SHEAR HELL HOLES AND ANISOTROPIC UNIVERSES

by

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SUMMARY

If the early Universe was highly anisotropic, primordial black holes may have formed prolifically (despite previous claims to the contrary) even if the initial density fluctuations were small. However, the holes would initially be endowed with an immense amount of shear, so it is not obvious that they would evolve into the conventional type of stationary black hole envisaged by the "No Hair" theorem. If they do settle down to a stationary state, it may only be on a considerable timescale; and in principle there might exist soliton-type solutions which represent holes with shear which persists indefinitely. Such "shear hell holes", as we term them, could have even more dramatic properties than the usual stationary holes: in particular, they might be prolific generators of gravitational radiation and they could be associated with interesting quantum effects.

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INTRODUCTION

Traditionally it is assumed that a black hole will rapidly settle down to a stationary state. Any initial perturbations in the shape of the event horizon, reflecting asymmetries in the matter configuration which originally collapsed to form the black hole, are assumed to be radiated away as gravitational waves. Since it is now known that stationary black holes can be characterized[1-4] by just three parameters - their mass M, angular momentum J and electric charge Q - this implies that, shortly after their formation, all black holes are described by one of the standard stationary solutions. (For simplicity we henceforth assume that J=Q=0.) However, the assertion that a black hole rapidly settles down to a stationary state has only been proved in the context of weak perturbations, when the unperturbed background geometry can be treated as nearly Schwarzschild. It has neither been proved analytically nor indicated by numerical experiments in the situation where the initial fluctuations are so large and non-linear that the background cannot be treated as Schwarzschild.

In this essay we will discuss a particular situation, gravitational collapse in an anisotropic universe, where the initial perturbation in the horizon configuration (associated with shearing motions) is very large indeed. In this context the assumption that a black hole rapidly settles down to a stationary state must be questioned very seriously. If it transpires that the shearing motion of the event horizon can persist for a long time, such a hole could have very bizarre properties indeed.

Imagine a black hole whose surface is distorting very violently. One envisages it behaving rather like a localised Mixmaster universe, puffing out first in one direction and then another because of extremely high anisotropy in the local three-space curvature. Such an object would pose a daunting threat to any astronaut who happened to stray too close to it. If he were located a few thousand kilometres from a solar mass Schwarzschild hole, he might consider himself reasonably safe. But his complacency would turn into a singular state of distress should the hole turn out to be violently shearing. The hole might suddenly squelch out in his direction, envelope him and suck him into its singularity before bouncing out towards the next unsuspecting astronaut!

If such objects existed in the real Universe, they might well be described as "hellish". Indeed the above, somewhat melodramatic, description is presented merely to motivate calling them "shear hell holes". We claim that such objects might have formed (and formed prolifically) if the early Universe were highly anisotropic and inhomogeneous. In fact, the relevant question is not whether such objects exist (since, in the absence of precisely spherically symmetric collapse, even the usual sort of black hole may go through a shearing phase), but rather how long such an object can persist before its oscillations are dissipated by gravitational radiation damping. In this essay we discuss, firstly, why shear hell holes might form in an anisotropic universe and, secondly, what their mathematical and physical characteristics might be.

GRAVITATIONAL COLLAPSE IN AN ANISOTROPIC UNIVERSE

The expansion of the Universe cannot be exactly isotropic. The existence of density fluctuations (as required for galaxy formation) necessarily induces local shearing motions and, in addition, there may be a global (homogeneous) shear field, perhaps a relic from an early chaotic phase[5] in the history of the Universe. Such a homogeneous shear can be described by a quantity σ, with units $(\text{time})^{-1}$, which specifies how much the cosmological expansion rate varies with direction. Although the shear is known to be small now, it may have been large at early times because σ increases with reshift like z^3 in the simplest models. Indeed the effective energy density associated with the shear $\sim \sigma^2/G$ must dominate the density of the Universe before some time t_S related to the present value of σ.

The simplest homogeneous anisotropy-dominated model, the Kasner solution[6] (Bianchi 1), contains only one more parameter than the Friedmann model: the three cosmological scale lengths R_i (i=1,2,3) go like t^{P_i} where $\Sigma P_i = \Sigma P_i^2 = 1$. The Kasner indices P_i (when suitably ordered) are thus constrained to lie in the range $-1/3 \leqslant P_3 \leqslant 0 \leqslant P_2 \leqslant 2/3 \leqslant P_1 \leqslant 1$, so the Universe expands in directions 1 and 2 but collapses in direction 3. Despite its simplicity, the Kasner model is a behavioural paradigm for more general anisotropic cosmologies. Extra kinematic features tend to be unimportant at early times and even the most complicated homogeneous models like Mixmaster (Bianchi 1X) display Kasner-type behaviour most of the time (although the values of P_i change periodically[9]). We thus confine attention to the Kasner model in the following considerations.

The problem of black hole formation in an anisotropic Kasner universe has been studied in a previous paper[8]. If one considers a region with an initial density fluctuation (be it in the shear or the matter density), one can show - by considering the perturbation to the equation for the expansion rate - that its volume will eventually stop increasing provided a quantity which can be interpreted as the total initial energy in the region is negative. However, density fluctuations grow at different rates in different directions[9], which means that a bound region will not in general stop expanding in every direction at the same time. Alternatively, if one considers regions which <u>are</u> binding in all directions at any particular time, they will not in general be spherically symmetric. The behaviour of a bound region in a Kasner universe is therefore much more complicated than that of a (spherically symmetric) bound region in an isotropic universe, essentially because the density inhomogeneity couples with background shear to first order. As the region collapses and bounces in various directions (perhaps passing through pancake and spindle configurations), it will squelch around violently until equipartition and virialization can be achieved. This reflects the fact that, even after the volume of the region has stopped increasing, it may still be endowed with a lot of shear energy. Indeed, just as in the Kasner background, the shear energy within the region may be much larger than the rest mass energy of its matter content.

The manner in which such a region could collapse to a black hole is far from clear. It might just collapse to a conventional stationary black hole by dissipating its shear energy first (although a large fraction of the dissipated energy must still go into the collapsing region). On the other hand, it might collapse without dissipating its shear at all to form what we have termed a shear hell hole. Still more dramatically, it might produce a naked singularity without any horizon. In any case, if collapse is to occur at all, the region must be bigger than the Jeans length when it stops expanding. The Jeans length is the scale on which the gravitational binding energy of a region at maximum expansion equals its internal energy, where we must account for the fact that both these terms include and are indeed dominated by a contribution which either derives from the dissipated shear kinetic energy or is the shear if it has not been dissipated. This naively implies that the Jeans length is of order the horizon size - or more precisely, since the region is not spherical, the Jeans volume is of order the horizon volume[8]. Since a bound region cannot be bigger than the horizon size in any direction (else it would close up in that direction and form a separate closed universe[10]), this means that a region can collapse to a black hole only if at maximum expansion it has of order the horizon size ~ $(1-P_i)^{-1}ct$ in all three directions. It is very unlikely that such a condition would be satisfied, and this led us to conclude in our previous paper that no black holes could form in the anisotropic period before t_S.

This argument is incomplete, however, because it neglects the fact that the violent space-time oscillations within a collapsing region will generate relativistic shock waves - indeed such shock waves might play a major role in dissipating the shear energy into internal energy. Simple energy arguments show that the Lorentz factor Γ of these shocks would be of order (shear energy in region/rest mass energy in region)$^{1/2}$. If the ratio of these energies at time t is the same as the ratio in the Kasner background, one infers $\Gamma \sim (t/t_S)^{(f-1)/2}$ where $p = f\rho$ $(0 \leq f \leq 1)$ is the equation of state of the matter. This means that the Jeans length is reduced by a factor of order $\Gamma^{-1/2} \sim (t/t_S)^{(1-f)/4}$, suggesting that black hole formation is easier than in an isotropic universe for $t << t_S$. (In an isotropic Universe, the Jeans length[10] is $\sim \sqrt{f}$ ct.) In this case, the conclusion of our previous paper is reversed: we would now claim that black holes can only form before t_S (because, if the initial density fluctuations were large enough for holes to form after t_S, they would be overproduced before t_S) with masses extending up to $10^5 \left(\frac{t_S}{1s}\right) M_\odot$. Furthermore black holes can form before t_S even if the initial density fluctuations are quite small.

SHEAR HELL HOLES

We now discuss the question: into what sort of black hole does such a region evolve? If one considers a star with small perturbations from spherical symmetry undergoing gravitational collapse, one does not expect these perturbations to grow dramatically as they fall within the event horizon[11]. Indeed, from the point of view of an external observer, all perturbations (either in the shape of the star or its accompanying fields), other than those associated with conserved multipole moments (i.e. mass, angular momentum, and charge), die out as the star approaches the horizon[12]. Any dynamic perturbations are radiated away as gravitational and electromagnetic waves and these waves merely leave behind a "tail" which, for an initial l-pole perturbation, decays like $t^{-(2l+2)}$. (For a shear perturbation $l=2$.) Any static perturbation, frozen into the star as it crosses the horizon, is prevented from propagating to an external observer because of the curvature of spacetime: the corresponding "infinite wavelength" perturbation is completely reflected by a gravitational barrier just outside the horizon[12]. An external observer therefore expects to see the hole settle down to one of the standard stationary solutions and this is the basis of the famous "No Hair Theorem". Certain types of perburation, the so-called quasi-normal-mode perturbations, damp out exponentially on a timescale[13] of order GM/c^3 and it has been conjectured (but not proved) that all perturbations will be channeled into these quasi-normal-modes[14]. In this case all black holes would settle down to a stationary state on the Schwarzschild timescale.

Now all these arguments assume that the background geometry is nearly Schwarzschild (or Kerr if the collapsing star is rotating). They do not apply if the initial perturbations are highly non-linear and if the background geometry is more complicated. It is not known what a shear hell hole background would look like but it certainly would not be Schwarzschild. Since the collapsing region is something like a Mixmaster universe, pulsating between a minimum and maximum radius along each axis, one envisages a black hole solution which bears a comparable relationship to Mixmaster as Schwarzschild does to the "k=+1" Friedmann universe (which, it is recalled, models the collapsing matter in a spherically symmetric situation). It is far from obvious that such a solution would quickly settle down to a stationary state. In particular, the notion that there are normal-mode perturbations which decline exponentially and that there is a potential barrier which perfectly reflects long wavelength perturbations is questionable. Despite these reservations, it is not inconceivable that gravitational radiation could damp large fluctuations relatively quickly: a simple order-of-magnitude argument implies[15] that the gravitational radiation damping timescale is of order $(GM/R^3)\ (R/R_S)^{-5/2}$, where $R_S \sim GM/c^2$ is the Schwarzschild radius, and this timescale becomes comparable to the Schwarzschild timescale when $R \sim R_S$ (i.e. as the region falls through its event horizon). However, this argument is also based on linear considerations and so is unreliable. While it is possible that a violently shearing hole would settle down to a stationary state in a Schwarzschild timescale (indeed, in the spirit of the No Hair theorem, most people would like to assume as much), it certainly has not been proved.

If a black hole does not quickly radiate away its shearing motion and can persist as a non-stationary shear hell hole, what does it do? One possibility is that it does settle down to a stationary state but not on a Schwarzschild timescale. (Since there are extra dimensionless parameters in the problem, like the ratio of the shear energy to the rest mass energy, the Schwarzschild timescale is not the only one which can be constructed.) A more dramatic possibility is that there may be non-linear solutions to Einstein's equations in which a shearing black hole can persist indefinitely. A shearing hole in a Kasner universe, for example, might be able to preserve its shear by feeding off the background. We have not yet found such a solution but there are, after all, many situations in which non-linear effects are reponsible for the existence of higher order stable structures. Indeed there is evidence that such structures are the final evolution products of arbitrary initial data sets[16]. What is important is that these non-linear structures are qualitatively different from their linear counterparts, no matter how small their amplitudes. An interesting clue in this direction may come from a consideration of the soliton solutions to Einstein's equations which have already been discovered. For example, it is known that certain types of perturbations to a Kasner universe can be represented as single-soliton solutions. It is also known that the Kerr black hole can be represented as a two-soliton solution[17]. (These solutions are metrics which, although inhomogeneous, are very closely related to the Mixmaster universe of Bianchi type IX.) This suggests that one may be able to represent a shearing black hole in a Kasner background by some sort of multi-soliton solution.

Finally, we speculate about some of the interesting astrophysical properties of shear hell holes. One possibility is that they might be prolific generators of gravitational radiation. The energy density of gravitational waves (in units of the critical density) generated by the usual sort of primordial black holes would be $\Omega_g \sim \beta\Omega_B z_B^{-1}$, where Ω_B is the density of the black holes, z_B is their formation redshift and β is the efficiency with which they generate gravitational waves. Since z_B is very large ($\sim 10^{10}$ if $t_B \sim 1$ s), the gravitational wave density is tiny even if $\Omega_B \sim 1$. However, if the generation of the gravitational waves is postponed (as it is in the shear hell hole scenario) until some redshift z_g, Ω_g could be quite large. Essentially the shear hell holes would be bottling up the initial shear energy of the Universe to prevent its rapid adiabatic decay and releasing it during the isotropic phase as gravitational radiation. Perhaps the most plausible possibility is that the holes would continue to shear until the background universe becomes isotropic at t_S. In this case, although the gravitational waves generated would have a small density today (unless they were channelled into Bianchi type modes which grow in time), they may have played an important cosmological role at early times. They may even have provided an important isotropizing effect; in this case, the presence of small inhomogeneities in an anisotropic universe could indirectly cause its isotropization!

Another interesting property of a shear hell hole might be its quantum mechanical effects. Because of such effects even stationary black holes emit particles and eventually completely evaporate[18]. However, a black hole whose horizon is highly non-stationary, where the gravitational field is changing even more rapidly than it does in spherically symmetric collapse, might be expected to have particle production effects of an even more

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