chicago, and myself.

Sincerely yours,

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SPACETIME FOAM AS THE UNIVERSAL REGULATOR

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ABSTRACT

A distribution of virtual black holes in the vacuum will induce modifications in the density of states for small perturbations of gravitational and matter fields. If these virtual black holes fill the volume of a typical spacelike surface then perturbation theory becomes more convergent and may even be finite, depending on how fast the number of virtual black holes increases as their size decreases. For distributions of virtual black holes which are scale invariant the effective dimension of spacetime is lowered to a noninteger value less than four, leading to an interpretation in terms of fractal geometry. In this case general relativity is renormalizable in the 1/N expansion without higher derivative terms. As the Hamiltonian is not modified the theory is stable.

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One of the oldest and most exciting ideas to be proposed concerning quantum gravity is that the gravitational field should act as a universal regulator, rendering all quantum field theories convergent through nonperturbative quantum gravitational effects below the Planck scale.[1-4] Although there are rather compelling arguments for this idea,[1,2] and effects of this sort are found by summing up certain classes of diagrams,[3,4] up till now no systematic realization of this idea in the context of quantum field theory has been found.

Another idea which has a long history in quantum gravity is that of spacetime foam.[5,6] The basic idea here is that Einstein's action, being of dimension two, does not effectively damp large quantum fluctuations in the metric at scales smaller than the Planck length. As a result one expects that, if Einstein's action governs the dynamics at smaller than Planck scales, the geometry and the causal structure of spacetime should be more and more complicated at smaller and smaller scales.

Recently we discovered that there is a simple mechanism by means of which the high energy behavior of perturbation theory will be regulated if the vacuum contains a distribution of black holes whose density increases with decreasing scale.[7] This mechanism has nothing to do with topology, or changes thereof, but arises from an alteration in the spectrum of perturbations of the metric and other fields induced by the presence of a distribution of black holes in the vacuum. This effect is understood directly in terms of a semiclassical expansion of the path integral over spacetimes with Minkowskian, rather than Euclidean, signature.

It might seem that to describe the physical properties of a vacuum state containing a distribution of virtual black holes we would have to first solve the difficult problems associated with how black holes, real or virtual, disappear, once the semiclassical evaporation process has brought them down to Planck scales. However, we find that these difficult problems can be sidestepped if we are willing to make one assumption concerning their eventual resolution. This is that the phase of the wavefunction of anything which comes out of a black hole is completely uncorrelated with the phases of the wavefunctions of everything which went into the formation of the black hole. In addition the phases associated with different particles which emerge from the black hole are uncorrelated. In other words, we assume that the disordering of the phase of the quantum state which Hawking discovered in the semiclassical theory for real black holes[8] is actually true in the full quantum theory, for both real and virtual black holes.[9,10]

Given this assumption no process involving something which falls into and then emerges from a black hole can contribute coherently to a sum over virtual intermediate states in perturbation theory. Then, if the background manifold contains black holes, only those small perturbation which vanish on the apparent horizons of the black holes can contribute coherently to perturbation theory. In other words, while black holes in the background geometry cannot contribute to perturbation theory directly as intermediate states, they have an indirect effect in that the presence of the black holes alters the boundary conditions that the states which do contribute coherently must satisfy.

Once we make this assumption we can be confident about the use of a semiclassical expansion of the functional integral to describe a vacuum state containing a distribution of virtual black holes. This is because, while it may not be correct to describe the geometry inside the horizon of a virtual black holes semiclassically, we have no such doubts concerning the evolution of the geometry and the fields in the region outside the black hole horizons. Thus, we can construct a reliable semiclassical model of spacetime foam in which the virtual black holes are represented by black holes in the background metric, and the quantum fields which correspond to small perturbations in the gravitational and matter fields satisfy the boundary condition that they vanish on and inside their apparent horizons.

Of course it is difficult to construct a manifold which has a distribution of black holes at all scales. Thus, we proceed by constructing a cutoff functional integral in which a short distance cutoff, a, is imposed on the background geometry such that all black holes in the background must have a radius greater than a, and must be spaced more than a distance a apart. Then 1/a is imposed as a high frequency cut off for the spectrum of perturbations and, after computing the Green's functions, we define the theory by taking the limit a ->0.

There is then a simple effect by which the distribution of black holes in the background induces an additional cutoff dependence in sums over virtual states in perturbation theory. As a result of the imposition of the boundary condition mentioned above the density of states for the perturbations of any field are multiplied by a factor A(a) which is the volume of a typical three surface which remains once the interiors of the black holes have been excluded.[7]

In order to compute A(a) we must know the density of black holes of various sizes in the vacuum. This may be specified by a function $\rho(b)$ which is defined such that $\rho(b)db$ is the number of black holes per unit volume in the vacuum with radii between b and b+db.

The naive arguments for spacetime foam, which are based on the uncertainty principle and the form of the Einstein action,[5,6] suggest that $\rho(b)$ should be negligible for $b > l_{Planck}$, but should diverge as $b \to 0$. Unfortunately, we cannot compute $\rho(b)$ by perturbative methods, so at the present time we are restricted to discussing the effect that various forms of $\rho(b)$ would have on the divergence structure of perturbation theory. At the present time we are investigating several ideas for computing $\rho(b)$, based on the strong coupling expansion[11] or on a Monte-Carlo simulation involving the Regge calculus.[12]

In order to compute the effect of $\rho(b)$ on the densities of states it is most likely sufficient to consider a model in which a typical three surface containing a distribution of black holes with some distribution of relative velocities is approximated by a three surface, $\Sigma(a)$ on which all the black holes are momentarily at rest with respect to each other. This is convenient because the metric of such a three surface, with an arbitrary distribution of black holes, is known to have the simple form,[13]

$$g_{ij}(x) = \Omega^4(x)\,\delta_{ij} \tag{1}$$

where

$$\Omega(x) = 1 + \sum_\alpha \frac{m_\alpha}{2|\vec{c}_\alpha - \vec{x}|} \tag{2}$$

Here m_α and $\vec{C}_\alpha$ are, respectively, the masses and the positions of the centers of the black holes. The surfaces $|\vec{C}_\alpha - \vec{x}| = m_\alpha/2$ are apparent horizons which are called the throats of the black holes.

These metrics are complicated inside of these horizons, but as we are only interested in perturbations which vanish inside of all horizons this is of no concern to us. This model does have the disadvantage that Lorentz and translational invariance are lost for the spectrum of perturbations on a given background $\Sigma(a)$. However we believe that this is not essential and that these symmetries may be restored in a more sophisticated version of this model, perhaps involving summing over a class of background manifolds, without changing the results which follow.

On $\Sigma(a)$ the density of states for any field will have the asymptotic form,

$$D_a(\omega)\,d\omega = K\,\omega^2 A(a)\,d\omega\,V \qquad (3)$$

where K is a collection of constants.* In this model it is simple to compute the relationship between $\rho(b)$ and $A(a)$. We find,

$$A(a) = \exp\left[-\frac{4\pi}{3}\int_a^\infty b^3 \rho(b)\,db\right] \qquad (4)$$

Since $1/a$ is the cutoff frequency we have,

$$D_a(\omega) \leq K\,\omega^2 V \exp\left[-\frac{4\pi}{3}\int_{1/\omega}^\infty b^3 \rho(b)\,db\right] \qquad (5)$$

Thus we see that if $\rho(b)$ is sufficiently divergent as $b \to 0$ the effect of the high frequency fluctuations will be regulated.

It is easiest to examine this by first considering some examples. Assume to begin that $\rho(b)$ diverges as $b \to 0$ as,

$$\rho(b) = \frac{c}{b^4}\left(\frac{l_{planck}}{b}\right)^q \qquad (6)$$

with $q>0$. Then we find that,

$$A(a) = \exp\left[\frac{4\pi c q}{3}\left(1 - \left(\frac{l_{planck}}{a}\right)^q\right)\right] \qquad (7)$$

Thus, an exponential cutoff is induced in the density of states at a frequency

$$\omega_0 = \left(\frac{3}{4\pi c q}\right)^{1/q} \omega_{Planck} \qquad (8)$$

and all sums over virtual states are convergent. Thus, for such distributions the proposal that nonperturbative gravitational effects at short distances render all quantum field theories finite is realized.

A somewhat different proposal for the high energy behavior in quantum gravity, known as asymptotic safety,[14] is realized if $\rho(b)$ is of the form of (6) with $q=0$. This proposal states that general relativity becomes

* and V is a volume, which has been introduced as an infrared cutoff.

renormalizable through a non-trivial fixed point of the renormalization group, the result being that Green's functions become asymptotically scale invariant in the limit of short distances. Now it has been known for several years that this proposal can be realized by adding to the Lagrangian terms of dimension 4.[15] However, because the metric is dimensionless these terms involve four derivatives so that when they are added to the theory the Hamiltonian is no longer bounded from below and there is no stable ground state.[16]

It has been suspected that this problem could be avoided, and the proposal of asymptotic safety rescued, if it were the case that nonperturbative effects induced an anomolous dimension for the gravitational field. It is remarkable that this is exactly the effect of a scale invariant distribution of virtual black holes, which is that given by (6) with q=0. In this case the density of states is found to have the form,[7]

$$D_q(\omega) = K\omega^2\left(\frac{q}{\ell_{planck}}\right)^{\varepsilon} V \le K\omega^{2-\varepsilon}\omega_{planck}^{\varepsilon} V \qquad (9)$$

where $\varepsilon=4\pi C/3$. In this case all fields behave at high energy as if they were in a spacetime of $4-\varepsilon$ dimensions. Given this one can show using the 1/N expansion that general relativity becomes renormalizable without any counterterms other than the usual Einstein and cosmological terms.[15,7] The renormalization proceeds through a nontrivial fixed point in the gravitational and cosmological constants, details are given in[7]. After renormalization the graviton propagator is found to have the form,

$$S(p^2)_{\mu\nu\alpha\beta} = \frac{i P^{(2)}_{\mu\nu\alpha\beta}}{p^2\left(M_{Pl}^2 - \frac{N B(\varepsilon)}{\varepsilon}(-p^2)^{1-\frac{\varepsilon}{2}} M_{Pl}^{\varepsilon}\right)} \qquad (10)$$

where $B(\varepsilon)$ is a finite constant. As $q^2 \rightarrow \infty$,

$$S(p^2) \longrightarrow \frac{\varepsilon}{N B(\varepsilon)(-p^2)^{2-\frac{\varepsilon}{2}} M_{Pl}^{\varepsilon}} \qquad (11)$$

showing that the anomolous dimension of the graviton is $\varepsilon/2$.

This behavior can also be understood in terms of a fractal picture of spacetime geometry, because the set of points which remains when one removes from R^3 a random scale invariant distribution of spheres, corresponding to the interiors of the black holes, is fractal.[7,17]

One thing which is very good about both of these cases is that the regularization induced by foam is universal and effects both gravitational and matter perturbations. This is necessary to get a sensible quantum theory involving gravity because even in the semiclassical theory logarithmic divergences in $\langle T_{ab}\rangle$ induce untenable instabilities.[18] If these are to be avoided the regularization must render logarithmic divergences finite.

Since $\rho(b)$ has yet to be calculated it is interesting to see what forms of $\rho(b)$ will accomplish this. To investigate this it is useful to define a scale dependent anomolous dimension,

$$\varepsilon(a) = \frac{\partial \ln A(a)}{\partial \ln a} = \frac{4\pi}{3} a^4 \rho(a) \qquad (12)$$

For $\varepsilon(a)$ small it may be shown that $d(a)=4-\varepsilon(a)$ is a generalization of Mandelbrout's scaling dimension, and that its limit, as $a\to 0$, is the Hausdorff dimension. It is easy to show that logarithmic divergences will be regularized if the integral,

$$J = \int_0^1 \frac{da}{a} \exp\left[-\int_a^{\ell_{planck}} \frac{db}{b}\varepsilon(b)\right] \qquad (13)$$

is finite. One can show that a necessary condition for this is that $A(0)$, the three volume of the set of points not contained in black holes of all scales, is zero.[7] However, it is not necessary that the Hausdorff dimension be less than four for J to be finite. For example, it is sufficient that $\varepsilon(a)>0$ for $a>0$ and that it go to zero as $a\to 0$ at least as slowly as $C/|\ln a|$ with $C>1$.[7]

The graviton propagator (10) clearly has a nonunitary piece. However, this cannot be a reflection of an instability because no new counterterms have been added to the Lagrangian. The Hamiltonian is still that of general relativity and, as the arguments which guarantee the positivity of the energy[20] still hold for $\Sigma(a)$ of the form of (1 & 2), perturbation theory should be stable. Instead, of being connected with an instability, the nonunitarity of (10) is a consequence of our boundary condition, which has forced us to truncate sums over virtual states by removing any which correspond to particles falling into black holes. Thus the loss of unitarity is connected with our original assumption, it is in fact a measure of the loss of quantum coherence induced by the presence in the vacuum of virtual black holes.

In spite of the loss of unitarity we believe that it will be possible to show that the theory conserves probability. This will require a framework which generalizes quantum theory in that it allows the loss of quantum coherence and the evolution of pure states to mixed states. Two possiblities for such a formalism are Hawking's dollar matrix formalism,[9] and stochastic quantization on curved manifolds.[21]

We close by noting that this idea, if correct, has important implications for particle physics. As all logarithmic divergences must be regularized to avoid instabilities, dimensionless coupling constants and fermion masses can suffer only finite renormalizations. In particular a number of formerly divergent mass differences now become computable. In addition four-fermi interactions now become renormalizable in the context of a 1/N expansion[22] and it is likely that ordinary supergravity becomes similarly renormalizable. Finally, the loss of quantum coherence due to the virtual black holes will lead to universal CP violation. This will be tiny on elementary particle scales, but it is not possible to avoid the speculation that this may be responsible for part or all of the CP violation observed in K-mesons.

What is most exciting is that by computing $\rho(b)$ we may be able to make definite predictions for these and other previously noncomputable quantities.

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