

# Physics with nonperturbative quantum gravity: radiation from quantum black holes

(Essay written for the 1996 Gravity Research Foundation  
Awards)

Marcelo Barreira<sup>1\*</sup>, Mauro Carfora<sup>2†</sup>, Carlo Rovelli<sup>1‡</sup>

<sup>1</sup> *Department of Physics and Astronomy, University of Pittsburgh,  
Pittsburgh PA 15260, USA*

<sup>2</sup> *Dipartimento di Fisica, Universita' di Pavia, Pavia, Italy*

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## Summary

We study quantum gravitational effects on black hole radiation, using loop quantum gravity. Bekenstein and Mukhanov have considered the modifications caused by quantum gravity on Hawking's thermal black-hole radiation. Using a simple ansatz for the eigenstates the area, they have obtained the intriguing result that quantum gravity affects the radiation considerably, yielding a non-thermal spectrum. We replace the simple ansatz with the eigenstates of the area computed using loop quantum gravity. We derive the emission spectra, using a classical result in number theory by Hardy and Ramanujan. We do not recover the Bekenstein-Mukhanov spectrum, but a Hawking's thermal spectrum. The Bekenstein-Mukhanov result is therefore likely to be an artefact of the naive ansatz. The result is an example of concrete application of nonperturbative quantum gravity.

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\*e-mail: marcelo@phyast.pitt.edu

†e-mail: carfora@pavia.infn.it

‡e-mail: rovelli@pitt.edu

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## Abstract

We study quantum gravitational effects on black hole radiation, using loop quantum gravity. Bekenstein and Mukhanov have recently considered the modifications caused by quantum gravity on Hawking's thermal black-hole radiation. Using a simple ansatz for the eigenstates of the area, they have obtained the intriguing result that the quantum properties of geometry affect the radiation considerably, yielding a definitely non-thermal spectrum. Here, we replace the simple ansatz employed by Bekenstein and Mukhanov with the actual eigenstates of the area, computed using the loop representation of

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\*e-mail: marcelo@phyast.pitt.edu

†e-mail: carfora@pavia.infn.it

‡e-mail: rovelli@pitt.edu

quantum gravity. We derive the emission spectra, using a classical result in number theory by Ramanujan and Hardy. Disappointingly, we do not recover the Bekenstein-Mukhanov spectrum, but –effectively– a Hawking’s thermal spectrum. The Bekenstein-Mukhanov result is therefore likely to be an artefact of the naive ansatz, rather than a robust result. Finally, we comment on the relevance of the above derivation for the understanding of black hole entropy. The result is an example of concrete (although somewhat disappointing) application of nonperturbative quantum gravity.

Quantum gravity research has traditionally suffered for a great scarcity of physical applications where theories and ideas could be tested, at least in principle [1]. One of the few areas in which ideas on quantum gravity may be tested is black hole physics [2]. The loop approach to quantum gravity [3] is now sufficiently developed that we may begin to probe it within “physical” applications. It is thus natural to investigate what loop quantum gravity asserts about black hole physics.

Recently, Bekenstein and Mukhanov [4] have suggested that the thermal nature of Hawking’s radiation may be affected by quantum properties of gravity (For a review of earlier suggestions in this direction, see [5]). As it is well known, Hawking derived the black hole thermal emission spectrum from quantum field theory in curved spacetime, therefore within the approximation in which the quantum properties of gravity are neglected. Attempts have been made to relate Hawking’s temperature with gravitational dynamics, but the problem of how quantum gravity affects black hole emission can be convincingly addressed only within a full theory of the quantum gravitational field. Bekenstein and Mukhanov observe that in most approaches to quantum gravity the area can take only quantized values [6]. Since the area of the black hole surface is connected to the black hole mass, black hole mass is likely to be quantized as well. The mass of the black hole decreases when radiation is emitted. Therefore emission happens when the black hole makes a quantum leap from one quantized

value of the mass (energy) to a lower quantized value, very much as atoms do. A consequence of this picture is that radiation is emitted at quantized frequencies, corresponding to the differences between energy levels. Thus, quantum gravity implies a discretized emission spectrum for the black hole radiation.

By itself, this result is not physically in contradiction with Hawking's prediction of a continuous thermal spectrum. To understand this, consider the black body radiation of a gas in a cavity, at high temperature. This radiation has a thermal Planckian emission spectrum, essentially continuous. However, radiation is emitted by elementary quantum emission processes yielding a discrete spectrum. The solution of the apparent contradiction is that the spectral lines are so dense in the range of frequencies of interest, that they give rise –effectively– to a continuous spectrum. Does the same happen for a black hole?

In order to answer this question, we need to know the energy spectrum of the black hole, which is to say, the spectrum of the Area. Bekenstein and Mukhanov pick up a simple ansatz: they assume that the Area is quantized in multiple integers of an elementary area  $A_0$ . Namely, that the area can take the values

$$A_n = nA_0, \tag{1}$$

where  $n$  is a positive integer, and  $A_0$  is an elementary area of the order of the Planck Area

$$A_0 = \alpha \hbar G, \tag{2}$$

where  $\alpha$  is a number of the order of unity ( $G$  is Newton's constant and  $c = 1$ ). Ansatz (1) is reasonable; it agrees, for instance, with the partial results on eigenvalues of the area in the loop representation given in [7], and with the idea of a quantum picture of a geometry made by elementary “quanta of area”. Since the black hole mass is related to the area by

$$A = 16\pi G^2 M^2, \tag{3}$$

it follows from this relation and the ansatz (1) that the energy spectrum of the black hole is given by

$$M_n = \sqrt{\frac{n\alpha\hbar}{16\pi G}}. \quad (4)$$

Consider an emission process in which the emitted energy is much smaller than the mass  $M$  of the black hole. From (4), the spacing between the energy levels is

$$\Delta M = \frac{\alpha\hbar}{32\pi GM}. \quad (5)$$

From the quantum mechanical relation  $E = \hbar\omega$  we conclude that energy is emitted in frequencies that are integer multiple of the fundamental emission frequency

$$\bar{\omega} = \frac{\alpha}{32\pi GM}. \quad (6)$$

This is the fundamental emission frequency of Bekenstein and Mukhanov [4] (they assume  $\alpha = 4 \ln 2$ ). Bekenstein and Mukhanov proceed in [4] by showing that the emission amplitude remains the same as the one in Hawking's thermal spectrum, so that the full emission spectrum is given by spectral lines at frequencies multiple of  $\bar{\omega}$ , whose envelope is Hawking's thermal spectrum.

As emphasized by Smolin in [5], however, the Bekenstein-Mukhanov spectrum is drastically different than the Hawking spectrum. Indeed, Hawking temperature is

$$T_H = \frac{\hbar}{8\pi kGM} \quad (7)$$

( $k$  is Boltzmann constant); therefore the maximum of the Planckian emission spectrum of Hawking's thermal radiation is at

$$\omega_H \sim \frac{2.82kT_H}{\hbar} = \frac{2.82}{8\pi GM} = \frac{2.82 \cdot 4}{\alpha} \bar{\omega} \approx \bar{\omega}. \quad (8)$$

That is: the fundamental emission frequency  $\bar{\omega}$  is of the same order as the maximum of the Planck distribution of the emitted radiation. It follows that there are only a few spectral lines in the regions where emission is appreciable. Therefore the Bekenstein-Mukhanov spectrum is drastically different than the Hawking spectrum: the two have the same envelope, but while Hawking spectrum is continuous, the Bekenstein-Mukhanov spectrum is formed by just a few lines in the interval of frequencies where emission is appreciable.

This result is of great interest because, in spite of its weakness, black hole radiation is still much closer to the possibility of (indirect) investigation than any quantum gravitational effect of which we can think. Thus, a clear quantum gravitational signature on the Hawking spectrum is a very interesting effect. Is this Bekenstein-Mukhanov effect credible?

One of the most definite results of loop quantum gravity is a calculation of the spectrum of the area from first principles [8]. Thus, following a suggestion in [5], we may use loop quantum gravity to check the Bekenstein-Mukhanov result, by replacing the naive ansatz (1) with the precise spectrum computed in this approach to quantum gravity.

Consider a surface  $\Sigma$  –in the present case, the event horizon of the black hole–. According to loop quantum gravity, the area of  $\Sigma$  can take only a set of quantized values. These quantized values are labelled by unordered  $n$ -tuplets of positive integers  $\vec{p} = (p_1, \dots, p_n)$  of arbitrary length  $n$ . The spectrum is then given by

$$A_{\vec{p}} = 16\pi\hbar G \sum_{i=1,n} \sqrt{\frac{p_i}{2} \left( \frac{p_i}{2} + 1 \right)}, \quad (9)$$

For a full derivation of this spectrum, see [8]. The spectrum (9) is not complete. There is an additional sector corresponding to a class of “degenerate” states, whose physical interpretation is not obvious to us. These degenerate states play no role in the present discussion, however.

If we disregard for a moment the term  $+1$  under the square root in (9), we obtain immediately the ansatz (1), and thus the Bekenstein-Mukhanov result. However, the  $+1$  is there. Let us study the consequences of its presence. First, let us estimate the number of Area eigenvalues between the value  $A \gg \gg l_0$  and the value  $A + dA$  of the Area, where we take  $dA$  much smaller than  $A$  but still much larger than  $l_0$ . Since the  $+1$  in (9) affects in a considerable way only the terms with low  $p_i$ , we can neglect it for a rough estimate. Thus, we must estimate the number of unordered strings of integers  $\vec{p} = (p_1, \dots, p_n)$  such that

$$\sum_{i=1,n} p_i = \frac{A}{8\pi\hbar G} \gg 1. \quad (10)$$

This is a well known problem in number theory. It is called the partition problem. It is the problem of computing the number  $N$  of ways in which an integer  $I$  can be written as a sum

of other integers. The solution for large  $I$  is a classic result by Ramanujan and Hardy [11]. According to Ramanujan and Hardy,  $N$  grows as the exponent of the square root of  $I$ .

$$N(I) \sim e^{\sqrt{I}}. \quad (11)$$

Applying this result in our case we have that the number of eigenvalues between  $A$  and  $A + dA$  is

$$\rho(A) \approx e^{\sqrt{\frac{A}{8\pi\hbar G}}}. \quad (12)$$

Now, because of the presence of the  $+1$  term, eigenvalues will overlap only accidentally: generically all eigenvalues will be distinct. Therefore, the average spacing between eigenvalues decreases exponentially with the inverse of the square of the area. This result is to be contrasted with the fact that this spacing is constant and of the order of the Planck area in the case of the naive ansatz (1). This conclusion is devastating for the Bekenstein-Mukhanov argument. Indeed, the density of the energy levels becomes

$$\rho(M) \approx e^{\sqrt{2G\hbar M}}, \quad (13)$$

and therefore the spacing of the energy levels decreases exponentially with  $M$ . It follows that for a macroscopical black hole the spacing between energy levels is infinitesimal, and thus the spectral lines are virtually dense in frequency. We effectively recover in this way Hawking's thermal spectrum (except, of course, in the case of a Planck scale black hole). The conclusion is that the Bekenstein-Mukhanov effect disappears if we replace the naive ansatz (1) with the spectrum (9) computed from loop quantum gravity. More generally, we have shown that the Bekenstein-Mukhanov effect is strongly dependent on the peculiar form of the naive ansatz (1), and it is not robust. In a sense, this is a pity, because we loose a possible window on quantum geometry.

We close with a comment on black hole entropy. From the point of view of an observer at infinity, a (non-charged and non-rotating) macroscopic black hole appears as an object completely characterized by its energy  $M$ . If we study the thermodynamics of a system

that contains the black hole, we have to include the microscopic black hole states in the count of the density of states. In other words, the black hole thermodynamical properties are characterized by the microcanonical partition function  $z(M)$  which gives the number of states between the energies  $M$  and  $M + dM$ . Notice that

$$z(M) = \nu(M)\rho(M), \tag{14}$$

where  $\rho(M)$  is the density of eigenvalues of  $M$  given in (13), and  $\nu(M)$  is the factor determined by the fact that  $M$  eigenstates may be degenerate. If we assume for a moment that the area energy levels are non degenerate, then we have immediately the full thermodynamics of the black hole from (13). Unfortunately, it is easy to see that this yields a dependence of the entropy from the area of the form  $S(A) \sim \sqrt{A}$ , which is likely to be incorrect. Therefore considering the area eigenstates as degenerate is incorrect. It is not difficult to understand why. The black hole surface is a sphere in which we may distinguish points from each other, in terms of the neighboring geometry. Thus, a microscopical analyses could in principle distinguish whether one patch of the surface has more area than another one. Consequently, we must consider two quantum states of the black hole as distinguishable (from the outside) if the two are related simply by a relabelling of the microscopical area elements. Two such states are characterized by different orderings of the same n-tuplet  $\vec{p} = (p_1, \dots, p_n)$ . Thus, in studying the thermodynamical properties of a black hole we must assign a multiplicity  $n!$  to a state characterized by the n-tuplet  $\vec{p} = (p_1, \dots, p_n)$ . In a remarkable recent paper [10], Krasnov has explicitly computed this density of state and the related entropy, using an elegant technique from statistical mechanics and obtaining a linear dependence of the entropy  $S$  from the area, as in the Bekenstein-Hawking formula. In [10], Krasnov motivates this computation in terms of the Bekenstein entropy-bound conjecture and the holographic hypothesis. We believe that such arguments are in fact superfluous, and that the Krasnov computation can be reinterpreted as a straightforward computation of the density of the microstates of the black hole distinguishable from the outside. Then, loop quantum gravity provides a direct root for deriving the Bekenstein-Hawking formula  $S \sim A$  from nonpertur-

bative quantum gravity. These considerations will be further developed elsewhere.

Summarizing, we have derived two consequences from loop quantum gravity. We have shown that the discretization of the spectrum derived by Bekenstein and Mukhanov disappears if we use quantitative result from the theory, and we have indicated that the density of states of a quantum black hole can be computed, yielding the correct Bekenstein-Hawking entropy. Our results indicate that loop quantum gravity is sufficiently mature to begin addressing concrete physical problems.

## REFERENCES

- [1] See for instance C. Isham: “Structural Issues in Quantum Gravity”, lecture at the GR14 meeting, Florence 1995, gr-qc/9510063.
- [2] For a review, see RM Wald: *Quantum Field Theory on Curved Spacetime and Black Hole Thermodynamics* (University of Chicago Press: Chicago 1994).
- [3] C Rovelli, L Smolin: Phys Rev Lett 61 (1988)1155, Nucl Phys B331 (1990) 80. For various perspectives on loop quantum gravity, see: C Rovelli, Class and Quantum Grav 8, 1613 (1991). A Ashtekar, in *Gravitation and Quantization, Les Houches 1992*, edited by B Julia and J Zinn-Justin (Elsevier Science: Paris 1995). L Smolin, in *Quantum Gravity and Cosmology*, ed J Perez-Mercader, J Sola, E Verdaguer (World Scientific: Singapore 1992).
- [4] JD Bekenstein, VF Mukhanov, “Spectroscopy of the quantum black hole”, gr-qc/9505012.
- [5] L Smolin: “Microscopic Deviations from Hawking radiation?”, Matters of Gravity 7, gr-qc/9602001.
- [6] LJ Garay: “Quantum Gravity and Minimal Length”, gr-qc/9403008.
- [7] A Ashtekar, C Rovelli, L Smolin: Phys Rev Lett 69, 237 (1992)
- [8] C Rovelli, L Smolin: Nuc Phys B 442 (1995) 593. R DePietri, C Rovelli: “Geometry Eigenvalues and Scalar Product from Recoupling Theory in Loop Quantum Gravity”, gr-qc/9602023. A Ashtekar, J Lewandowski: “Quantum Theory of Geometry I: Area Operator”, gr-qc/9602046.
- [9] JD Bekenstein: Phys Rev D7 (1973) 2333. SW Hawking: Nature 248 (1974) 30.
- [10] K Krasnov, “The Bekenstein bound and non perturbative quantum gravity”, gr-qc/9603025.

- [11] G E Andrews *The theory of partitions; The Encyclopedia of Mathematics and its applications, Vol.2* (Addison-Wesley: New York 1976).