On the Phase Transition to Space-Time in String Cosmology

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Closed string models have recently been constructed in lower than their critical space-time dimensions $D \leq D_{cr}$. An ideal gas of closed strings with $D \geq 4$ undergoes a phase transition at a universal point (Hagedorn temperature). We argue that non-trivial configurations on the string world-sheet (vortices) drive the system into a high temperature phase where the vacuum is dominated by vortex condensates. Flat space-time is identified with the dipole low-temperature phase of vortex anti-vortex pairs. This is a "Kosterlitz-Thouless" transition on the string world-sheet. It is suggestive of a "stringy" realization of the inflationary universe paradigm.

String theories provide a framework for a consistent quantum theory unifying all known particle interactions including gravity.[1] The last two years have witnessed an explosion of closed string theories formulated in arbitrary space-time dimensions $D \leq D_{cr}(D_{cr} = 26$ bosonic string, $D_{cr} = 10$ type II and heterotic strings). More specifically four dimensional consistent string models were constructed either by compactification or by the utilization of different versions of the underlying two dimensional (super) conformal field theory of the string world-sheet.[2]

By the first method (torus or orbifold construction), both left and right moving degrees of freedom on the string world-sheet are compactified. In this way the original uniqueness of the gauge groups $E_8 \times E_8$ and $S0(32)$ is lost. There now exist numerous string models with simply or non-simply laced groups of rank $r \leq 22$ $(S0(Z_r), SU(r+1), E_r)$. Fermionic constructions, on the other hand, utilize the boson-fermion equivalence in the two-dimensional world-sheet. Space-time dimensionality is reduced by the introduction of free world-sheet fermions instead of bosons consistently with (super) reparametrization, (super) conformal and modular invariance ("fermionization" of the bosonic space-time coordinates). In this way one may dispense completely with the idea of extra space-time dimensions as well as of their compactification interpretation.

All string models correspond to perturbatively stable vacua of a second quantized string theory and they should be treated on an equal footing. The plethora of closed string models raise important new questions. Are we dealing with different string theories or with distinct ground states of the same string theory? In the case of an enormous "vacuum degeneracy" what is the physical mechanism that picks the ground state we live in?

It has recently been realized that the thermodynamics of an ideal string gas at high densities and temperatures can effectively classify closed string models in terms of their space-time dimensions D.[3,4,5] For one thing such a system exhibits a critical behavior at the characteristic temperature T_H (Hagedorn temperature). This critical point can be found by a direct computation of the one loop string free energy[3] or through an evaluation of the asymptotic density of string states.[4,5] Remarkably it was discovered that modular invariance constrains the Hagedorn temperature to be universal. It only depends on the (super) reparametrization properties of the world-sheet and the value of the string mass scale α' (the Regge slope). In this essay, rather than go over the detailed technical arguments,[4,5] we will discuss further aspects of this remarkable property of closed strings.

More specifically the universal Hagedorn temperature suggests a classification of all closed string models on the basis of the number of world-sheet supersymmetries as follows:[4,5]

i) *Type II strings* with left and right superconformal invariance

$$T_H = \frac{1}{2\sqrt{2}\pi\sqrt{\alpha'}}\,. \tag{1}$$

ii) *Heterotic strings* with left world-sheet supersymmetry, left superconformal invariance and right conformal invariance

$$T_H = \frac{1}{(2+\sqrt{2})\pi\sqrt{\alpha'}}\,. \tag{2}$$

iii) *Bosonic strings* with left and right conformal invariance. There is neither left nor right world-sheet supersymmetry

$$T_H = \frac{1}{4\pi\sqrt{\alpha'}}\,. \tag{3}$$

A $D = 10$ heterotic string, in effect, cannot be identified as an asymmetric ground state of the $D = 26$ bosonic string. Ideal string gases with $D \geq 4$ undergo a phase transition at T_H with a characteristic power low critical behavior that depends crucially on D. The distinct thermodynamic properties of the $D = 4$ system, in particular (e.g. diverging specific heat), seem to suggest a unique four-dimensional universe. Alas, if string vacua with different values of D are ground states of the same string theory, the universality of T_H allows it to be a multi-critical point. Our observable universe may have emerged at T_H as a membrane embedded in a higher dimensional space-time manifold. Such amusing geometrical constructions have recently been made by Gibbons et al.[6] It must be noted that coexistence of distinct "phases" at a multi-critical point is a common phenomenon among many fluid and magnetic systems with multiple ground states.[7]

Our discussion so far considered the high temperature behavior of an ideal string gas. Higher order of the string topological expansion (loop expansion in α') and non-perturbative effects are likely to play an essential role at the Planck scale of energies and temperatures. In what follows we will attempt to take some of these effects into account.

It is well known[1] that the configuration space of a closed non-orientable string compactified on a circle of radius R contains solitonic states with non-zero winding number. A bosonic coordinate has left and right moving states with discretized momenta (P_L, P_R) where $P_L + P_R = \frac{\eta}{R}$ (η = integer) and $P_L - P_R = 2mR$ (m = integer winding number). In the equivalent fermionic language one Dirac fermion will possess similarly states of discrete momenta and winding numbers.[8] More precisely the winding number of a single fermionic state is given by $([A]-[B])/2 + N_L - N_R$. Here we take $([A],[B])$ to denote the spin structure of the (left, right) Dirac spinor components. In our notation $[A] = A\ mod2$ with $[A]\epsilon(1,-1)$. The left-right net fermion numbers of the state are N_L and N_R respectively. We will now examine the physics of these configurations in the most convenient case of the bosonic string gas.

The compact part of the closed bosonic string action in flat Euclidean 26-dim. space is given by

$$S = \frac{R^2}{4\pi\alpha'} \int d^2 z \partial_a \theta \partial_a \theta \,. \tag{4}$$

Here θ is compact ($\theta \equiv \theta + 2\pi n$) and α' is the usual string mass scale. The circle S with radius R is identified with the Euclidean imaginary time. We also identify the temperature $T = \frac{1}{2\pi R}$. The theory, as argued before, admits states with non-trivial winding numbers (vortices) due to non-trivial maps of the world-sheet's edge on $S(\pi_1(S) = Z)$.

More importantly we recognize, in the form of the action of Eq. (4), the continuous limit of the low temperature phase of the XY model (Villain model).[9] It is well known that it undergoes a phase transition due to vortex condensation (Kosterlitz-Thouless transition). These configurations are irrelevant at low temperatures but they nevertheless drive a phase transition at a critical point T_{KT}. Moreover they dominate the ground state of the system at $T > T_{KT}$.

The action of a single vortex is $\sim 2\pi \left(\frac{R^2}{4\pi\alpha'}\right) \ln\left(\frac{R}{\beta}\right)$ where β is an ultraviolet cut-off associated with the 2-dim. lattice spacing. The entropy of a vortex has a similar form $\sim \ln\left(\frac{R}{\beta}\right)^2$. Our action of Eq. (4) is defined up to a temperature T_{KT} where the free energy of the vortex $\propto \left(\frac{R^2}{4\alpha'} - 1\right) \ln\left(\frac{R}{\beta}\right)$ vanishes. Indeed we trivially observe this to occur at $R = 2\sqrt{\alpha'}$ and $T_{KT} = \frac{1}{4\pi\sqrt{\alpha'}}$. Remarkably we have rediscovered the Hagedorn temperature of the bosonic string gas (Eq. (3)). The phase diagram and coupling constant hyperbolic trajectories (Kosterlitz flows) are depicted in the $X - Y$ plane of Fig. 1. The fugacity $Y = \exp - \left(\frac{\text{chemical potential}}{T}\right)$ measures the importance of vortices in the system. Our argument ignores interactions between the vortices as they would give small corrections to T_{KT} of order $\exp -1/T$. In the low temperature phase $X > Y$ $\left(X = \frac{R^2}{4\alpha'} - 1\right)$, the conformal string action is identified with the line of renormalization group fixed points on the positive X-axis where it can only be meaningfully defined. In the high temperature phase $(X < Y)$ vortices become increasingly important as the trajectories lead us away from the line of fixed points to larger values of the fugacity Y. We do not know whether there are non-trivial fixed points in this region and hence any consistent string theories. Our action of (Eq.(4)), defined in a flat background, is certainly invalid in this regime. Strings conceivably propagate in a non-trivial background which would surely contribute to the energy momentum tensor and hence to curvature. This leads us naturally to speculate that the inflationary phase in the early universe has a "stringy" origin.[10] In such a case the Hawking temperature of the De-Sitter phase finds a natural interpretation as the Hagedorn temperature of the hot string gas. This possibility would reconcile inflation with topological defects (cosmic strings) that appear in GUT phase trnsitions ($10^{15} - 10^{16}$ GeV) and are candidate seeds for galaxy formation and the observed large scale structure of the universe.

In conclusion, we identified a mechanism that drives the Hagedorn phase transition by taking into account non-trivial configurations (vortices) on the string world-sheet. This is certainly the "tip of the iceberg". A better understanding of the high temperature and energy phase in string theory will entail the development of new theoretical tools. They should be able to deal with infinite genus fluctuations due to string interactions and string world-sheet interactions, i.e. fluctuations of the sigma model action on the world-sheet.

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Figure Caption

1. Renormalization flows for the periodic Gaussian (Villain) model. A dashed line denotes a locus of initial conditions. The low temperature phase $(X > Y)$ is a domain of attraction of the line of fixed points $Y = 0$. Only the fixed points $(X > 0, Y = 0)$ can be reached. For $X < Y$ one is driven away from the X axis. Here $X \sim \frac{J}{T} - 2$($J \equiv$ spin coupling) and $Y = \exp -\left(\frac{\text{chemical potential}}{T}\right)$.

Figure 1