The Irreversible Thermodynamics of Black Holes

1. G+
2. G+
3. E

P. Candelas

Center for Relativity
University of Texas at Austin

and

Balliol College, Oxford

AND

D. W. Sciama

Department of Astrophysics
South Parks Road
Oxford

Abstract

The action of quantum fluctuations of the gravitational field may be regarded as the origin of the dissipative processes associated with Hawking radiation. In this picture the black hole possesses internal coherence by virtue of the localization of its mass. The cumulative effect of the quantum fluctuations in the geometry is that this coherence is corrupted and the mass is sapped away.

Recent work on black holes, culminating in Hawking's [1] remarkable discovery of their quantum radiance, has shown that they obey the laws of thermodynamics as applied to equilibrium states and reversible processes.[2] We wish to argue that they conform also to the principles of irreversible thermodynamics, in the form of a fluctuation-dissipation theorem.[3] The dissipation is associated with the absorption of ordered energy by the black hole and its subsequent reradiation by the Hawking process. It has been shown[4] that Hawking radiation has the same stochastic properties as black body radiation, and so is completely disordered. A black hole is thus a perfect dissipator.

To help understand this property of black holes we apply the theory of dissipative processes.[5] The theory is based on the following ideas:

a) A dissipative system D possesses a large number of closely spaced energy levels lying near to the ground state.

b) In consequence, when this system is coupled to another system S, it exerts a force on it which fluctuates in time and is usually uncorrelated with the natural fluctuations of S. The cumulative effect of this fluctuating force is to dissipate the excess energy of S by distributing it among the many energy levels of D.

c) The effect of S on D, in linear approximation, is to produce a deviation from its equilibrium state which cannot be distinguished from a purely spontaneous fluctuation of D.

d) The fluctuating force exerted by S represents a source of noise power as well as of dissipation. The fact that S and D can come into equilibrium depends on both the dissipative and exciting aspects of the force. The rate at which equilibrium is approached is thus determined by the statistical properties of the fluctuations. The formal statement of these relations are the fluctuation-dissipation theorems. The first of which was discovered by Einstein.[6]

We now apply these ideas to the dissipative action of a black hole, confining ourselves in the main to the dissipation of gravitational disturbances.

It has been known since the work of Callen and Welton that it is possible to ascribe radiation damping to a coupling between the radiating charge and the electromagnetic vacuum. In order to proceed in this same spirit we shall briefly examine the effect that the vacuum fluctuations that are present in the Minkowski vacuum $|0\rangle$ have on a charge that is uniformly accelerated. The simplest example of this type is that of a scalar charge contained within a rigid box that is subject to uniform acceleration, this calculation was first performed by Unruh[7] and has since been elaborated by DeWitt.[8]

The analysis is facilitated by the introduction of accelerated (Rindler) coordinates defined in terms of standard Minkowski coordinates by

$$t = \xi \operatorname{sh}\tau \qquad x = \xi \operatorname{ch}\tau$$

which have the property that a worldline of constant ξ is a path of uniform acceleration ξ^{-1}. It is supposed that the coupling of the particle to the scalar field ϕ is achieved by an interaction Lagrangian of the form

$$L_{int} = \varepsilon m(x)\phi(x)$$

where $m(x)$ is a monopole charge and ε is a small coupling constant. If we adopt the convention of denoting $\phi(x)$ evaluated at $x(\tau) = (\tau,\xi,y,z)$ by $\phi(\tau)$ and similarly for $m(x)$, then it is found that the rate at which the detector makes transitions from an energy eigenstate corresponding to a frequency ν_1 to another eigenstate corresponding to another frequency $\nu_2 = \nu_1 + \nu$ is

$$R(\nu|\nu_1) = \varepsilon^2|\langle\nu_1+\nu|m(o)|\nu_1\rangle|^2 \int_{-\infty}^{\infty} d\tau e^{i\nu\tau}\langle 0|\phi(\tau)\phi(o)|0\rangle \quad (1)$$

$$= \varepsilon^2|\langle\nu_1+\nu|m(o)|\nu_1\rangle|^2 \frac{1}{2\pi\xi^2} \frac{\nu}{e^{2\pi\nu}-1} \quad (2)$$

and the last equality follows by explicitly evaluating the integral, and is just Unruh's result. We wish to emphasize that (1) is a fluctuation-dissipation relation since we might wish to consider an ensemble of such boxes which are uniformly accelerated. The particles will then make transitions among their various energy levels at a rate given by (1). $R(\nu,\nu_1)$ will determine not only the equilibrium distribution of the particles between the energy levels but it will also determine the dissipation rate of any correlations that might initially be present. We note also that (1) shows that $R(\nu,\nu_1)$ is essentially determined by the Fourier tranform of the auto-correlation function of the scalar field which, by the Wiener-Khinchin theorem, is related to the power spectrum of the noise along the worldline of the box.

The observation that (1) is a fluctuation-dissipation relation does not of itself explain the remarkable fact that the spectrum in (2) should be Planckian. This property appears to be intimately related to the causal and analytic properties of the manifold.[9]

Let us now consider a similar ensemble that is constrained to remain near the horizon of a Kerr black hole and to corotate with it. That is, we take the ensemble to follow the path

$$x(\tau) = (t+\tau, r, \theta, \phi+\Omega\tau)$$

where (t,r,θ,ϕ) are Boyer-Lindquist coordinates and Ω is the angular velocity of the horizon.

Since we are mainly concerned with gravitational disturbances we shall consider the particles as weakly coupled to the fluctuations in the gravitational field rather than those of a scalar field. The path of the boxes follows the trajectory of a Killing vector so the regime inside each box is static. We would anticipate that the particles would make transitions between their various energy levels at a rate dictated by the level of the vacuum fluctuations in much the same way as in our previous example.

As a measure of the level of vacuum activity we shall take the Fourier transform of the autocorrelation function of the gravitational shear σ (in the Hawking Hartle tetrad). We choose σ since it determines all the non trivial perturbations of the metric,[10] moreover, since it is gauge invariant it is a suitable variable to quantize. The problem is, of course, not well posed until we define the 'vacuum state' about which the gravitational field is fluctuating. Of the three 'vacua' usually considered, namely those of Boulware,[11] Hawking and Hartle,[12] and Unruh,[7] only that of Unruh meets the requirements that the renormalised value of physical observables are well behaved on the future horizon and that at large radii it corresponds to an outgoing flux of black body radiation.[13] In this sense then, the

Unruh vacuum seems to approximate best the state that would obtain following the gravitational collapse of a star. We shall therefore compute the expectation value of the auto-correlation function in the Unruh vacuum. A direct calculation shows that asymptotically as $r \to r_+$

$$\int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle U|\sigma^*(\tau)\sigma(o)|U\rangle \sim \frac{4}{5} \frac{1}{r-r_+} \frac{\omega(\omega^2+4\kappa^2)}{e^{2\pi\omega/\kappa}-1} \tag{3}$$

κ is the surface gravity of the hole, r_+ is the radius of the outer horizon and by $\sigma(\tau)$ we mean $\sigma(x)$ evaluated at $x(\tau)$.

The factor $\omega^2 + 4\kappa^2$ that occurs in the numerator is a density of states factor[14] (for a field of spin s the corresponding quantity would be $\omega^2+s^2\kappa^2$). So that we see that (3) is in fact a blackbody spectrum.

Let us now consider the rate at which a black hole dissipates a gravitational perturbation. If the disturbance is purely gravitational and $\sigma\sigma^*$ is slowly varying then to lowest order the area of the horizon increases at a rate given by[15]

$$\frac{dA}{dv} = \frac{2}{\kappa} \int \sigma\sigma^* \, dA \tag{4}$$

where v is a suitably defined time coordinate on the horizon. Since the black hole entropy is proportional to the area this formula determines the dissipation rate in a macroscopic process such as the slowing down of a rotating black hole by a moon. The same equation may be shown to govern the dissipation in a microscopic process provided that the symbols are interpreted appropriately. (4) shows that the dissipation rate is quadratic in the perturbation, so that the black hole is a linear system in the sense of Callen and Welton.

In virtue of this we can fit into our picture the emission by a black hole of Hawking gravitational radiation. We know from macroscopic theory that a non-stationary black hole with a non-vanishing shear on its horizon would radiate gravitational waves to infinity, and in consequence would reduce the shear of the horizon and approach a stationary state. Now according to our point of view a linear system (that is one with no memory) behaves in the same way in a given configuration whether it reached that configuration by a spontaneous fluctuation or by an externally induced perturbation. Accordingly we would expect that the quantum fluctuations of the shear would also lead to the emission of gravitational radiation, and since the shear fluctuations have the stochastic properties of black body radiation at a temperature $2\pi/\kappa$ we would expect the gravitational radiation to have the same properties. This is, of course, just Hawking's result. There is an important proviso, however. There will be a net emission of gravitational radiation provided phase relations have been chosen which do not suppress the flux. If the black hole is the result of stellar collapse then one would not expect the collapse to respect correlations that might initially be present. This is essentially Hawking's point of view. By contrast, if we are dealing with an eternal Kruskal black hole, it is not clear what are the 'correct' initial conditions to take. The problem is an academic one and is usually phrased in terms of choosing a particular 'vacuum state'; if one were to choose either the Boulware or the Hawking vacuum there would be no net flux at infinity since in these cases correlations have been chosen which exactly suppress the radiation.

The resulting situation is then essentially the same as for an atom in its ground state coupled to the electromagnetic vacuum. There is no real exchange of energies between the atom and the electromagnetic field because the zero point fluctuations of the electromagnetic field drive those of the atomic moments and produce complete interference.[16]

Acknowledgements

It is a pleasure to acknowledge many fruitful discussions with Paul Chrzanowski, David Deutsch and Bryce DeWitt. The authors are especially indebted to Paul Chrzanowski for an extensive series of discussions regarding gravitational perturbations of the Kerr metric.

References

1. S. W. Hawking, Nature 248, 30 (1974); Commun. Math. Phys. 43, 199 (1975); in Quantum Gravity: an Oxford Symposium, edited by C. J. Isham, R. Penrose, and D. W. Sciama (Clarendon, Oxford, 1975) pp 219-267.

2. J. D. Bekenstein, Phys. Rev. D7, 2333 (1973); Phys. Rev. D9, 3292 (1974).

   J. M. Bardeen, B. Carter and S. W. Hawking, Commun. Math. Phys. 31, 161 (1973).

   S. W. Hawking, Phys. Rev. D13, 191 (1976).

   An introductory survey of black hole thermodynamics is given by D. W. Sciama, Vistas in Astronomy 19, 385 (1976).

3. H. B. Callen and T. A. Welton, Phys. Rev. 83, 34 (1951).

   L. Landau and E. M. Lifshitz, Statistical Physics (Pergamon 1958)

   R. Kubo, Rep. Prog. Phys. 29, 255 (1966)

4. R. M. Wald, Commun. Math. Phys. 45, 9 (1975)

   L. Parker, Phys. Rev. D12, 1519 (1975)

   S. W. Hawking, Phys. Rev. D14, 2460 (1976)

5. L. Landau and E. M. Lifshitz, Statistical Physics (Pergamon 1958)

6. A. Einstein, Ann. Phys. Lpz. 17, 549 (1905)

7. W. G. Unruh, Phys. Rev. D14, 870 (1976)

8. B. S. DeWitt, Review in preparation.

9. G. W. Gibbons and M. J. Perry, "Black Holes and Thermal Green's Functions" University of Cambridge preprint.

10. P. L. Chrzanowski, Phys. Rev. D11, 2042 (1975)

11. D. G. Boulware, Phys. Rev. D11, 1404 (1975)

12. J. B. Hartle and S. W. Hawking, Phys. Rev. D13, 2188 (1976)

13. P. Candelas, paper in preparation.

14. P. Candelas and D. Deutsch, paper in preparation.

15. See, for example, the lectures of S. W. Hawking in Black Holes (edited by B. S. DeWitt and C. DeWitt, Gordon and Breach 1973)

16. V. M. Fain and Ya. M. Khanin, Quantum Electrodynamics Vol 1. (Translated by H. S. H. Massey, Pergamon 1969) p. 211.

Curriculum Vitae of P. Candelas

Name: CANDELAS, Philip

Date of Birth: 24 October 1951

Place of Birth: London, U.K.

Nationality: British

Marital Status: Married, no children

Address: 1200 Broadmoor, #364, Austin   Telephone: 454-1412

Education:

1963-1970	Christ's College, Finchley	'O' Levels in French, German, Hebrew, English, Religious Knowledge, History, Physics, Chemistry, Mathematics and Additional Mathematics. 'A' Levels in Spanish AA, French BA, Pure Mathematics A, Applied Mathematics A1, and Further Mathematics A.
1970-1972	Christ's College, Cambridge	First Class Honours in Mathematics Part IB " " " " Part II
1971-1972	Christ's College, Cambridge	Scholar
1972-1973	Wadham College, Oxford	Private work in Relativity and Astrophysics supervised by Dr. D.W. Sciama.
1973-1975	Wadham College and the Dept. of Astrophysics, Oxford	Research in Relativity and Astrophysics. Senior Scholarship at Wadham College.
1975-	Balliol College, Oxford	Junior Research Fellowship.
1976-	University of Texas, Austin	Research Associate I, Center for Theoretical Physics Research and Center for Relativity (On leave of absence from Balliol College).

D.W. Sciama was born on November 18th 1926 and educated at Trinity College, Cambridge, where he was elected to a Prize Fellowship in 1952. He has since been a lecturer at the Department of Applied Mathematics and Theoretical Physics, Cambridge University and a Fellow of Peterhouse. Currently he is a Senior Research Fellow at All Souls College, Oxford, and in 1977-8 will be Henry R. Luce Professor of Cosmology at Mount Holyoke College.