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Essay on the Possibility of an Insulator of Gravity

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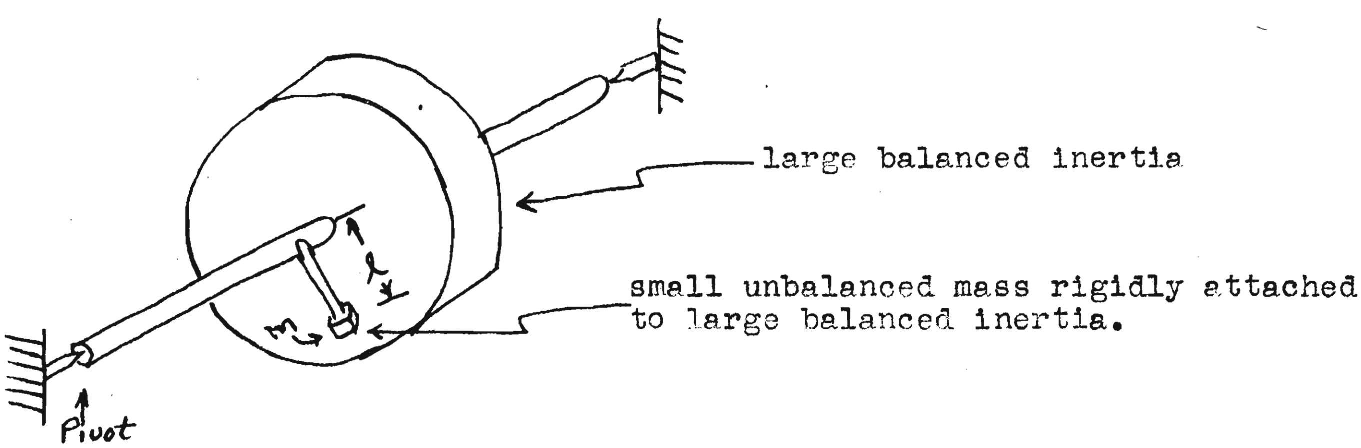
When one inquires into the possibility of creating or discovering an insulator of gravity, it is quite likely that the mind is prejudiced by familiarity with the techniques of magnetic or electrical insulation and so is lead off in search of some material substance having magical properties. If, however, one returns to the more classical definition of an insulator which includes "to place in a state of isolation", the search may be broadened to include devices which will isolate something from the effects of gravity. A device which in this broader sense effectively insulates an accelerometer from the gravitational force is described in this paper.

It is frequently necessary to measure the horizontal acceleration of moving bases such as ships and airplanes. Einstein's laws of relativity establish the fact that it is impossible to distinguish between the specific force of gravity and the specific force due to any other acceleration. It follows that it is impossible to construct any physical acceleration sensitive instrument which, by itself, can distinguish between the accelerations related to movement over the earth's surface and the gravitational acceleration. It is possible, however, to construct a pendulous platform with the proper dynamics which will maintain the orientation of an accelerometer always normal to the local vertical and, hence, through this device effectively "insulate" an accelerometer from the gravitational acceleration.

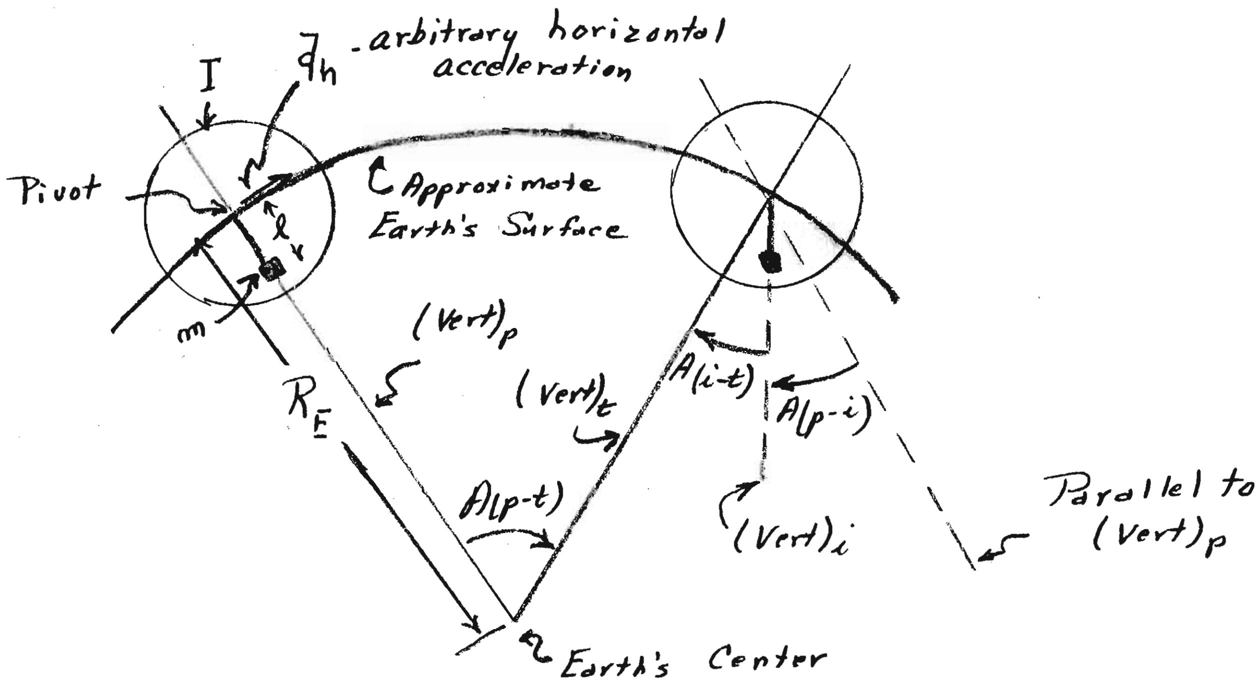
In order to simplify this presentation of the problem, it is assumed that the vehicle maintains a substantially constant altitude with respect to the earth. The required dynamics of the pendulous platform may be derived by considering a very simple instrumented pendulum which operates in a single plane; the ultimate device must be gimballed and two orthogonal axes similarly instrumented.

Consider, then, the instrumented pendulum consisting of a large balanced inertia to which is attached a very small unbalanced mass as shown in the sketch.



In what follows, we consider that the arm supporting the small mass m is initially lined up along the local vertical and ask if it is possible to adjust the three parameters available, I, l, and m such that this arm remains along the instantaneous local vertical regardless of any arbitrary horizontal acceleration normal to the pivot axis of the system.

In the sketch below, the system is shown at the left with the arm of the unbalanced mass (or accelerometer) lined up with the vertical at some arbitrary point of departure, $(Vert)_p$ and experiencing a horizontal acceleration a_h. At the right, the system is shown after some elapsed t time displaced from the new true vertical, $(Vert)_t$. The line along the arm of the unbalanced mass is called the indicated vertical, $(Vert)_i$. The equations are derived from the geometry of the problem and Newton's law that the sum of the torques acting on a body must be zero. Equation (7) is the differential equation which relates the actual motion of the pendulum, $A_{(p-i)}$ to the actual movement of the system over the earth, $A_{(p-t)}$. Equation (8) expresses the desired condition that these two angles be equal for the perfect system and it follows that the condition expressed by Equation (9) must then be met.



R_E = radius of the earth

g = gravitational acceleration (32.2 ft/sec²)

Torque applied $= d_h m l \cos A(i-t) + mgl \sin A(i-t) \approx d_h ml + mgl A(i-t)$ (1)

Inertia reaction $= -I \frac{d^2}{dt^2} A(p-i) = -I \ddot{A}(p-i)$ (2)

Σ Torque $= 0$: $d_h ml + mgl A(i-t) - I \ddot{A}(p-i) = 0$ (3)

Geometry: $A(p-t) = A(p-i) + A(i-t)$ (4)

$$d_h = R_E \frac{d^2}{dt^2} A(p-t) = R_E \ddot{A}(p-t) \quad (5)$$

Substitute (4) and (5) into (3):

$$ml R_E \ddot{A}(p-t) + mgl[A(p-t) - A(p-i)] = I \ddot{A}(p-i) \quad (6)$$

$$I \ddot{A}(p-i) + mgl A(p-i) = ml R_E \ddot{A}(p-t) + mgl A(p-t) \quad (7)$$

To make: $A(p-i) = A(p-t)$: or $A_{i-t} = 0$ (8)

then:

$$I = ml R_E \quad : \quad \text{or} \quad I/ml = R_E \quad (9)$$

The period at which a pendulous device will oscillate when disturbed from the local vertical is the single characteristic which completely describes its dynamics. In the following equations, it is shown that when this pendulum, adjusted as required by equation (9), is so disturbed, it will oscillate with the period $T = 2 \, R_E/g$ = 84 minutes approximately. This is exactly the period of a simple pendulum having a length equal to that of the radius of the earth.

Torque applied $= -mgl \sin A(i-t) \approx -mgl A(i-t)$ (10)

Inertia reaction $= -I \ddot{A}(i-t)$ (11)

Σ Torque $= 0 =$ $I \ddot{A}(i-t) + mgl A(i-t) = 0$ (12)

This is recognized as an oscillatory system whose natural frequency,

$$\omega_n = \sqrt{mgl/I}$$

or whose natural period is:

$$T_n = 2\pi \sqrt{I/mgl}$$

But for the adjustment required by equation (9):

$$I/ml = R_E$$

$$T_n = 2\pi \sqrt{R_E/g} = 2\pi \sqrt{\frac{4000 \times 5280}{32.2}} \times \frac{1}{60} \quad (13)$$

$$= 84 \text{ minutes Approximately}$$

We have now established that a pendulous platform which is adjusted to have an 84 minute period will always remain in the instantaneous local horizontal plane if initially so established. An accelerometer mounted on such a platform will read only horizontal accelerations and will be effectively insulated from gravity. Of course, the crude device just described is not realizable because of the large friction inevitably encountered in supporting such a large mass. In order to realize a practical instrument, we must substitute a gyroscope for the large balanced inertia to obtain in effect a very large inertia without an unduly high mass.

We now imagine a gimballed platform on which are mounted a gyroscope with its spin axis normal to the platform and two orthogonal accelerometers. In addition, we must provide torque motors at the gimbal bearings in order to apply precessional torques to the gyroscopic element. If now we connect the output of each accelerometer to the appropriate torque motor (the associated accelerometer and torque motor must be orthogonal) so that a precessional torque exactly proportional to the accelerometer output is produced, we will have a "first order erection system" for the vertical gyro. The instrumentation just described is the standard method of obtaining a vertical reference in an airplane autopilot system. If disturbed from the vertical and the vehicle is not accelerating, such a device will settle onto the vertical exponentially. In the presence of horizontal accelerations, such a device will settle onto the line which is the resultant of the gravitational acceleration and the average horizontal acceleration.

In order to impart to this common device the second order oscillatory characteristic, it is only necessary to integrate the output signal of the accelerometer with respect to time and apply this integrated signal to the torque motors. If now we adjust the available parameters: accelerometer, integrator, and torque motor sensitivites, and the gyro angular momentum so that the system has the required 84 minute oscillatory period, we can actually realize a practical instrument having the required characteristics.

The constant angular velocity of the earth which the system must account for is provided by setting the proper inital integrator settings and by giving the system directional stability with a second gyroscope. Additional refinement is required to compensate the system for Coriolis acceleration. The instrument is complete in itself for the purpose of "insulating" the accelerometers from the gravitational acceleration. The outputs of the same accelerometers which are used in the basic mechanization can be connected to suitable indicating or recording instruments and will read the accelerations of the vehicle which are related to its actual motion in space and not include any component of the gravitational acceleration.

From the literature, it appears that the required 84 minute characteristic of a pendulous device to achieve a perfect vertical indicating instrument for use on a moving base was first recognized by Schuler and reported by him in 1923 (1). Further, Schuler recognized the difficulties of realizing such a device and actually proposed a design utilizing a pendulous gyroscopic element. However, Schuler's proposed device was a simple pendulous gyroscope which would precess around on a conical surface requiring 84 minutes for one complete revolution. It can be shown that the mean position of two such gyroscopes rotating in opposite directions exhibits the same characteristic as that of the correctly adjusted second order system described herein. On this basis, the 84 minute adjustment of vertical indicating and compass devices is sometimes referred to as the Schuler tuning of such instruments. This early work with some of Schuler's proposed designs is described in a paper by Wrigley (2).

Thus we see, that it is quite possible to "insulate" an instrument such as an accelerometer from gravity using well established theory and instrumentation techniques. We may say that here is a specific example of an insulator of gravity.

References:

1. Schuler, M., "Die Störung von Pendel - und Kreiselapparaten durch die Beschleunigung des Fahrzeuges," Physikalische Zeitschrift, Band 24, 1923.

2. Wrigley, W. "Schuler Tuning Characteristics in Navigational Instruments," Navigation Journal of the Institute of Navigation. Vol. 2. No. 8 December 1950.

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