Errata Sheet For Manuscript:

"Gravitational Radiation Antennas Using The Sagnac Effect"

J. Anandan and R.Y. Chiao

p. 4, the line after Eq.(6) (This error occurs twice.)

"ημν" should read "$\eta_{\mu\nu}$"

p. 5, the first unnumbered equation

"Rozxz = 0" should read "$R_{ozxz} = 0$"

p. 6, the second line after the first unnumbered equation

"This inertial mass also acts as an active gravitational mass in producing gravitational radiation in the reciprocal process."

should read

"This inertial mass is an extremely rigid mass due to the rapidity with which helium atoms are exchanged.[16] Its sound speed is equal to the speed of light. It should therefore be an efficient antenna for gravitational radiation."

p. 6, the third line of the third paragraph, reference 16 should read 17.

p. 10, reference 16 should read

"16. R.Y. Chiao (submitted for publication)" and the old reference "16" becomes reference "17".

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GRAVITATIONAL RADIATION ANTENNAS USING THE SAGNAC EFFECT

J. Anandan*
Department of Mathematics
University of California
Berkeley, California 94720

and

R.Y. Chiao**
Department of Physics
University of California
Berkeley, California 94720

Summary

A new class of gravitational antennas that utilize the general relativistic Sagnac effect is proposed. These antennas are more efficient than the Weber bar by a factor of $(c/v_s)^4 \sim 10^{19}$, where v_s is the velocity of sound in the bar. A specific case of such an antenna consisting of a superfluid helium Josephson interferometer is considered. A general relativistic theory of the interaction of the superfluid with the gravitational field is given. Using this theory, the phase shift due to a gravitational plane wave on one such antenna is obtained. More generally, the proposed interferometer involves the interplay of general relativity and quantum theory and affords the possibility of testing general relativity in the laboratory at the quantum mechanical level. The possibility of detecting gravitons, assuming nearly unit coupling efficiency for the antenna, is explored.

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In this essay we propose a novel method of detecting and generating gravitational waves which uses the general relativistic Sagnac effect,[1-6] instead of the tidal forces due to the gravitational waves. These antennas are fast in the sense that their time of response is of the order of only the time ℓ/c taken by the gravitational wave to pass over them, where ℓ is the typical linear dimension of the antenna and c is the velocity of light. Hence for a wave of period T, $\ell \leq \frac{cT}{2} = \frac{\lambda}{2}$. This is in contrast with the Weber bar where the length of the bar is limited to $\ell \leq v_s \frac{T}{2}$, v_s being the velocity sound in the bar. Since the efficiency of quadrupole antenna is proportional to ℓ^4, the present type of detectors are more efficient than the Weber bar by the factor

$$\left(\frac{c}{v_s}\right)^4 \sim 10^{19} .$$

Such antennas can be classical or quantum mechanical. The quantum mechanical antennas, however, rely on interference, with the phase shift being proportional to the ratio of the size of the antenna ℓ to the Compton wavelength of the interfering particle, which makes it much larger than the corresponding classical effect. For the special case of an antenna consisting of superfluid in a tube wound in a closed curve with a Josephson junction, which we shall consider, there is an additional advantage due to the fact that since the superfluid has zero entropy and viscosity, once the gravitational energy is transferred to the fluid there is very little dissipation so that the process is almost reversible. Such a Josephson interferometer, more generally speaking, is exciting from an experimental point of view because it raises the possibility of testing general relativity in the laboratory at the quantum mechanical level for the first time and from a theoretical point of view because it involves an interplay of general relativity and quantum theory.

The essence of the relativistic Sagnac effect is illustrated by the following gedanken experiment. Consider a rigid disc with observers seated on its circumference, facing outwards, and separated by infinitesimal distances. All the observers, with the exception of a single observer 0, have agreed that each will always synchronize his clock to agree with the clock of the observer immediately to his left, while 0's clock is allowed to run normally. When the disc is at rest relative to a local inertial frame, there is no difference in the times of 0's clock and that of his neighbor on his left. However, if the disc begins to rotate then a time difference Δt between these two observers begins to build up.[7] If the rotation is uniform, then it is clear that if a wave is split into two waves at 0, which are sent around the disc in opposite directions, then the phase difference between them at their arrival at 0 is $\Delta\phi=2\omega\Delta t$ where ω is the common frequency of the beams at 0, relative to 0.

But clearly any device that enables synchronization of clocks around a rotating frame would enable its angular velocity to be measured very rapidly even when the rotation is non-uniform. Such a device is the phase of the wave function of a massive particle. In the rest frame of the particle, the surfaces of constant phase have constant time, and the time and phase are related by the Einstein-Planck law[8]

$$mc^2 = h\nu \qquad (1)$$

Superfluid helium, for instance, which is described by a macroscopic wave function, can therefore be used to test the Sagnac effect. This can be done by placing superfluid helium in a toroidal tube with a Josephson junction, i.e. a weak link at which ψ vanishes and consequently the phase is not defined. If the tube is now rotated together with superfluid then there would be a phase difference $\Delta\phi$ across the Josephson junction. It can then be shown that, analogous to the Josephson current in superconductors,[9] there must be a mass current

$$I = I_0 \sin \Delta\phi \tag{2}$$

across the Josephson junction. The phase difference can now be determined by measuring the recoil due to this current on the Josephson junction.[10]

The phase shift $\Delta\phi$ can also develop as a result of the application of a gravitational field such as the Lense-Thirring field or a gravitational wave. We shall compute this by using a general relativistic theory of interaction of the superfluid with the gravitational field. At present, superfluid described by an order parameter ψ or effective wave function which is assumed to satisfy the phenomenological non-relativistic Gross-Pitaevskii equation.[11] A general relativistic generalization of this equation is

$$\Box\psi + \frac{m^2c^2}{\hbar^2} = - \frac{2mg}{\hbar^2} |\psi|^2 \psi \tag{3}$$

where $\Box = g^{\mu\nu} \nabla_\mu \nabla_\nu$, ∇_μ being the covariant derivative $g^{\mu\nu}$ the inverse of the usual pseudo-Riemannian metric on space-time, $2\pi\hbar$ is Planck's constant, m is the mass of helium atom and g is a constant. The last term in (3) incorporates the many body effects. Writing $\psi = \alpha e^{i\phi}$, α and ϕ are real, and $v_\mu \equiv \frac{\hbar}{mc} \partial_\mu \phi$ in the interior of the fluid, the real and imaginary parts of (3) are

$$v^\mu v_\mu = 1 + f(\alpha) \tag{4}$$

where $f(\alpha) = \lambda_c^2 \frac{\Box\alpha}{\alpha} + \frac{\lambda_c^2 \alpha^2}{\xi^2 \alpha_0^2}$ ($\lambda_c = \frac{\hbar}{mc} \sim 10^{-16}$m is the Compton wavelength and

$\xi = \left(\frac{\hbar^2}{2m\alpha_0^2 g}\right)^{1/2} \sim 10^{-10}$m is the coherence length) and

$$\nabla_\mu (\alpha^2 v^\mu) = 0 .$$

Hence in the W.K.B. approximation $f(\alpha) << 1$.

Consider now, a tube filled with superfluid helium, wound in a closed curve, and interrupted by a Josephson. The phase difference across the Josephson junction is

$$\Delta\phi = \frac{mc}{\hbar} \oint_\gamma v_\mu \, dx^\mu \tag{5}$$

where γ is a closed curve through the tube beginning and ending at the Josephson junction. Since γ is in a multiply connected region, and $\psi=0$ at the junction, (5) need not be zero nor quantized. We shall assume that as a result of the interaction between the apparatus and the superfluid, the superfluid is dragged so that $v^\mu = \Lambda\, t^\mu$ along a two-dimensional submanifold σ formed by propagating a closed curve γ, going around the tube, along the integral curves of v^μ, where t^μ is the four-velocity field of the apparatus ($t^\mu t_\mu = 1$). From (4), $\Lambda=(1+f(\alpha))^{\frac{1}{2}}$. Now choose a Fermi-normal coordinate system around the world-line of the center of mass of the apparatus such that $t^\mu = g_{00}^{-\frac{1}{2}} \delta_0^{\ \mu}$, i.e. the apparatus is quasi-rigid, with the particles constituting the apparatus being at constant distances and angles from the center of mass. We neglect the deviations arising from tidal forces which are measured in the Weber bar, because they are very slow compared with the Sagnac effect. In this coordinate system, the contribution to $\Delta\phi$ from the gravitational perturbation, can be shown, using (4) and (5) to be[12]

$$\Delta\phi = \frac{2mc}{3\hbar} \oint_\gamma R_{o\ell ik}\, x^\ell x^k \, dk^i + O(h^2) + O(f^2) + O(hf) + O(x^3) \tag{6}$$

where $h_{\mu\nu} = g_{\mu\nu} - {}_{\mu\nu}$, ${}_{\mu\nu}$ being the Minkowski metric, R is the curvature tensor and x^ℓ the spatial coordinates.[13]

Consider now as the apparatus, the gravitational antenna, which consists of a tube containing superfluid helium, wound around N times in the shape shown in Figure 1,[14] so chosen because of the quadrupole nature of the radiation. Suppose that a plane gravitational wave, with its plane of polarization in the x-y plane and the wave vector in the z-direction, is incident on the antenna in the x-z plane as shown. The independent curvature components of this wave near the center of mass in the chosen Fermi-normal coordinate system, that are needed to compute (7) for this antenna are

$$R_{oxxz} = \frac{\omega^2}{2c^2} A_+ \cos \omega(t - \frac{z'}{c}), \; R \qquad = 0.$$

It should be noted that the "magnetic" component R_{oxxz} is as strong as the "electric" components R_{oioj} measured by tidal forces such as in the Weber bar. If a, $b \ll \frac{c}{\omega}$ (see Figure 1) then (6) yields

$$\Delta\phi_G = \frac{2\, m\omega^2 a^2 bN}{\hbar c} A_+ \cos\omega t \qquad (7)$$

Expression (7) is the prototype for a whole class of antennas, both classical and quantum mechanical, which have the general shape shown in Fig. 1.[15]

The Josephson current (2) in the antenna can be detected by the recoil it produces at the Josephson junction which can be picked up by means of an electromechanical transducer. For weak fields,

$$I(t) = I_0 \Delta\phi(t) \qquad (8)$$

The linearity of (8) implies that the process of absorbing gravitational radiation by our device can be reversed to produce gravitational radiation with the same efficiency, which is close to unity, based on an analogy with electromagnetic antennas. Due to the coupling between the Josephson junction and the metric, described by Eq. (8), the junction acquires a nonclassical coupling mass

$$M_c = I_0 L^2 m/\hbar$$

in the limit $L \ll \lambda$, where L is the length of the closed curve γ, i.e. that of the tubing. This inertial mass is an extremely rigid mass due to the rapidity with which helium atoms are exchanged.[16] Its sound speed is equal to the speed of light. It should therefore be an efficient antenna for gravitational radiation. Communication by gravitational radiation should therefore be possible. If gravitational waves can be produced with the appreciable intensity, a very important experiment would be to observe the effects of interfering two gravitational waves.

An antenna for 10 MHz described in Figure 1, can be constructed with two sets of 3000 turns of tubing of opposite senses on two 100 cm diameter drums placed side by side. The upper frequency limit for this interferometer is that for the Josephson effect, or approximately 2×10^{11} Hz.

The linear dependence of the phase shift of the interferometer on $A_+ \cos \omega t$, which is a measurement of amplitude and phase of the gravitational plane wave given by Equation (7), has some interesting implications.[17] When the number of gravitons is very large, the semi-classical approximation used in deriving these equations is appropriate. However, there exists an uncertainty principle relating the fluctuations in graviton number δN and in the phase $\delta\Phi$ of a plane wave given by

$$\delta N \ \ \delta\Phi \geq 1. \qquad (9)$$

Now a measurement of the Sagnac phase shift constitutes a measurement simultaneously of the amplitude and the phase of the gravitational wave. Therefore, fluctuations will be induced in the gravitational wave, which are intimately connected to fluctuations in the motion of the Josephson junction, assuming unit coupling efficiency between the two, which is a good approximation in the present case. These fluctuations are due to spontaneous emission of gravitons from the apparatus. Experimental observation of an increase of noise over that of the zero-point motion of a Josephson junction due to the presence of another nearby spontaneously emitting antenna would constitute detection of the graviton. Radio frequency amplifiers, such as masers whose noise is limited to zero-point fluctuations when coupled with nearly unit efficiency to the antenna, should allow such an observation.

Acknowledgements

It is a pleasure to thank R.K. Sachs for his help and guidance during the course of this work. One of us (J. A.) wishes to thank G.A.J. Sparling for stimulating his interest in graviton detection using superfluid helium.

Footnotes

1. G. Papini, Phys. Lett., 24A 32 (1967).
2. A. Ashtekar and A. Magnon, J. Math. Phys., 16, 342 (1975).
3. B. Linet and P. Tourrenc, Can. J. Phys., 54, 1129 (1976).
4. J. Anandan, Phys. Rev. D, 15, 1448 (1977).
5. L. Stodolsky, in Proc. of the International Conf. on Neutron Spectroscopy, eds. U. Bonse and H. Rauch, (Oxford University Press, New York (1980)).
6. J. Anandan, to be published in Phys. Rev. D, July 15 (1981).
7. $\Delta t \simeq \frac{2\Omega A}{c^2}$ where Ω is the instantaneous angular velocity, and A the area of the disc provided $\frac{d\Omega}{dt} << \frac{c^2}{A}$. For gravitational wave detection, $\frac{d\Omega}{dt}$ is proportional to $\frac{dh_{oi}}{dt}$ where h_{oi} is a small perturbation of the metric (in the Fermi-normal coordinate system mentioned in the text) and this condition is satisfied. This makes a gravitational antenna, based on the Sagnac effect, much faster than any previously proposed antenna.
8. It has been pointed out that the existence of accurate clocks is ultimately due to the relation (1) (R. Penrose, in Batelle Recontres, eds. C.M. DeWitt

and J.A. Wheeler; Benjamin, New York (1968); see also J. Anandan, Foundations of Physics, 10, 601 (1980). Hence, the measurement of Δt by a "classical" clock can only be less accurate than the method being proposed here, which uses the phase of the wave function much more directly.

9. See for instance, R.P. Feynman, The Feynman Lectures in Physics (Addison-Wesley, 1965), III, 21-9. The Josephson current in superconductors due to the Sagnac effect has been observed by J.E. Zimmermann and J.E. Mercereau (Phys. Rev. Lett., 14, 887 (1965)).

10. Since $\Delta\phi = \Delta t\ mc^2/\hbar$ by (1), the observation of $\Delta\phi$ through the Josephson effect constitutes an operational procedure for an accurate measurement[8] of the time interval Δt mentioned earlier. No violation of the time-energy uncertainty principle is involved, since the Josephson current arises form the superposition of different energy eigenstates.[9]

11. E.P. Gross, Nuovo Cimento, 20, 454 (1961); L.P. Pitaevskii, Sov. Phys.-JETP, 13, 451 (1961).

12. See C.W. Misner, K.S. Thorne and J.A. Wheeler, Gravitation (Freeman 1973) (13.73), p. 332 for the expression for h_{oi} in a Fermi-normal coordinate system used here. It should be noted that we are not using the transverse traceless gauge usually used.

13. A non zero contribution to (6) would be due to the Lense-Thirring field of the earth. In principle this can be detected by a Josephson superfluid interferometer in the form of a "figure eight" so that the Sagnac effect of the earth's field is canceled. If the two loops are separated by a height H then the phase shift due to the Lense-Thirring field is $\frac{4}{5}\frac{GM}{c^2R^2}\frac{mH\Omega A}{\hbar}$ where M, R and Ω are the mass, radius and the component of the angular velocity of the earth normal to the area A of each loop.

14. A gravitational antenna in the form of "figure 8" was proposed by W.H. Press.

(W.H. Press and K.S. Thorne, Ann. Rev. Astron. Astrophys. 10, 335 (1972)). However, he considers only the classical circulation of the fluid due to the "magnetic" component of the tidal force as a means of detecting gravitational waves. This is less efficient than the Weber bar by the factor $\left(\frac{v_{sound}}{c}\right)$ unlike the present detectors which use the Sagnac effect and consequently are much more efficient than the Weber bar.

15. For instance, $\frac{\hbar}{mc^2} \Delta\phi_G$ is the time delay in synchronization of clocks around the antenna, which is very difficult to measure classically. Alternatively two laser beams may be sent around the antenna by means of mirrors or an optical fiber. In this case m in (7) must be replaced by $\hbar \frac{\omega}{c^2}$ where ω is the frequency of light relative to the apparatus at the point of interference. For optical frequencies $\frac{\hbar\omega}{mc^2} \sim 10^{-9}$, so this device would be much less sensitive than the superfluid interferometer. For neutron waves, we do not have the advantage with superfluid helium that the tube could be wound around a large number of times. Also, because neutrons obey Pauli exclusion principle, the intensity of the neutron beam must remain low whereas a large concentration of superfluid helium atoms can be obtained. As for the superfluid in a superconductor, it is difficult to isolate the gravitational effects from the electromagnetic effects which are much larger. Hence neutral superfluid helium seems to be the best possible means of detecting gravitational waves.

The possibility of detecting gravitational waves by neutron interference was suggested in references 3 and 5. However, they considered only the contribution to the phase shift given by (3·2·3) of reference 3 which is v/c times smaller than the Sagnac phase shift, where v is the speed of neutrons, as a means of detecting gravitational waves. Also this is not as fast as the present antenna. The Sagnac effect, for neutrons has been

detected by Werner et al., Phys. Rev. Lett., 42, 1103 (1979).

16. R.Y. Chiao (submitted for publication).

17. R. Serber and C.H. Townes in Quantum Electronics, ed. C.H. Townes (Columbia University Press (1960)), p. 223.

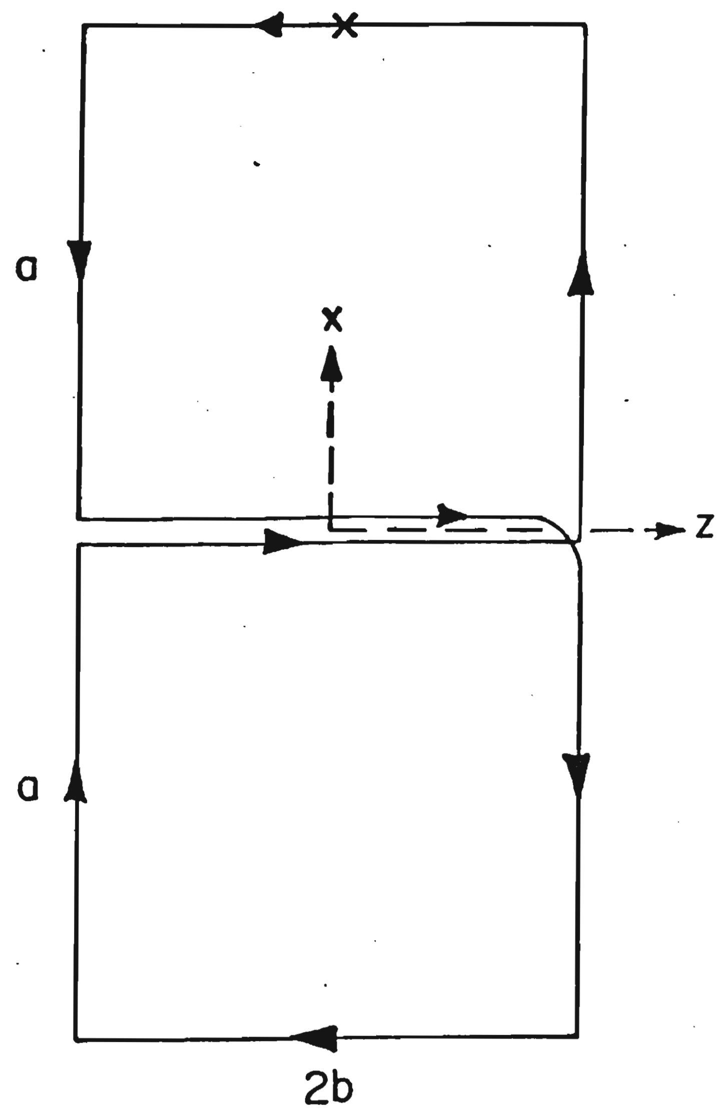

Figure 1

A schematic representation of the "figure-eight" superfluid gravitational antenna. The arrows indicate the direction in which vorticity is set up in the apparatus, which is then transferred to the superfluid when a plane polarized gravitational wave, with its wavevector in the z-direction passes the apparatus. x denotes the Josephson junction. When 2b=half wave-length, the antenna is ideally tuned.