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ON THE PHYSICAL NATURE OF GRAVITATIONAL WAVES

by

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1. INTRODUCTION

The general theory of relativity has a very definite and strict conception of empty space. The field equations for empty space not only exclude any possible presence of matter at rest or in motion, but also exclude electromagnetic fields of force, and with it electromagnetic radiation, as well as nuclear fields of force. The question then arises as to whether there is any form of energy that can travel across a region of space which is empty in the relativistic sense. Clearly, the only possibility is gravitation, because in the whole conception of general relativity gravitation is not a field of force that exists _in_ space, and so makes it non-empty, but is merely a property _of_ space itself. It is, therefore, of considerable interest to investigate whether gravitational energy can be transferred across a region of space which is empty in the relativistic sense.

In electromagnetic theory one distinguishes between near-field interaction (i.e. interaction due to Coulomb forces) and radiative transfer of energy. The gravitational forces corresponding to Coulomb forces are, of course, well known, and it is also well established that the Newtonian approximation gives an excellent description of the relativistic situation. Accordingly, we may infer from certain idealized experiments that in general relativity, too, energy can be transferred across empty space by near-field interaction. However, transfer by radiation is unknown by experiment. It would clearly be very small (if it exists at all) in any practical situation but it is nevertheless of interest to examine this possibility in general relativity. As we know from Newtonian theory, the empty space condition cannot be taken to exclude energy transfer by gravitation altogether. Not only is it intrinsically important to find hitherto unknown consequences of general relativity as an otherwise successful theory, but the synthesis of general relativity and quantum theory is still, despite great efforts, in an unsatisfactory state. It has been suggested (various speakers, Chapel Hill Conference Report 1957) that the quantization of the gravitational field is necessary only if the gravitational radiation field possesses independent degrees of freedom. Since this synthesis is expected to have important consequences for quantum theory, it is important to investigate gravitational field dynamics. For these and other reasons a good deal of work has been done on this subject, but in view of its indecisive nature a reexamination of the problem is indicated.

In carrying out an investigation of gravitational waves it is essential to realise that the variety of familiar properties possessed by electromagnetic waves need not be possessed by other waves satisfying normal hyperbolic equations.

The non-linearity of gravitational waves is only the most obvious difference, a difference that is moreover known from the case of soundwaves of finite amplitude to raise the possibility of such physical features as the formation of shocks.

2. PROPAGATION OF INFORMATION

It is an essential part of the nature of normal hyperbolic equations that their solutions need not be analytic functions but may possess singularities which, however, must be confined to the characteristics. In fact, it may well be argued that an artificial restriction to analytic functions obscures the nature of hyperbolic equations. In physical language this mathematical property is expressed as the propagation of new information at a finite velocity. An analytic function may after all be described as a function whose entire future (and past) is implicit in an arbitrarily short interval of the present. A physical system in which new information never appears is therefore unsuitable for an exposition of the basic properties of any radiation field.

Insuperable difficulties of interpretation may arise in examining such a pre-determined system.

In electromagnetic theory a transmitter of waves is a system containing one or more <u>arbitrary</u> functions of the time in its description. Only such a system is a full representation of, say, a broadcasting station. However, the definition of such a transmitter is somewhat more difficult in the gravitational than in the electromagnetic case.

An electromagnetic transmitter generally necessitates in its construction the use of electrically neutral* machinery, operators, etc., whose description may be omitted since they do not affect the wave emitted. Owing to the empirically established non-existence of gravitationally negative matter (and hence of neutral matter which would have to combine positive and negative matter) the description of a gravitational transmitter must be complete.

In spite of this difficulty, a self-contained transmitter can be constructed that transmits <u>new</u> information, i.e., whose future transmission cannot be forecast. For consider an unpredictable trigger mechanism consisting of a radioactive atom whose decay moves a counter, or of a diode valve, run by local batteries, so connected to a series of amplifiers that a peak of shot noise of sufficient size activates the system etc.

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* Combining positively and negatively charged matter.

The trigger mechanism can control the movement of masses vastly exceeding its own through activating a local store of energy such as a clock-spring, an electrical battery, or an explosive. In this manner, movements of arbitrarily large masses may be made to occur at unpredictable times in a closed self-contained system.

3. THE CONSERVATION LAWS

Transmitters of electromagnetic waves need only satisfy one conservation law, that of charge. In the gravitational case there are several of these (mass, linear momentum etc.) and their relativistic formulation is not fully understood. In the Newtonian approximation the potential due to a finite isolated system may be expressed outside the system in the form

$$V = \sum_{n=0}^{\infty} A_n S_n r^{-n-1}$$

where the S_n are suitably normalized surface harmonics, r is the distance from the origin of coordinates, supposed interior to the system, and the A_n are coefficients. The coefficient A_0 is the gravitating mass of the system and the law of conservation of matter implies that A_0 is constant in time. Given $A_0 (\neq 0)$, the coefficient A_1 defines the distance of the centre of mass of the system from the origin of coordinates. By the law of conservation of momentum, the centre of mass must

must be unaccelerated, and so a suitable transformation of the velocity and the origin of the frame of reference will reduce A_1 to zero. The higher coefficients describe the shape of the mass distribution. They are arbitrary apart from certain inequalities due to the positive sign of mass. A change in the distribution of mass will change these coefficients, so that the variable part of the Newtonian field is of symmetry S_n, with $n \gg 0$. Energy transfer by such variability may occur in Newtonian theory. In relativity we might therefore expect the simplest radiation to be of quadruple symmetry (Landau & Lifshitz).

The firmness of this conclusion is however upset by the obscurity of the relativistic conservation laws. The concept of mass, clear and unique in Newtonian theory, disintegrates in general relativity. We have inertial mass (in $T^{\mu\nu}$) and active gravitational mass (the Schwarzschild 'm'). The absence of tensorial integrals over extended regions means that in a finite isolated system neither of these masses need necessarily be constant, though in some specially simple systems this may be so. The Newtonian arguments, therefore, are unlikely to be a good guide to the relativistic problem. The non-linearity of the equations, together with energy considerations, suggests as a serious possibility that in relativistic theory the field component corresponding to A_0 need not be constant. For consider the following problem: An isolated spherically symmetric distribution of mass is divided in two halves by an imaginary plane through its centre. The two hemispheres are connected through a mechanism that allows the distance between them to be varied in time, the mechanism, including its energy store (say, a battery), being so arranged with counter-weights etc. that in the closed position the system is spherically symmetrical. Suppose now that before $t=0$ the system is in the closed position. The external gravitational field will then be completely characterized by the parameter m of the external Schwarzschild solution. Suppose further that in $0 \leq t \leq t_1$ the system is in motion, the variable separation of the hemispheres producing a variable quadruple moment, but that at and after $t=t_1$, the system has an isotropic equation of state, such as that of a perfect fluid. The important question then arises of the nature of the external field for $t \to \infty$. Confining our attention to a bounded spherical region of space surrounding the mass we may distinguish three possibilities:-

(i) The field in this region eventually tends to a Schwarzschild field with the same m as for $t<0$.

(ii) The field eventually tends to a Schwarzschild field with an m different from that for $t<0$.

(iii) The field never tends to a Schwarzschild field.

Possibility (iii) is scarcely acceptable on physical grounds. It would imply that the field of a system depends on local conditions in the arbitrarily remote past and so would effectively cut the connection between distribution of mass and gravitational field. If the theory did indeed lead to such a situation it would virtually rule out the theory. The mathematical question involved has not, to my knowledge, been solved, but the possibility of (iii) applying in general relativity seems to be remote, though disturbing, and constitutes a problem well worth solving.

If then we anticipate the probable answer and so reject (iii) we are left with the first two possibilities. The distinction between them involves essentially the question: Do gravitational waves in general relativity carry energy? If they do, then the transmitter must be in a different state after emission than it was before. We must have less energy, and it is almost inescapable that any such change must involve m. For otherwise we would have to accept that m has nothing to do with actual available energy (as e.g. the charge in a battery undoubtedly is), a possibility too repugnant to contemplate seriously. We must therefore conclude that if energy carrying gravitational waves can be sent out, a variation of m and with it of the spherically symmetrical part of the field is a necessary consequence. Birkhoff's theorem, it must be remembered, only denies the possibility of <u>purely</u> spherically symmetrical variations, and the non-linearity of the equations rules out an extension of this denial to the spherically symmetrical part of a more general disturbance. As a by-product our consideration shows the danger of applying Newtonian symmetry arguments to relativity, a danger known and understood in the case of cylindrical symmetry (Marder 1958). Moreover, this consideration shows that the most interesting physical features of the problem cannot become apparent in the linear approximation even if it is mathematically valid. Note also the contrast with electromagnetism where charge is conserved unconditionally.

4. NEAR-FIELD AND WAVE-FIELD

In Newtonian theory any transfer across empty space is clearly transfer by the near-field (Coulomb field), just as in the corresponding electromagnetic case. There the two outstanding differences between near-field and wave-field transfer are:

(i) In near-field transfer the energy loss of the transmitter depends entirely on the position and nature of the receiver and vanishes when there is none, whereas in wave-field transfer the energy loss of the transmitter is determined independently of the specification of the receiver.

(ii) The rate of absorption of energy by a given receiver depends on its distance from the transmitter through the inverse square law (for sufficiently large distances) if it is wave-field transfer and more steeply if it is near-field transfer.

There is no reason why this distinction should not be applied equally in the case of gravitational waves but there are some points where more thorough definition is required. In the Newtonian approximation difficulty is caused by the non-localisable nature of potential energy. Although this difficulty does not occur in the same way in general relativity no statement about energy has a clear physical meaning unless it is strictly localised energy such as the charge of a battery or the heat of a material body.

In relativity it is not immediately clear which of the variables (metric, 3-index symbols, curvature tensor) can be converted into localised energy by a suitable receiver. In electro-magnetic theory the answer to the corresponding question is obvious since an imperfect conductor (or, with optical frequencies, a black body) will turn $\underset{\sim}{E}$ into heat. Such a body will not move as a whole because the bulk of its material is not directly affected by the incident field. The principle of equivalence thus has an important effect on the construction of a gravitational receiver.

The simplest kind of gravitational receiver would appear to be a particle constrained to move in a freely falling rough tube of appreciable rigidity. The tube will fall according to a suitable average of the field over its length, and the test particle will thus tend to move relative to the tube according to the equation of geodesic variation. In this movement heat will be produced and accordingly the curvature tensor is responsible for energy production, but it requires consideration whether the relation is linear or by square law.

In near-field cases it is often useful to employ a receiver kept in motion by internal machinery which does no work in a cycle in the absence of external excitation. With a suitable phase of operation the work done may then be proportional to the incident field. However, in a radiation theory the machinery is bound to do work in the absence of external excitation since it is bound to radiate to space. The advantage of such an 'active' receiver over a 'passive' one is thus at most marginal if there is radiation in the theory.

In a 'passive' receiver in which the entire motion is due to the incident field, the velocity will be proportional to the field and thus the energy to the square of the field. Accordingly, in our case the energy is proportional to a suitable evaluated square of the curvature tensor. Thus for proper wave transfer we require the leading terms of the curvature tensor to vary like r^{-1}.

However, this analysis, though plausible, is not entirely satisfactory. The receiver employed is a 'test' receiver, i.e. one whose reaction on the wave is ignored. The total amount of <u>available</u> energy in the wave cannot be found by the use of such a receiver. It is not easy to see how to construct a relativistic receiver whose reaction on the wave can be studied. A material with a visco-elastic equation of state might be employed, corresponding as it does to an imperfect conductor. Absorption by a shell of such material entirely surrounding the transmitter would be of interest, as the dependence of the amount absorbed on the separation would give a clear indication of the wave or near-field character of the perturbation.

5. CAUSALITY

It has so far been assumed implicitly that, wherever there is an ambiguity, the retarded solution of the field equations is to be taken. This is in accord with the classical views of causality entirely appropriate to a theory like general relativity. The time symmetry of the field equations implies that there should also be advanced solutions, and presumably a one-parameter series of solutions corresponding to the linear superposition of the two solutions in electromagnetic theory. In general relativity, just as in electromagnetic theory, a

Wheeler-Feynman type of argument is required to account for the causality principle in terms of actual distributions of matter. The non-linearity raises difficulties which are probably purely technical but the absorber at infinity is almost bound to have cosmological consequences of the greatest importance which are likely to be connected with Mach's principle.

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