# PREGEOMETRIC ORIGIN OF THE BIG BANG

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ABSTRACT

The temperature-dependent effective action for gravity is calculated in pregeometry. It indicates that the effective potential for the space-time metric has the minimum at the origin for extremely high temperature. The origin of the big bang can be taken as a local and spontaneous phase transition of the space-time from the pregeometric phase to the geometric one.

A fundamental theory from which Einstein's gravitation is derived is called "pregeometry". In 1967, Sakharov [1] proposed to take the Einstein action as quantum fluctuations of matters. Later, many authors [2] proposed various composite models of the graviton. None of them, however, could induce the Einstein action without any approximation. Therefore, they are not really models of pregeometry. About three years ago, we [3] presented a field-theoretical formulation of pregeometry where the metric (and, therefore, the graviton) appears as a composite of fundamental matters, inducing the Einstein action. More recently, Adler [4] has proposed a new programme of pregeometry that a possible dynamical breakdown of scale invariance is expected to induce a finite Einstein action in pregeometry.

In order to see whether pregeometry is appreciated by Nature, one must find physical phenomena which can occur in pregeometry but not in the usual Einstein geometrical picture of gravitation. In this paper, we shall discuss such astonishing phenomena as phase transitions of the space-time between the "geometric phase" and the "pregeometric one". To this end, we shall first calculate the temperature-dependent effective action for gravity in pregeometry. Our result will indicate that the effective potential for the space-time metric has the minimum at the origin for extremely high temperature. We shall then propose to take the origin of the big bang of our Universe as a local and spontaneous phase transition from the pregeometric phase to the geometric one in the over-cooled space-time manifold. We shall further suggest that even in our present Universe there may exist "pregeometric holes" where the space-time metric absolutely vanishes and/or "space-time discontinuities" where the metric discretely changes. Before discussing such temperature-dependent effects on gravity in pregeometry, let us briefly review for later convenience what pregeometry means in the limit of zero temperature [3].

For simplicity, we first assume that the fundamental matters are N scalar particles $\phi^i$ ($i = 1 - N$). A fundamental Lagrangian for the matter fields in "scalar pregeometry" is given by

$$\mathcal{L} = \sqrt{-\det(\partial_\mu\phi^i\partial_\nu\phi^i)}\ F \quad , \tag{1}$$

where F is an arbitrary constant. This is equivalent to another Lagrangian for the matter fields $\phi^i$ with the space-time metric $g^{\mu\nu}$ as an auxiliary field

$$\mathcal{L}' = \sqrt{-g}\,(g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^i - F^{-1}) \quad , \tag{2}$$

where $g = \det g_{\mu\nu}$ , since $\mathcal{L}'$ leads to the following "constraint equation of motion" for the metric $g_{\mu\nu}$ :

$$g_{\mu\nu} = \partial_\mu\phi^i\partial_\nu\phi^i\ F \quad . \tag{3}$$

The effective action for the metric $g^{\mu\nu}$ due to quantum fluctuations of the matter fields is given by

$$S^{eff} = -i\ \ell n\left[\int \prod_i (d\phi^i)\ \exp\left(i\int d^4x\ \mathcal{L}'\right)\right] \quad . \tag{4}$$

The path integration over $\phi^i$ can be formally performed to yield

$$S^{eff} = i\ \mathrm{Tr}\ \ell n\left(\partial_\mu\sqrt{-g}\ g^{\mu\nu}\partial_\nu\right) - \int d^4x\ \sqrt{-g}\ F^{-1} \quad . \tag{5}$$

As was first demonstrated in Ref.3, for a small scalar curvature R this action can be calculated to be

$$S^{eff} = \int d^4x\ \sqrt{-g}\ \left[\lambda + \frac{1}{16\pi G}R + c(R^2 + d\ R^{\mu\nu}R_{\mu\nu}) + \cdots\right] \tag{6}$$

with

$$\lambda = \frac{N\Lambda^4}{8(4\pi)^2} - F^{-1} \quad , \tag{7}$$

$$\frac{1}{16\pi G} = \frac{N\Lambda^2}{24(4\pi)^2} , \tag{8}$$

$$c = \frac{N \ln \Lambda^2}{240(4\pi)^2} , \quad \text{and} \quad d = 2 , \tag{9}$$

where $\lambda$ , $G$ and $\Lambda$ are the cosmological constant, the Newtonian gravitational constant, and the momentum cut-off of the Pauli-Villars type, respectively. Because of the presence of the arbitrary constant $F^{-1}$ in Eq.(7), we can make the cosmological constant as small as Nature wants. Notice also that Eq.(8) indicates that the induced gravity in scalar pregeometry is the attractive one and that the momentum cut-off must be of order of the Planck mass ($G^{-1/2} \sim 10^{19}$ GeV). Furthermore, Eqs.(8) and (9) indicate that the third term on the right-hand side of Eq.(6) is practically negligible. Also, the remaining term which is of order $R^2$ is finite even for $\Lambda \to \infty$ and completely negligible.

For the cases where the fundamental matters are spinor and gauge fields, similar formulations and calculations are possible. We only present our results in Table I. See the details in Refs.3 and 5. Notice that "gauge field pregeometry", the pregeometry with gauge fields as the fundamental matters leads to antigravity.

One of the most remarkable consequences of the pregeometry combined with a pre-gauge theory [6),7)], a composite model of gauge fields which induces a gauge theory, is the $G$-$\alpha$ relation [8),9)], a simple relation between the fine structure constant $\alpha$ and the Newtonian gravitational constant $G$. The $G$-$\alpha$ relation in a "spinor pregauge-pregeometry", for example, is given by

$$\alpha = 3\pi \Big/ \sum_i Q_i^2 \ln(4\pi/N\kappa G m_i^2) , \tag{10}$$

where $Q_i$ and $m_i$ are, respectively, the charge and the mass of the fundamental fermions which are either leptons and quarks or subquarks (or preons)[10], and $\kappa$ is a calculable constant which is of order unity and depends on the type of momentum cut-off.

Now we are ready to discuss the temperature-dependent effects on gravity in pregeometry. Our general expectation is the following: under certain physical conditions, the space-time metric which is a composite of the fundamental matters would dissociate into its constituent matters, just as ordinary objects do. Then, the metric would disappear or vanish although the fundamental matters still remain in a manifold of the space-time. We call it the "pregeometric phase" of the space-time. Namely, the pregeometric phase is the phase of the space-time in which the metric and, therefore, the distance disappear. There, the space-time still exists as a mathematical manifold for the presence of the fundamental matters. Such an extraordinary phase may be realized in such regions as that beyond the space-time singularity, i.e. before the big bang and that far inside a black hole where the temperature is extremely high (as high as the Planck mass). It may also be realized in regions where the matter density is extremely high. It should be noticed, however, that the effective Einstein action given in Eq.(6) would no longer be valid in pregeometry either where the curvature is large or where the temperature (or the density) is high. There, the gravitation is controlled, instead, by the non-local effective action given in Eq.(5).

In order to calculate the temperature-dependent effective action for gravity in pregeometry, we apply the technique developed by Dolan-Jackiw, Weinberg and Kirzhnits-Linde[11] to our effective action in (5). For simplicity, we restrict ourselves to the specific case where the metric is parametrized with two parameters, $b$ and $\xi$, as

$$g^{\mu\nu} = b^2 \,\mathrm{diag}(1, -\xi^2, -\xi^2, -\xi^2) \quad . \qquad (11)$$

After a simple calculation, we finally obtain the following effective potential $V^{eff}$ for the metric $g^{\mu\nu}$ at high temperature:

$$V^{eff} \cong \frac{\xi^3 T}{2\pi^2} \int_0^\infty d\zeta\, \zeta^2\, \ln\left[1 - \exp\left(-\sqrt{\zeta^2 + m^2 b^2}/T\right)\right] , \qquad (12)$$

where $T$ and $m$ are the temperature and mass of the fundamental matters, respectively. Notice that the temperature-dependent part of $V^{eff}$ given in Eq.(12) is free from any divergence, thanks to the presence of $T$ which plays a role of momentum cut-off both in the infra-red and ultraviolet regions. The behaviour of $V^{eff}$ as a function of $b$ for a fixed $\xi$ is illustrated in Fig.1(a). It looks like a deep well with the minimum at the origin where $b = 0$. The bottom of the well where

$$V^{eff}\big|_{b=0} \cong -\pi^2 \xi^3 T^4/90 \qquad (13)$$

gets deeper and deeper as $T^4$ as the temperature increases. The pregeometric phase would then be realized at the bottom of this well where $b = 0$ or $g^{\mu\nu} = 0$. This might indicate that the only stable phase at finite temperature in pregeometry would be the pregeometric one. This expectation seems, however, too naive since no other interactions among the fundamental matters than the pregeometric one have not yet been taken into account.

In order to obtain a more realistic effective potential for the metric, we phenomenologically assume that due to the other interactions among the fundamental matters there exist some effects that prevent the space-time metric from vanishing at least at low temperature. These effects may add to the effective potential (12) such a contribution as is illustrated in Fig.1(b). Furthermore, we assume that these effects of the other interactions on gravity are less dependent of the temperature than those of the pregeometric ones. A typical behaviour of the sum of these different contributions is illustrated in Fig.1(c). In this figure, one can then naturally find that although only the pregeometric phase is stable at very high temperature, the geometric phase where the metric is finite and non-vanishing will turn out to be stable as the temperature goes down. This remarkable possibility of phase transitions of the space-time metric between the geometric and pregometric phases will exhibit a characteristic feature of pregeometry, if it is found.

Where can we find such phase transitions of the space-time? It seems very attractive to interpret the origin of the big bang of our Universe as such a local and spontaneous phase transition of the space-time from the pre-geometric phase to the geometric one in the over-cooled space-time manifold which had been present in the "pre-big-bang" era for some reason. The enormous energy of the big bang can be taken as the latent heat liberated by the phase transition of the space-time.

In conclusion, we would also like to suggest a more astonishing possibility that even in our present Universe there may exist "pregeometric holes", the local spots in the pregeometric phase with an extremely high temperature where the space-time metric disappears, liberating enormous latent heat, and/or "space-time discontinuities", the local plains where the metric discretely changes due to the phase transitions of the space-time. How to observe these exotic and yet physical objects in future astronomical experiments are subjects for future investigations.

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## TABLE CAPTION

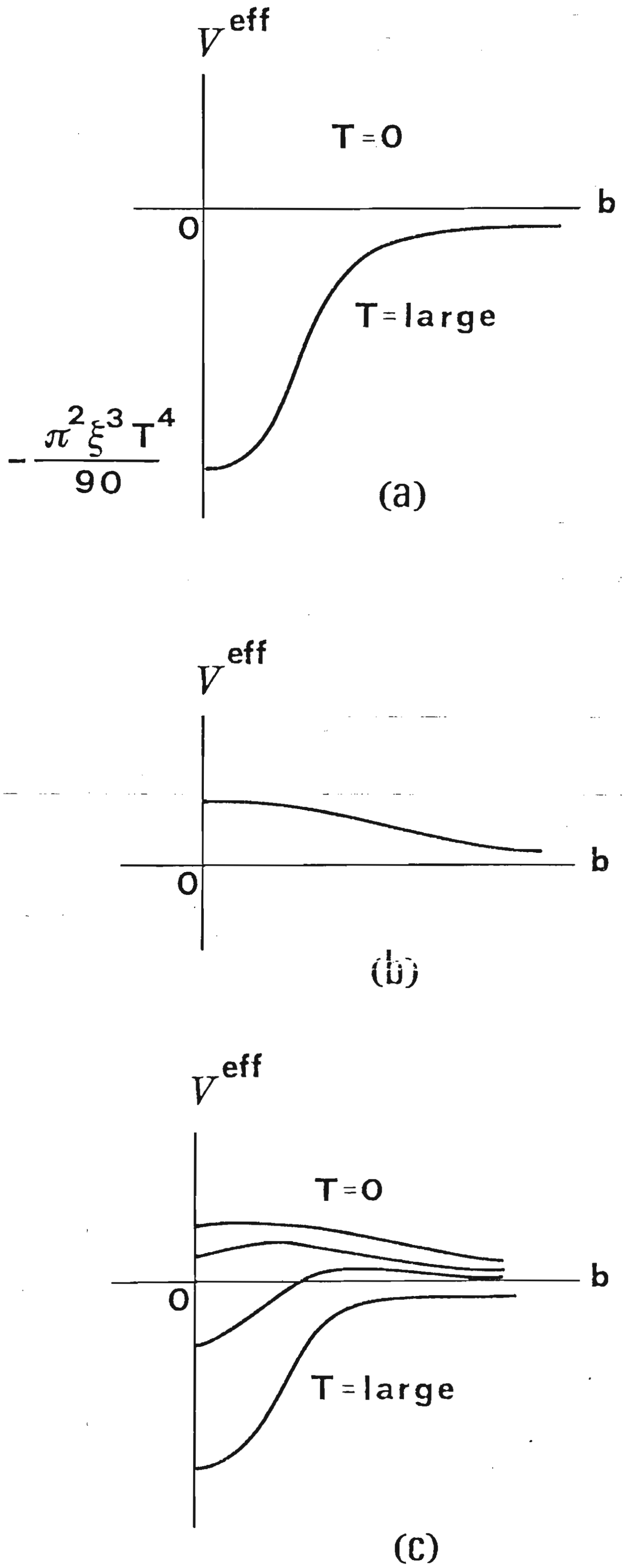
Table I — The calculated coefficients in the effective action for gravity defined in Eq.(6) for various types of the matter fields and for different types of the momentum cut-off.

## FIGURE CAPTION

Fig.1 — The effective potentials $V^{eff}$ for the space-time metric as a function of the one parameter $b$ for the other $\xi$ fixed: (a) $V^{eff}$ given in Eq.(12) in the absence of the other interactions than the pregeometric one, (b) the assumed contribution of the other interactions to $V^{eff}$, and (c) the sum of (a) and (b) for various temperatures $T$.

Table I

| Type of cut-off (Reference) | Type of matter field | $\lambda$ | $1/16\pi G$ | c | d |
|---|---|---|---|---|---|
| Pauli-Villars cut-off (Ref.3) | scalar | $\frac{\Lambda^4}{128\pi^2} - F^{-1}$ | $\frac{\Lambda^2}{384\pi^2}$ | $\frac{\ln\Lambda^2}{3840\pi^2}$ | 2 |
| | spinor | $-\frac{\Lambda^4}{32\pi^2} - 3F^{-1}/3$ | $\frac{\Lambda^2}{192\pi^2}$ | $-\frac{\ln\Lambda^2}{960\pi^2}$ | -3 |
| Scale cut-off (Ref.5) | scalar | $\frac{\Lambda^4}{4} - F^{-1}$ | $\frac{\Lambda^2}{48\pi}$ | $\frac{\ln\Lambda^2}{3840\pi^2}$ | 2 |
| | spinor | $-\frac{\Lambda^4}{4} - 3F^{-1}/3$ | $\frac{\Lambda^2}{48\pi}$ | $-\frac{\ln\Lambda^2}{960\pi^2}$ | -3 |
| | gauge field | $-\frac{\Lambda^4}{2}$ | $-\frac{\Lambda^2}{8\pi}$ | $-\frac{\ln\Lambda^2}{480\pi^2}$ | -3 |

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