might be ok but much hard to follow OK

To
Gravity Research Foundation, New Boston, N.H.

From
Dr. F.J.Belinfante, 1809 Ravinia Road, West Lafayette, Indiana.

March 17, 1960.

Dear Mr. Rideout:

With this letter I send, with two carbon copies of everything: an essay on gravity especially written for your Award, preceded on page 1 by title and a long summary (about 200 words); but, on a separate sheet, I wrote the same title and an abbreviated summary of about 100 words, which you may substitute for the other summary. The last page of the essay contains the Figure, to which there is reference in the text, and it carries the various footnotes (including literature references).

The "neutralizing gravity" which you want discussed in the essay, is discussed both for more or less uniform motion, in which case it can be done (see the underlined sentence on page 3), and for vibrations and oscillations, in which case it cannot be done (according to theory), for reasons explained in the essay.

The last paragraph of page 5 and all of page 6 was added because I think it will interest the scientists among your readers, as well as the teachers of relativity theory. However, if my essay otherwise would be too long (which I don't think it is, though because of the many formulas I did not count words), you could leave it off, in as far as it does not discuss the problem of "harnessing gravity". (It is just another application of the same method used in the first part of the essay in discussing this harnessing or neutralizing etc. by means of motion.)

I enclose also a biographic sketch, which newspaper men may abbreviate as they see fit.

I also enclose a set of the literature references in this essay (footnotes 1 through 4). Reprint 4 unfortunately is defective, as two of the important pages are missing. I have no ~~other~~ undefective printed copy, though I could send you a dittoed preprint with the missing material if you want it. Also I can send duplicates on request, but I think I sent you these publications already in the past.

With kind regards,

Yours sincerely,

F.J.Belinfante

F.J.Belinfante

Title and abbreviated summary for an essay on gravity presented to the Gravity Research Foundation by Dr. F. J. Belinfante, 1809 Ravinia Road, West Lafayette, Indiana.

Title:

"On the question whether fast motion or fast rotation or vibration of an object can decrease the effect of gravity on it."

Abbreviated summary (almost 100 words:)

The gravitational acceleration is velocity-dependent. Consequently, fast particles may be repelled rather than attracted by gravity. Can this fact be utilized by fast rotation or vibration of objects? For Einstein's theory of gravitation the answer is "no", because of the equivalence principle. We show how this fact is consistent with the velocity-dependence of the gravitational acceleration. Our method may be used for deriving the well-known measurable effects of general relativity. For the linear theory of gravitation of this author, the equivalence principle is not postulated; nevertheless, the same conclusions hold in a very good approximation.

On the question whether fast motion or fast rotation or vibration of an object can decrease the effect of gravity on it

by Dr. Frederik J. Belinfante
professor of theoretical physics
Purdue University, Lafayette, Indiana

SUMMARY

People have suggested that fast rotation or vibration of an object may decrease the gravitational acceleration of its center. The velocity-dependence of the gravitational acceleration of a free particle would seem to substantiate this claim. This velocity dependence may be used not only for neutralizing the pull of gravity toward heavy bodies, but even for changing it into a repulsion. For a fast-rotating hoop, a blind application of the formulas would seem to predict a vanishing of the gravitational acceleration as the speed of the rim of the hoop approaches the speed of light.

The equivalence principle shows that for Einstein's theory of gravitation the argument somehow must break down for rotating or vibrating objects. We show here how the velocity dependence of the gravitational acceleration and the absence of an effect of the speed of parts of an object on the acceleration of its center are consistent with each other. Our method of calculation can also be used for explaining the various "experimental proofs" of Einstein's general theory, without talking about geodesics in a curved space.

For the linear theory of gravitation proposed by this author, the equivalence principle is not postulated. Nevertheless also for it the gravitational acceleration of the center of a rotating body seems to be substantially (though probably not rigorously) independent of the speeds of the outer parts of the body.

The linear theory of gravitation[1,2,3] predicts a velocity dependence of the gravitational pull. It has been asked[4] whether this fact would make the gravitational acceleration of an atomic nucleus of, say, copper different from the acceleration of a hydrogen nucleus, as the latter is a single proton practically at rest, while the former contains nucleons moving fast within it.[5]

Although the linear theory does not assume the validity of the equivalence principle, which would forbid such a dependence of the gravitational acceleration on the nature of the nucleus, nevertheless it has been shown by consideration of an electro-[redacted] dynamic model of a rotator that the effects of the high speed of the outer parts of a rotator also in the linear theory of gravitation are substantially undone by compensating effects in the mechanism that keeps the rotator together.[2,4]

It is interesting to remark that the velocity dependence of the gravitational acceleration is a feature not merely of the linear theory, but of Einstein's theory as well. Using "isotropic coordinates", the static spherically symmetric gravitational field around a star may be described by a line element given by

$$ds^2 = Q\,(dx^2 + dy^2 + dz^2) - \Omega c^2\,dt^2, \qquad (1)$$

where Q and Ω are functions of $r = \sqrt{x^2 + y^2 + z^2}$ only.
A particle moving in this field with a velocity

$$\mathbf{v} = d\mathbf{x}/dt \quad \left[\text{with speed } v = \sqrt{Q\,(v_x^2 + v_y^2 + v_z^2)} = \sqrt{Q\,\mathbf{v}^2}\,\right] \qquad (2)$$

~ means "boldface print"

will experience an acceleration

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -\frac{c^2}{2Q}\nabla\Omega + \frac{\mathbf{v}^2}{2Q}\nabla Q + \frac{\mathbf{v}}{\Omega}\frac{d\Omega}{dt} - \frac{\mathbf{v}}{Q}\frac{dQ}{dt}, \qquad (3)$$

where

$$\frac{dQ}{dt} = \mathbf{v}\cdot\nabla Q, \qquad \frac{d\Omega}{dt} = \mathbf{v}\cdot\nabla\Omega\,. \qquad (4)$$

Einstein's gravitational equations give

$$Q = \left(1 + \tfrac{1}{2}\iota\right)^4, \qquad \Omega = \left(\frac{1 - \frac{1}{2}\iota}{1 + \frac{1}{2}\iota}\right)^2, \qquad (5)$$

where $i = \frac{GM}{c^2 r}$, with G = gravitational constant, c = speed of light, and M = mass of the star. In practice (for planets etc.), i is very small, and

$$\left.\begin{aligned} &Q \approx 1 + 2i + \ldots, \quad \Omega \approx 1 - 2i + 2i^2 + \ldots, \\ &\qquad \text{so } \frac{1}{\Omega} - 1 \approx 2i + 2i^2 + \ldots . \end{aligned}\right\} \quad (6)$$

(This approximation is good enough for the following.) Then, [redacted] by Eq. (3), the gravitational acceleration of a particle at rest is

$$\mathbf{g} \equiv \mathbf{a}_{(v=0)} = -\frac{c^2}{2Q}\nabla\Omega = -\frac{GM\mathbf{x}}{r^3}(1 - 4i \ldots), \qquad (7)$$

and for a moving particle we find[6]

$$\mathbf{a} = \mathbf{g}\left(1 + \frac{v^2}{c^2}\right) - \frac{4\mathbf{v}(\mathbf{v}\cdot\mathbf{g})}{c^2} + \ldots. \qquad (8)$$

where we neglect terms of the order $i^2\mathbf{g}$ and $\frac{v^2}{c^2}i\,\mathbf{g}$, but we neglect no powers of $\frac{v}{c}$ in the terms linear in $\mathbf{g}$.

If we resolve $\mathbf{a}$ and $\mathbf{g}$ into components $\parallel \mathbf{v}$ and $\perp \mathbf{v}$, we find

$$\mathbf{a}_{\parallel} = \mathbf{g}_{\parallel}\left(1 - 3\frac{v^2}{c^2}\right), \qquad (9)$$

$$\mathbf{a}_{\perp} = \mathbf{g}_{\perp}\left(1 + \frac{v^2}{c^2}\right). \qquad (10)$$

Figure 1 shows this relation for the case $v = c$ (for photons), [redacted] for which $\mathbf{a}_{\perp} = 2\,\mathbf{g}_{\perp}$. This explains why Einstein's theory gave twice the bending of light rays passing the sun predicted according to Newton's theory.

Equation (9) shows that, for an object approaching or leaving a star ($\mathbf{v} \parallel \mathbf{g}$), <u>the gravitational attraction changes into a repulsion</u> for $v > c\sqrt{\frac{1}{3}} \approx 0.58\,c$. This is not much of a help in interstellar travel, as such high speeds do not occur just after take-off or just before landing. For objects bypassing a star, the effects of $\mathbf{a}_{\parallel}$ before and after passing cancel each other, and only the bending of the path according to Eq. (10) is important. *If a synchrotron beam ($v \approx c$) is shot up vertically, it will experience a gravitational acceleration $2g$ up (instead of g down), but this would be difficult to measure.*

A blind application of Eqs. (8) - (10) to a rotational motion

$$x \approx A \cos \omega t, \qquad y \approx A \sin \omega t \qquad (12)$$

in a plane passing through the star would seem to yield a gravitational acceleration which on the average would be decreased. If g is in the x direction, we would find (putting $\beta = \frac{v}{c} = A\frac{\omega}{c}$)

$$\left.\begin{aligned} a_x &= g(1 + \beta^2) - 4\beta^2 g \sin^2 \omega t, \\ a_y &= 4\beta^2 g \sin \omega t \cos \omega t. \end{aligned}\right\} \qquad (13)$$

The time average of this would give

$$\overline{a_x} = g\,(1 - \beta^2), \qquad \overline{a_y} = 0, \qquad (14)$$

as if the pull of gravity (in the x direction) would b e decreased by a factor $(1 - v^2/c^2)$, and therefore would vanish as v approaches c.

This would directly contradict the equivalence principle, according to which the accelerations of all free objects are zero in a freely falling frame of reference, and therefore must be equal among each other also in different frames of reference, independent of whether the objects rotate or not. Therefore, the center of the rotator, when at rest, should experience the acceleration g, and not $g(1 - \beta^2)$.

How Eq. (8) is invalidated for a rotator is most easily understood if we derive this equation directly from the equivalence principle. In the gravitational field with the line element (1) in our original world frame of reference Σ, we can introduce a local freely falling frame of reference Σ' near the point $x \approx y \approx t \approx 0$, $z \approx R$, by a transformation

$$\left.\begin{aligned} x' &= x\sqrt{Q}, \qquad y' = y\sqrt{Q}, \qquad t' = t\sqrt{\Omega}, \\ z' &= \zeta\sqrt{Q} + \frac{1}{4\sqrt{Q}}\frac{dQ}{dr}(x^2 + y^2 + \zeta^2) + \frac{c^2}{4\sqrt{Q}}\frac{d\Omega}{dr}t^2, \end{aligned}\right\} \qquad (15)$$

where $\zeta = z - R$. This transformation makes ds^2 of Eq. (1) equal to the Minkowskian

$$ds'^2 = dx'^2 + dy'^2 + dz'^2 - c^2 dt'^2 \qquad (16)$$

in Σ', but for terms at least quadratic in x', y', z', t' in the coefficients of dx'^2 etc.

In Σ' then a free particle travels according to

$$x' = v_x't', \qquad y' = v_y't', \qquad z' = v_z't', \qquad (17)$$

with constant velocity $\underset{\sim}{v}'$. By transforming back to Σ, we find $\underset{\sim}{x}$ as a function of t. Thence, we calculate $\underset{\sim}{v} = d\underset{\sim}{x}/dt$ and $\underset{\sim}{a} = d\underset{\sim}{v}/dt$. From the latter equations we eliminate $\underset{\sim}{v}'$, and we obtain $\underset{\sim}{a}$ as a function of $\underset{\sim}{v}$. This yields Eq. (3), and hence Eq. (8). The terms quadratic in v are found to arise in this derivation partially from the terms in (15) quadratic in the coordinates, as $x \approx x' = v_x't' \approx v_xt$, when squared, will contribute to a second time derivative; and partially from the fact that, for instance in $x = x'/\sqrt{Q} = v_x't\sqrt{\Omega}/Q$, a factor like $\sqrt{\Omega}/Q$ by Eq. (4) introduces another time dependence. All these $\underset{\sim}{v}^2$ contributions to $\underset{\sim}{a}$ therefore arise from the motion of the particle to points $\underset{\sim}{x}' = \underset{\sim}{v}'t'$ away from its starting point.

For a [redacted] rotational or vibrational motion, on the other hand, in Σ' Eq. (17) must be replaced by something different, and, if the center instantaneously was at rest, this on the average will keep the particle where it came from, and in the mean no v^2 contributions to $\underset{\sim}{a}$ will arise. If the center moves, then $\underset{\sim}{a}$ on the average will be the function (3) or (8) of the velocity $\underset{\sim}{v}$ of the center rather than of some jittering parts.

The method of derivation of Eq. (3) proposed here has not only the advantage of being easily adapted for a discussion of oscillators and rotators. This method, which avoids the explicit mentioning of geodesics in curved space which scares people of little mathematical sophistication, can also be used for deriving the formulas for the experimentally verifiable predictions of Einstein's theory. We mentioned already the bending of light (Eq. 10 and Fig. 1). The gravitational red shift follows from [redacted] $t' = t\sqrt{\Omega}$ in Eq. (15): An observer at infinity in Σ, using for comparison an emitter of light of frequency ν_0, observes the frequency ν of a similar emitter falling freely near a star, and emitting the frequency ν_0 with respect to the freely falling frame of reference Σ'. Therefore,

$\nu/\nu_0 = (dt)^{-1}/(dt')^{-1} = \sqrt{\Omega} \approx 1 - i,$

so

[redacted] $\dfrac{\delta\nu}{\nu_0} \simeq -i =$ [redacted] $-\dfrac{GM}{rc^2}$ $\Big\}$ (18)

For deriving the perihelion motion of the planets, we must integrate the equation of motion (3). It is easily verified that integrals of motion are given by

$$F = \frac{v^2}{c^2\Omega^2} + 1 - \frac{1}{\Omega} = \frac{Q\,v^2}{c^2\Omega^2} + 1 - \frac{1}{\Omega} \tag{19}$$

and by

$$\underset{\sim}{\Lambda} = \frac{Q}{\Omega}\,\underset{\sim}{r} \times \underset{\sim}{v} \,. \tag{20}$$

By considering what values these constants take [redacted] faraway from the star ($r \to \infty$, $Q \to 1$, $\Omega \to 1$), we find that F and $\underset{\sim}{\Lambda}$ are related to the energy E and the (orbital) angular momentum $\underset{\sim}{L}$ of the planet of rest mass μ by

$$E = \frac{\mu c^2}{\sqrt{1 - F}} = \frac{\mu c^2 \Omega}{\sqrt{\Omega - \frac{v^2}{c^2}}} \,, \tag{21}$$

$$\underset{\sim}{L} = \frac{E\underset{\sim}{\Lambda}}{c^2} = \frac{\mu\, Q\, \underset{\sim}{r} \times \underset{\sim}{v}}{\sqrt{\Omega - \frac{v^2}{c^2}}} \,. \tag{22}$$

By the conventional methods, Eq. (8) is now brought into the form

$$i + \frac{d^2 i}{d\varphi^2} = \frac{m^2 c^2}{2\Lambda^2}\frac{d}{di}\left[Q\left(F - 1 + \frac{1}{\Omega}\right)\right] , \tag{23}$$

with $i = m/r = GM/c^2 r$. Using a trial solution

$$i = B(1 + e\cos\Gamma\varphi) \,, \tag{24}$$

we find

$$B = \frac{m}{a(1 - e^2)} \tag{25}$$

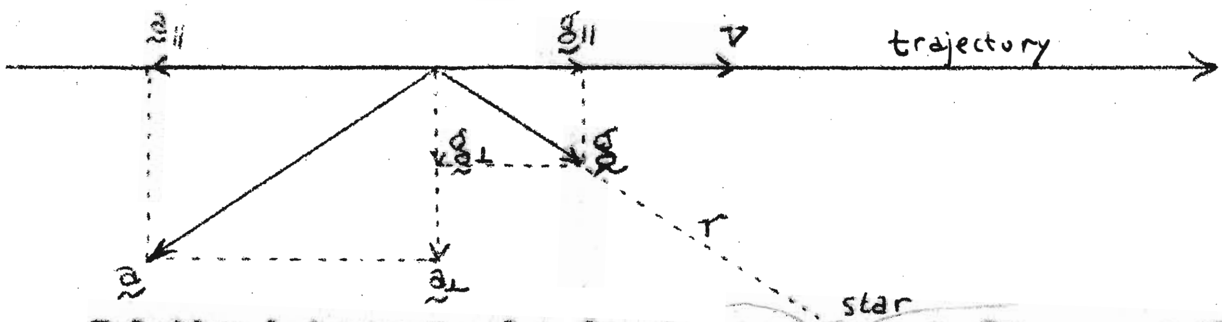
and, in first approximation,

$$\Gamma \approx 1, \quad \Lambda^2 \approx mc^2 a(1 - e^2), \quad \text{and} \quad F \approx -\frac{m}{a} \,. \tag{26}$$

The motion in this approximation is along a Kepler ellipse of major semiaxis a and excentricity e. The next approximation yields Einstein's result for the advance of the perihelion per period,

$$\eta = 2\pi\left(\frac{1}{\Gamma} - 1\right) \approx \pi(1 - \Gamma^2) \approx \frac{6\pi G M}{c^2 a(1 - e^2)} \,. \tag{27}$$

Figure 1.



Relation between $\underset{\sim}{g}$ and $\underset{\sim}{a}$ for v = c, so $\underset{\sim}{a}_{\parallel} = -2\underset{\sim}{g}_{\parallel}$, $\underset{\sim}{a}_{\perp} = +2\underset{\sim}{g}_{\perp}$.

Footnotes:

1) F.J.Belinfante and J.C.Swihart, Annals of Physics 1, 168 (1957).
2) F.J.Belinfante & J.C.Swihart, Annals of Physics 1,196 (1957).
3) F.J.Belinfante & J.C.Swihart, Annals of Physics 2, 81 (1957).
4) Revista Mexicana de Fisica 4, 202-205 (1955).
5) Also on less scientific grounds, it has been suggested that rotation of vibration might undo gravity. For instance, Joseph D. Stites expounds this idea in his proposals for shielding from gravity.
6) This particularly simple dependence of $\underset{\sim}{a}$ on $\underset{\sim}{v}$ is due to our choice of isotropic coordinates. With Schwarzschild's coordinates, r and therefore $\underset{\sim}{v}$ and $\underset{\sim}{a}$ have different meanings and therefore depend differently upon each other.

Biographical sketch of Dr. Frederik J. Belinfante, author of an essay on the question whether fast motion or fast rotation or vibration of an object can decrease the effect of gravity on it.

Frederik J. Belinfante was born in The Hague, Netherlands, on January 6, 1913. He got his elementary and highschool education in The Hague, and obtained his degrees in physics, mathematics, and astronomy at the University in Leiden, Netherlands. (B.S. 1933, M.S. 1936, Ph.D. in theoretical physics, 1939, doctor's thesis on meson theory.) In 1938 he attended the Seminar on Theoretical Nuclear Physics during the Summer session at Michigan State University in Ann Arbor, Mich. From 1936 until 1946 he was Assistant for Theoretical Physics at Leiden University, under Professor H. A. Kramers of international fame. Since 1945 he was also Lecturer of Thermodynamics. From 1946 until 1948, he was Associate Professor at the University of British Columbia, Vancouver, B.C., Canada. At Purdue University, Lafayette, Indiana, he was visiting professor in May-June, 1947; Associate Professor from 1948 until 1951, and Professor since 1951. Author of many papers in Physica, The Physical Review, American Journal of Physics, Progress of Theoretical Physics, Annals of Physics, Reviews of Modern Physics, etc.Main specialties: relativistic quantum theory of fields, gravitational theory. He was recipient of the first award of the Gravity Research Foundation in 1956.

He married in 1937 Wilhelmina F. M. Beukers, born in Groningen, Netherlands, and he has three children and one grandchild. In 1955, he and his wife became U.S. [redacted] citizens. He is a Fellow of the American Physical Society, member of the American Association of Physics Teachers, of the American Association of University Professors, of the Indiana Academy of Science, of Sigma Xi, of Sigma Pi Sigma, of the Netherlands Physical Society, of the Universal Esperanto Association, of Internacia Scienca Asocio Esperantista, and several other organizations. He is an active member of the First Methodist Church in West Lafayette, Indiana.

Hobbies: Physics; stamp collecting; tourism; taking stereo photos; Esperanto correspondence with foreign countries.