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QUANTUM HAIR AND QUANTUM GRAVITY

Sidney Coleman*
Lyman Laboratory of Physics, Harvard University, Cambridge MA 02138

Lawrence M. Krauss#
Center for Theoretical Physics and Department of Astronomy, Yale University, New Haven CT 06511

John Preskill^
Lauritsen Laboratory of High Energy Physics, California Institute of Technology, Pasadena, CA. 91125

Frank Wilczek%
School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton N.J. 08540

ABSTRACT:

The recent discovery that black holes may harbor a type of quantum hair, invisible classically, but measurable via quantum interference experiments, can have important dynamical effects which alter the process of black hole evaporation. A consideration of the effects of quantum hair on such processes as black hole evaporation and black hole scattering may lead to important new insights into such issues as: the origin of black hole entropy, the loss of quantum coherence, and the existence of topology changing processes---all of which must be addressed any theory of quantum gravity.

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It is one of the wonders of classical general relativity that in spite of the complexities associated with gravitational collapse and the formation of black holes, the objects which eventually emerge from this process appear remarkably simple. Powerful uniqueness theorems assert that a stationary black hole can be completely characterized by its mass, angular momentum, and electric charge [1]. Our ignorance about the other microphysical parameters which might characterize the black hole internal state has been suggested as an explanation of the large intrinsic black hole entropy[2]. This entropy is in turn related to an effective temperature associated with quantum field configurations calculated in a classical black hole background--leading to the phenomenon of black hole radiation.[3]

In spite of the apparent connection between black hole radiance and the no-hair theorems, a fundamental tension between these ideas looms beneath the surface. If a large number of internal states exist for the black hole it is difficult to conceive of a quantum mechanical framework in which these states cannot be excited in any measurable way. Any S matrix description, for example, should keep track of the initial and final states, including the possible excitation of internal degrees of freedom. This situation is exacerbated when one considers the logical extreme of a black hole which is sufficiently small so that one must treat it as a quantum mechanical object. If this is to be possible, logic suggests that the no-hair theorem must eventually break down. As elementary particles possessing arbitrarily internal quantum numbers become sufficiently heavy, surely they become indistinguishable from sufficiently small black holes. After all, their Compton radius becomes much smaller than their Schwarzchild radius, and thus for all operational purposes they exist inside their event horizons. It is difficult to imagine any smooth transformation in which such internal quantum numbers slowly become unmeasurable.

While any complete theory of quantum gravity must address such issues, one might hope to resolve them, at least in part, even without such a theory. In this regard, a recent development seems promising. It has been shown [4,5] that when quantum mechanical effects are properly taken into account, even macroscopic black holes can have detectable quantum "hair", which is invisibly classically. This demonstrates that quantum mechanical effects can effectively obviate the no-hair theorem [4]. More important perhaps, quantum hair has dynamical effects which give us a new handle on some central questions of quantum gravity. We describe in this essay how the process of black hole evaporation can be altered. This leads us to speculate on the possible implications of quantum hair on such outstanding issues as the role of topology changing processes and the loss of quantum coherence in quantum gravity, the origin of black hole entropy, and the relationship between thermodynamics and gravity.

We begin by reminding the reader of the canonical example of a theory with quantum hair introduced in [4]. If a simple U(1) gauge theory is spontaneously broken to a Z_n subgroup by the vacuum expectation value of a scalar field with charge N, then there are topological defects which trap magnetic flux in integer multiples of $2\pi/e$. A charge 1 particle can undergo Aharonov-Bohm scattering off of such defects. This scattering occurs in spite of the fact that no long range gauge fields mediate the interaction. Thus, a Z_n charge can be detected at long range, an effect which persists even if it falls into a black hole. In fact it has been verified formally that there are well defined non-local operators which can measure, by surface effects alone, the existence of discrete Z_n charges inside a volume.[5]

While the existence of observable Z_n charge follows directly from well understood physics, it is remarkable nonetheless. The classical metric outside the event horizon of a black hole is unaffected by the existence of a finite Z_n discrete charge inside of it. The fact that a Z_n charge can be attributed to the black hole indicates that the metric alone does not convey sufficient information to calculate all the properties of static black hole solutions. This can be true for dynamical properties as well. In particular, the properties of radiation emitted by a black hole will alter in the presence of Z_n charge.

The process of black hole evaporation can be understood in a number of complementary ways, all of which appear equivalent. It is simplest to begin with the following heuristic picture: virtual particle pairs are created near the event horizon and if the gravitational potential at this surface is sufficiently great, so as to exceed the rest energy of the particles, one of these may escape to infinity, while the other falls into the black hole. Seen in this way, particle creation in the strong background field at the event horizon is analogous to particle creation in a strong electric field. To probe the thermal aspect of the outgoing radiation in the former case, however, one must extend this simple picture. Two alternative strategies exist, one involving a Minkowski space calculation, and one requiring an analytic continuation to Euclidean space. In the former, a vacuum state defined by a free-falling observer near the event horizon is shown to be expressible, via a Bogoliubov transformation, in terms of N particle states at infinity, with coefficients appropriate to a thermal spectrum of particles [6]. In the latter case, for the Euclidean section of the Schwarzchild geometry, the time coordinate τ, turns out to be an angular variable, periodic with period β, where $\beta^{-1}=T_{bh}$, is the Hawking temperature characterizing the outgoing radiation.

Endowing a black hole with classical electric charge changes both these results by explicitly altering the metric in a way which changes the Hawking temperature T_{bh}. How does this picture alter in the presence of Z_n quantum hair? Because the metric is unaffected, it is clear that one must go beyond the leading semi-classical approximation represented by the techniques described above. Moreover, because the phase information encoded in the Z_n hair is explicitly non-local, considerations of local particle pair creation alone will not be expected to be sufficient to reveal its impact.

Both of these factors can be accommodated via Euclidean path integral techniques. [7,8] Here, effects which are nonperturbative in $\hbar$ can be incorporated into a systematic small $\hbar$ instanton approximation. Moreover, global effects enter directly into the specification of the path integrals. In particular, while virtual pair production appears insensitive, at least locally, to the Z_n charge on a black hole, virtual cosmic strings are not. A string which winds k times around an object with Z_n charge Q acquires the Aharonov-Bohm phase $\exp(2\pi ikQ/N)$.

If one does not specify its charge, then the free energy F of a black hole in equilibrium with a heat bath is specified by [9],

$$Z(\beta) = e^{-\beta F} = \int_\beta e^{-S_E/\hbar}$$

where S_E denotes the Euclidean action, and the path integral is restricted to geometries with topology $R^2 \times S^2$ that are periodic in τ with period $\beta\hbar$. By expanding S_E about its saddle point, one can, in the $\hbar \to 0$ limit, extract the standard results of black hole thermodynamics.

If one wishes to compute the free energy $F(\beta,Q)$ in the charge Q sector of a Z_n theory [7,8], then one must weight the different field configurations in the path integral by different phases. In particular, configurations involving the world sheet of a thin cosmic string will be specified by the winding number k introduced above. A string with tension κ_{string} has an action κ_{string} times the area of the worldsheet, which will be minimized if this worldsheet hugs the event horizon. If the string tension is small in Planck units, so that the back reaction of the string on the background geometry can be neglected, the presence of such a minimal worldsheet increases the action by $\approx A_{bh}\,\kappa_{\text{string}}$. Incorporating the necessary phase associated with this worldsheet gives the dominant charge-dependent correction to the partition function Z, of the form

$$Z_Q(\beta) \approx Z_0(\beta)[\,1 + C(\beta)\cos(2\pi Q/Ne)e^{-A_{bh}\kappa_{string}}]$$

where A_{bh} is the area of the event horizon, and $C(\beta) > 0$ is a numerical factor coming from small fluctuations about the minimal worldsheet, and $Z_0(\beta)$ is the partition function calculated in the absence of instanton effects.

This alters the combination βF, and yields a correction to β of the form

$$\beta \approx 8\pi GM\,[\,1 - 4G\kappa_{string}C(8\pi GM)\cos(2\pi Q/Ne)e^{-16\pi M^2\kappa_{string}}]$$

Thus, the presence of quantum hair due to a non-zero Z_n charge lowers the temperature of a black hole of given mass, just as classical U(1) hair due to electric charge does! One can substantiate these results and go beyond the thin string limit with more formal machinery [8]. However the central point is clear already. The dynamics of black hole evaporation in the presence of quantum hair is altered!

We might hope to use this result to resolve a problematic discontinuity in the classical description of black holes in gauge theories. When the gauge theory is unbroken, and the vector meson mass is zero, the black hole may carry hair with impunity. However, no matter how small the scale of symmetry breaking, and how small the vector meson mass, absolutely no classical hair is allowed on a stationary black hole once the symmetry is broken. In the classical theory, there is no smooth limit as the vector meson mass goes to zero. One might hope that the existence of quantum hair would allow such a smooth limit to be recovered. As the mass scale of the symmetry breaking μ goes to zero, the virtual strings become thick, so that one must extend the simple analysis discussed above.[8] Aside from this technicality, it appears at first that a path integral approach could be used to show at least formally, that in the N-> infinity limit, the string world sheet instantons which dominate the saddle point evaluation of the Euclidean path integral generate the Euclidean Reissner-Nordstrom solution. In this way the thermodynamic behavior of a black hole with quantum hair would match smoothly to that of a black hole with classical U(1) hair in this limit. Unfortunately, in the semiclassical small $\hbar$ limit, in which the instanton approximation is valid, this hope is not fulfilled. The Z_n charge is quantum mechanical, in the sense that, in order not be be screened, a Z_n charge Q must satisfy, $Q< Ne\hbar/2$, where here we include $\hbar$ explicitly. Thus, Z_n charge does match onto finite electric charge in the classical limit. Any attempt to resolve the discontinuity between the symmetric and broken phases of the theory must therefore go beyond the semiclassical approximation.

Before proceeding to consider other possible effects of quantum hair, we pause to discuss the challenge for recovering the above result in a Minkowski space formalism. After all, while the Euclidean result seems unambiguous, the Euclidean formulation of quantum gravity is notorious for the ambiguities it presents when one tries to continue back to Minkowski space [10]. It would be desirable to have a real time formulation demonstrating the same leading behavior. It should be clear from our earlier discussion however, that local particle creation calculations are insensitive to Z_n charge. Sensitivity to global effects might be obtained if boundary conditions necessary to define the vacuum state in the black hole background are carefully considered. It is clear, for example that on a surface located just outside the event horizon, massive vector meson fields (associated with the virtual strings in the Euclidean formulation) may have different boundary conditions if non-zero Z_n charge is contained inside the black hole. How this might explicitly be reflected in the definition of the vacuum state for a free-falling observer near the event horizon, and hence in the character of the outgoing radiation, remains to be determined however.

We believe there are other ways in which investigations of quantum hair may facilitate the transition from a classical to a quantum (i.e. elementary particle) description of small black holes. The thermodynamic relations suggested by black hole radiance and black hole entropy have raised the question, whether thermodynamics is more fundamental than quantum mechanics.[11] If a pure state collapses to form a black hole, whose subsequent evaporation results in a mixed state, an S matrix approach seems doomed to failure. On the other hand, it is quite possible that quantum coherence arises from the existence of quantum hair, particularly in its nonabelian version [12]. Such hair allows some aspects of the internal configuration of the black hole to be determined via scattering experiments. It is unclear at present whether enough about the internal state could be inferred to establish the existence of correlations between radiation emitted at different times.

A more radical possibility is that the leading semi-classical result changes dramatically when quantum effects become significant, and therefore that small black holes do not radiate as semi-classical results suggest. For example, for N sufficiently large, and for finite $\hbar$, there can be cases in which the objects with the largest ratio of charge to mass are black holes, so that evaporation is kinematically forbidden, and black holes are stabilized [7,8]. The possible stabilization of small black holes is relevant for the final, nagging, issue in the present debate over the proper approach to quantum gravity. If black holes evaporate completely, then the inclusion of topology changing processes seems required in any Euclidean path integral approach to quantum gravity. On the other hand, if black holes are stabilized by the something like quantum hair, the inclusion of topology changing configurations in the path integral would be less well motivated.

We believe that these issues, raised by the possible dynamical consequences of quantum hair, are exciting enough to encourage further work. In any case, the central claim of this paper---that quantum hair can alter the classification of black holes and their dynamics in dramatic ways---seems well established.

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