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ON THE EXISTENCE OF GRAVITATIONAL INSULATORS

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SUMMARY

Gravitational insulators depend on the existence of negative mass or mass dipoles. The possible existence of such material is discussed from general relativistic principles. The conclusion is reached that, although rejected in the conventional treatment, in the general theory there is so far nothing to prevent the existence of negative mass or mass dipoles. The argument depends on motion in the general theory of relativity but is not of a technical character. The scanty experimental evidence for the theory is reviewed briefly. The application of high-speed computing techniques to the study of gravitation is particularly suggested.

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It is reasonable to ask why gravitation has not found more practical application. It used to be thought that the neglect of gravitation in many situations was justified because of its weakness compared with nuclear or electromagnetic forces. This argument is not binding because, although weak, the gravitational attraction is universal and gravitating matter is abundant. Indeed, the Fermi interactions that were also considered negligibly weak, have recently appeared[1] to be closely related to interactions important in nuclear events.

The truth is that much of the study of gravitation has passed into the hands of the pure mathematicians. At the recent Berne conference three subjects of principally physical interest were considered: (1) unified field theories, (2) quantization of gravitation, and (3) equations of motion. The first two subjects have been treated in earlier papers in the "Essays on Gravity" series. The present essay will attempt to show that the basic study of motion may illuminate the possibility of negative mass or of mass dipoles.

Negative mass or mass dipoles are necessary in order that there be gravitational insulators. For if a gravitating system is surrounded by what is to be a gravitational shield, the system plus shield will itself produce a gravitional field unless there exists negative mass so that the collection can be gravitationally neutral.

Without negative mass there cannot be even a weakening of gravitation. Electrical shields are possible because there exist polarizable materials in which positive and <u>negative</u> charges can flow so as to neutralize the electric field. Partial magnetic shields are possible because there exist diamagnetic materials in which magnetic <u>dipole moments</u> can orient themselves so as to partly nullify the magnetic forces.

Our argument depends on the theory of general relativity, but does not need to be of a technical character; mathematical details can be found in the accompanying references. Our conclusion will be that although the conventional treatment arbitrarily rejects negative mass, there is nothing in the general principles that would prevent the existence of such material.

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WHY USE GENERAL RELATIVITY?

The theory of relativity will be employed, although the experimental confirmation of this theory has not been overwhelmingly compelling. Of the theory's three quantitative predictions--the precession of Mercury, the gravitational deflection of light passing near the sun, and the galactic red shift--the first effect has been known quantitatively for a hundred years. Many corrections have to be applied to the photographic data on the deflection of light, but Trumpler concluded[2] that the eclipse observations agree with Einstein's predictions within 5%. The authenticity of the galactic red shift is much more open to question. (The theory also predicts, according to some theoretical calculations, radiation of gravitational energy by oscillating masses. An experiment to detect gravitational radiation in the laboratory by conversion into an electrical signal has been considered at the Radiation Laboratory. The experiment has not been done because it was concluded that the expected signal would be too weak for observation.)

Since the precession of Mercury and the gravitational deflection of light can actually be obtained[3] without the Einstein theory, the attractiveness of the general theory lies not in the experimental evidence but rather in the point of view suggested. The crux of the general theory of relativity is that, in the formulation of physical problems, all coordinate systems are equivalent. This requirement has the important effect of making the gravitational field equations <u>nonlinear</u> and subject to four <u>identities</u> among themselves.

These two features, as will be explained, restrict the motion of masses and may determine the kinds of mass that can appear. Gravitation is unique in that "something is gotten for nothing"; the equations of motion follow from the field equations.

NONLINEARITY AND IDENTITIES

Suppose that in the presence of masses 1 and 2 separately, some field equations possess solutions, corresponding to motions, f_1 and f_2. If the equations are <u>linear</u> this means that the superposition, $f_1 + f_2$, is a solution. This says that when together, the two masses each move in the sameway as they did separately. The masses do not effect each other so long as the field equations are linear.

Nonlinearity of the field equations, while necessary, is not sufficient. It is the identities in general relativity, along with the nonlinearities, that constrain the motion in a definite way and cause mass to be conserved.[4]

An identity like those in gravitation is illustrated in the simple problem of finding the shortest distance between two points in a plane (a straight line). The differential equation for this

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problem can be derived from the requirement that the distance

$$D = \int \sqrt{1 + (dy/dx)^2} \; dx$$

be a minimum. The field equation is

$$\frac{d}{dx} \frac{(dy/dx)}{\sqrt{1 + (dy/dx)^2}} = 0$$

whose solution, dy/dx = constant, is known to be the equation of a straight line.

Suppose now x and y were expressed in terms of a parameter τ so that

$$D = \int \sqrt{\dot{x}^2 + \dot{y}^2} \; d\tau \quad ,$$

where

$$\dot{x} = dx/d\tau \; , \qquad \dot{y} = dy/d\tau \; .$$

The minimization of the distance D now requires two equations

$$\frac{d}{d\tau} \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = 0 \; , \qquad \frac{d}{d\tau} \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = 0 \; . \tag{1}$$

Because the parametrization of the line is completely arbitrary, there is invariance under arbitrary changes in the "coordinate" τ. This leads to

$$\dot{x} \frac{d}{d\tau} \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} + \dot{y} \frac{d}{d\tau} \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} \equiv 0 \; , \tag{2}$$

an identity (expression identically or obviously true) or constraint between the two field equations (1).

The problem in general relativity is quite similar to this minimization of the distance between two points. Four space-time coordinates are "parameters" subject to arbitrary transformations, and four identities like (2) follow among the field equations. These identities determine the motion of the masses.

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THE PROBLEM OF MOTION

Two methods, both approximate, have been used in deriving the motion of mass particles from the gravitational field equations. One method[5] is not applicable to the motion of finite masses of material. In the second method,[6] the exact gravitational equations are written in a form separating out the linear and the nonlinear properties of the field. These equations cannot be solved exactly, but are solved in successive approximations by a method whose significance has heretofore been unclear. (This method of approximation is to regard the changes of the field in time as small compared with the changes in space. The field equations in each order of approximation then are integrable in the higher orders only if certain conditions are satisfied. These conditions turn out to be precisely that the masses move in a definite way.

That the equations of motion follow from the field equations was originally quite unexpected. One wonders whether the field equations, which have heretofore been so incompletely studied, may not also contain other important restrictions on the nature of gravitating matter. A curious but important feature appears in the Einstein-Infeld approach.[6] In order that the equations be integrable to find a solution, mass dipoles must be temporarily introduced. But in order that the motion be determined, the condition must be imposed that the total field contains no net mass dipoles. The approximation procedure brings mass dipoles temptingly before our eyes, and then rejects them out of hand! This treatment proves only that if positive mass particles are assumed, then their motion follows from the field equations. It proves nothing about the possible existence of negative mass particles, of multipoles, or of insulators!

This poses the question of the meaning of the Einstein-Infeld approximation and the possibility of alternative treatments that will reveal negative mass. Consider again the profitable electromagnetic analogy. In the solution of the electromagnetic field equations there are three cases: (1) In cases of especially simple symmetry,[7] the field here and now can be given exactly; (2) In general, a closed solution can be given in terms of retarded quantities, in terms of the motion of charges at <u>another</u> time; (3) When a solution is demanded in terms of what is going on at <u>this</u> time, then it can be developed only in a series of successive approximations.[7]

The Einstein-Infeld approximation in gravitation consists of exactly such an expansion (3). Because the gravitational equations are so much more complicated than those in electrodynamics, no general closed solution like (2) is known. All too few exact solutions, corresponding to case (1), are known.

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A SUGGESTION

High-speed computing machines are now becoming available that can solve complicated systems of nonlinear partial differential equations like those in gravitation. (Hydrodynamical systems of precisely this character have recently been treated successfully on the Radiation Laboratory UNIVAC and elsewhere.) The use of these fast computing machines may eventually do as much for gravitation as the development of logarithms and trigonometric functions did for navigation and engineering. Fast computers can be used to find precise solutions to problems in gravitation that are otherwise impossible, and also as a form of numerical experimentation on the types of solutions possible. This does not imply that a better approximation than the Einstein-Infeld expansion is immediately at hand. On the contrary, experience shows that a great deal of basic physical work is still required before a problem can be economically placed on a computing machine.

The point to be emphasized is that the essence of gravitation, the nature of the gravitating(materials possible, has not been considered definitively in the conventional treatment. The use of modern methods of computation admits at least the possibility of searching for solutions corresponding to negative mass or mass dipoles. These are needed in order that there be any gravitational insulators.

The recent experience of industry and of government has been that physical research precedes development, and that fundamental physical understanding presupposes precise theoretical calculation. When fundamental research is undertaken, the practical outcome is unpredictable, but regarded as investment the cost of research on the purest and most basic of problems is cheap indeed.

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FOOTNOTES

1. S. A. Bludman and M. A. Ruderman, Physical Review 101, 910 (1956) and S. A. Bludman, Physical Review (in press).
2. R. J. Trumpler and others at the recent conference on gravity in Berne, Switzerland.
3. See the work of A. G. Walker in E. T. Whittaker, History of the Theories of Aether and Electricity, 1900-1926, London, Nelson, 1951.
4. Important consequences would indeed follow if nonlinearities and identities like those in general relativity were found in electromagnetic and nuclear theory. See S. A. Bludman, Physical Review 100, 372 (1955).
5. L. Infeld and A. Schild, Review of Modern Physics 21, 408 (1949).
6. A. Einstein and L. Infeld, Canadian Journal of Mathematics 1, 209 (1949).
7. L. Page and N. I. Adams, Electrodynamics, New York, Van Nostrand, 1940; and S. A. Bludman, Physical Review 95, 654 (1954).