FEEDBACK MECHANISM FOR THE COSMOLOGICAL CONSTANT.

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ABSTRACT

We study the dynamics of the true vacuum in a cosmological phase transition governed by a Higgs field. When the mass of the field depends on the temperature there is a feed back mechanism for relaxing the cosmological constant near zero.

The standard cosmological model is faced with a notorious, by now, hierarchical problem: the vacuum energy ρ_v, of the actual Universe is extremely fined tuned to zero : if μ_0 is the scale of a spontaneous symmetry breaking, the scalar potential energy is of order $\mu_0{}^4$ and the ratio ρ_v/μ_0^4 is less than 10^{-56} for, say $\mu_0 \sim 300$ MeV or 10^{-122} for the Planck mass. Put in other words the actual cosmological constant(C.C.) expressed in Planck units is less than 10^{-122} and the problem is how such a tiny number could appear in a local theory. There are mainly two attitudes toward the resolution of this problem: one of them assumes the C.C. is strictly zero due to some symmetry[1]. The difficulty with this approach is that there is no known symmetry, at least at low energy, to realize the desiderate. The second approach consists to finding a natural way to bring the C.C. close to zero[2]. By the words "a natural way" we mean a mechanism for which the C.C. relaxes near a vanishing value whatwever its initial value would have been. In this paper we study the dynamics of the true vacuum in a phase transition together with a tentative proposal for relaxing the CC The tentative lies in the framework of the second approach.

Let us consider a phase transition governed by the scalar potential energy density of a Higgs field :

$$V(\phi) = \lambda\phi^4 - \mu^2(T)\,\phi^2 \,; \qquad \mu^2(T) = \mu_0^2 - cT^2 \qquad (1)$$

The coefficient c for the first order (in λ) temperature correction of the mass depends on the fact the ϕ field is a gauge singlet or an n-plet; in general, c is of order λ. The phase transition takes place for temperature smaller than $T_{cr} = (\mu_0^2/c)^{1/2}$, when a new minimum forms at $\phi = \sigma \neq 0$ (true vacuum). The C.C. in the new phase is determined in part by $V(\sigma) = \mu^4/4\lambda$ and depends naturally on temperature. The idea of a temperature dependent C.C. is not new. However, the possibility of a continuous decaying vacuum with the temperature, all the history of the universe, as in Ref.3 is severely restricted in order not to interfere with the success of nucleosynthesis[4]. Here we limit ourselves to the temperature dependence of C.C. for phase transitions only as predicted by the field theory at finite temperature[5]. Our further assumptions are: i) the two phases are well described by a flat (k = o) homogeneous spacetime ii) the dynamics of the broken phase is determined by a cosmological term (either positive or negative) and a perfect fluid stress energy tensor which includes the kinetic energy of the field and other forms of matter. Let us write the Einstein equations for such a case; the relevant ones are:

$$3\dot{a}^2 = (\Lambda + 8\pi K\rho)a^2 \qquad (2)$$

$$\dot{\rho}a = -3(p+\rho)\dot{a} \qquad (3)$$

where a is the Robertson Walker scale factor, we use units such as $G = c = h = 1$ and the MTW[6] sign conventions. To solving (2) and (3) we must supply also an equation of state

$$p = \gamma\rho \qquad (4)$$

where γ is a number between 0 (for dust) and $\gamma = 1/3$ (for radiation). Any mixtures of this cases can be accurately fitted with a $\gamma \in [0, 1/3]$. There is a variation of γ with time but enough slowly and in the following we shall neglect it. In this situation, inserting (4) and (3) in (2) we obtain:

$$3(\dot{a}/a)^2 = \Lambda + 8\pi K\rho_0 (a/a_0)^{-3(\gamma+1)} \qquad (5)$$

which has analytic solutions. For a positive cosmological constant it is :

$$a = a_0 \left[sh\, t(\gamma+1)\sqrt{6\pi K \rho_0} \right]^{\frac{2}{3(\gamma+1)}} \qquad (6)$$

The scale factor being normalized so as $a = a_0$ at $t = 2\ln(1+\sqrt{2})/[\sqrt{3}(\gamma+1)\sqrt{\Lambda}]$ and $\Lambda = 8\pi K \rho_0$. For late times relation (6) represents a quasiexponential de Sitter expansion.
For a negative cosmological constant eq.(5) gives an oscillatory solution :

$$a = a_0 \left[\sin t(\gamma+1)\sqrt{3|\Lambda|}/2 \right]^{\frac{2}{3(\gamma+1)}} \qquad (7)$$

It can be ontained by direct solving (5) or by analytic continuation of the solution (6). Note, the solution is viable for $|\Lambda| \leq 8\pi K \rho$. Finally, for $\Lambda \to 0$, the solution is found by series expansion from (6) or (7) as :

$$a = a_0 \left[t(\gamma+1)\sqrt{6\pi K \rho_0} \right]^{\frac{2}{3(\gamma+1)}} \qquad (8)$$

This reproduces the usual cosmologies for radiation dominated ($\gamma = 1/3$, $a \sim t^{1/2}$) and for matter dominated ($\gamma = 0$, $a \sim t^{2/3}$) Universe.

We are now in position to explain, at least qualitatively, the feed back mechanism announced in the title. The symmetric phase possesses an initial C.C. Λ_0, which, we shall asume, has any arbitrary value, between zero and $\mu_0^4 K\pi/\lambda$. Once the temperature drops below T_{cr}, a global minimum forms(see fig.1); the assymmetric vacuum has an effective C.C. $\Lambda = \Lambda_0 - K\pi\mu_0^4/\lambda$

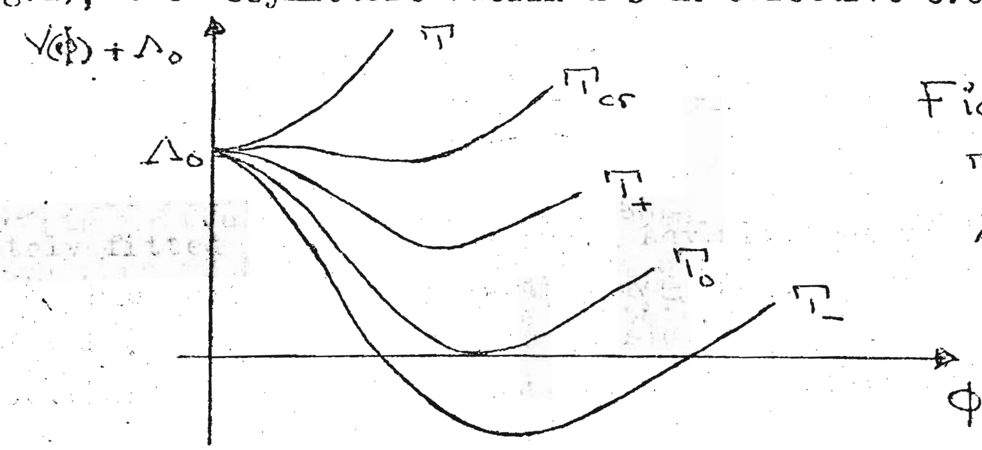

Fig. 1.
$T > T_{cr} > T_+ >$
$> T_0 > T_-$

Its value can be positive, zero or negative (corresponding to the curves T_+, T_0, T_-) depending on the magnitude of the initial Λ_0 and the time necessary for the transition to take place. For a delayed phase transition and a small Λ_0 (compared with $V(T,0) = \kappa T \mu_0^4/\lambda$, we expect the effective C.C. to be negative and the bubbles of the new phase to behave accordingly to (7). It will eventually expand to a maximum value (when $|\Lambda| = 8\pi\kappa\rho$) but then it recontracts, the temperature raises, and with certitude it will attain the value T_0 because as $a \to 0$, $T \to \infty$. On the other hand if the effective C.C. is positive (e.g. the T_+ curve) the new phase expands quasi-exponentially, the effect being a decrease of the temperature. Given an enough long time we expect the temperature to settle down to a value near T_0 corresponding to an effective $\Lambda \simeq 0$. It is clear, also that, the most of the time, the bubble will spend in a state with $\Lambda \leq 8\pi\kappa\rho$. The cT^2 plays the rôle of a retroaction: when C.C. is positive, the cT^2 term, via (6), diminishes itself and the C.C. (for normal matter when a and T are inversely correlated). Conversely for a negative C.C. the cT^2 term increases and so does the C.C. (after some lapse of time when $|\Lambda| < 8\pi\kappa\rho$). Till now we tacitely assumed the evolution of the effective C.C. is enough slowly and we can treat it as a real constant like in eq.(5). To see if this approximation is justified we must provide one more equation linking, say, the energy density and the temperature

$$\rho = \sigma T^n \tag{9}$$

with $n=4$ for radiation and $n=1$ for matter. As before, we argue that any combination matter-radiation can be accurately described by any $n \in [1,4]$; however the variation with time of the indice n may be important. The dynamical and conservation equation (2),(3) read now:

$$3(\dot{a}/a)^2 = 8\pi\kappa\sigma T^n + \Lambda_0 - \frac{\pi\kappa c^2 T_{cr}^4}{\lambda}\left(1 - \frac{T^2}{T_{cr}^2}\right)^2 \tag{10}$$

$$3\dot{a}(\gamma+1)\sigma T^n = -\dot{T}\left[\sigma n T^{n-1} + \frac{c^2}{2\lambda}(T_{cr}^2 - T^2)\right] \tag{11}$$

The equation of conservation can be readily integrated for any n but for definitess we take $n=4$. It is convenient to introduce the adimensional quantities $x \equiv T/T_{cr}$,

$C_1 \equiv c^2/8\lambda\sigma$ and $r \equiv \Lambda_0/8\pi K \rho_{cr}$. The equation (11) gives

$$a/a_{cr} = x^{C_1-1} \exp C_1 \frac{1-x^2}{2x^2} \qquad (12)$$

while eq.(11) gives :

$$(\dot{x}/x)^2 \left(C_1 + \frac{1-C_1}{x^2}\right)^2 = \left[x^4 + C_1(1-x^2)^2 + r\right] \omega \qquad (13)$$

where $\omega \equiv 8\pi K \rho_{cr}/3$. Note that C_1 is typically, a small number: with $c=\lambda/3$, $\lambda \sim 10^{-1}$, ϕ singlet and $\sigma=\pi^2/15$, $C_1 \sim 10^{-3}$; if ϕ is responsible for the breaking of SU(5), $\lambda \sim 1$ and $\sigma = N\pi^2/15$ with $N \sim 200$. So C_1 is at most $O(10^{-2})$. If we neglect C_1, eq (13) reduces to eq(5) via (9). So our previous approximation, treating Λ as a constant function, is justified. We did not succeed to solve the system (10,11) for any n, (some numerical simulations are in progress), let apart we do not know how vary n with the temperature. However for pure radiation there is an exact solution. The temperature is given implicitely by :

$$2(\omega t + \text{const}) = \sqrt{(r-C_1) + 2C_1 x^2 r (1-C_1) x^4}/(r-C_1)x^2 + \frac{r-C_1-C_1 r}{r-C_1} I \qquad (14)$$

where

$$I = \begin{cases} \dfrac{1}{\sqrt{r-C_1}} \operatorname{arsh} \dfrac{x^2 C_1 + r - C_1}{x^2\sqrt{r-C_1-C_1 r}} & \text{for } r > C_1/1-C_1 \\[2ex] -\dfrac{1}{\sqrt{-r+C_1}} \arcsin \dfrac{x^2 C_1 + r - C_1}{x^2\sqrt{C_1 - r + rC_1}} & \text{for } r < C_1 \end{cases}$$

The constant of integration is fixed demanding that at t = 0, x = 1.

The interesting feature is that, in the case $r < C_1$, there is a minimum temperature attainable. This is due to the theorem of positivity of energy: the system can not evolve into a state with $8\pi K\rho - |\Lambda| < 0$. Although the existence of a minimum temperature is general, its value depends on n. A smaller n gives a slower decreasing of the temperature with the time and the scale factor (e.g. for n = 2, $a \sim T^{-1} \exp C_2 T^2$)

Finally we argue that our restriction, $\Lambda_0 \leq \mu_0^4 K \pi/\lambda$ is not so drastic, if, in the history of the Universe, there was a sequence of phase transitions patterned after (1), then $\Lambda_0 \leq \mu^4_{0\,max} K\pi/\lambda$ say $\Lambda_0 \lesssim \mu^2$(Planck)

We think the mechanism discussed is quite simple and has the advantage it does not need the introduction of any extra physics. We publish it with the hope that somebody will find a more precise way to formulate it.

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