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Dr. George M. Rideout, Jr.
President
Gravity Research Foundation
P.O. Box 81389-0004
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Dear Dr. Rideout,

We wish to submit our essay, "New Approaches for Inflationary Cosmology," to the Gravitational Essay Competition sponsored by the Gravitational Research Foundation. In addition to reviewing some exciting, new developments in inflationary cosmology, the essay discusses for the first time in print an important effect that inflation can have in altering the nature of gravity. We hope that this aspect makes the essay especially enjoyable to the Foundation.

Please send all correspondence to Paul Steinhardt at the address above.

Sincerely,

Frank S. Accetta

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NEW APPROACHES FOR INFLATIONARY COSMOLOGY

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Abstract: New approaches for inflationary cosmology have been developed which avoid the extreme fine-tuning required by all previous models and which generate a new source of inhomogeneities that could influence the large-scale structure of the universe. The most surprising feature of the new approaches is the role that inflation can play in altering the nature of the gravitational force.

Inflationary cosmology[1], now more than a decade old, has reached a peculiar status. On the one hand, the inflationary scenario is appealing because it offers elegant solutions to longstanding cosmological problems. On the other hand, implementing the scenario in accordance with astrophysical constraints has required an extreme and unnatural fine-tuning of parameters. Recently, new approaches for inflationary cosmology — *extended*[2] and *hyperextended*[3] inflation — have been developed which may remove the fine-tuning problem. The new approaches also suggest new effects that inflation can have on gravitation and the large-scale structure of the universe.

The goal of every inflationary model is to generate a brief period in which the scale factor of the universe, $R(t)$, increases superluminally, $R(t) > t$. If $R(t)$ grows by e^{60} or more during this period, the cosmological horizon, flatness and monopole problems can be resolved. In addition, inflation generates density fluctuations which may be seeds for galaxy formation.

The simplest mechanism for inducing superluminal expansion is a strongly first-order phase transition in which the universe supercools into a metastable, "false" vacuum phase. The false vacuum phase is prevented from evolving to the stable true vacuum phase by an energy barrier. While the universe is trapped in the false vacuum, the energy density ρ_F is nearly constant. In standard general relativity, constant energy density means that the Hubble parameter, $H \equiv \dot{R}/R = (8\pi G\rho_F/3)^{1/2}$ is time independent and $R(t)$ grows as e^{Ht}. The desired expansion occurs if the universe is trapped in the false vacuum for more than 60 Hubble times (60 H^{-1}).

As shown in the original, "old" inflationary model[4], sufficient superluminal expansion is easily achieved in typical phase transitions, but it is impossible to end the transition afterwards. Regions of the universe might tunnel through the energy barrier to form expanding bubbles of true vacuum. However, bubbles can never coalesce to complete the transition because the false vacuum separating them continues to expand superluminally[5]. The "new" inflation model[6] (quite distinct from recent extended models) was designed to avoid this failure: a special phase transition is assumed in which the energy barrier disappears during supercooling so that the entire universe can evolve continuously from the false to the true vacuum phase. However, the special transition requires extreme fine-tuning of parameters, especially to ensure an acceptable distribution of density fluctuations after the transition[7]. Subsequent approaches, such as "chaotic" inflation or "power-law" inflation rely on similar principles and encounter the same fine-tuning problem[1].

To understand the "extended" approaches, the failure of old inflation bears closer scrutiny[5]. During inflation, nucleation of true vacuum bubbles causes the fractional volume of false vacuum at time t, $p_F(t)$, to decrease as $e^{-\epsilon H t}$, where $\epsilon \equiv \lambda / H^4$ is the number of bubbles nucleated per Hubble four-volume. Here, λ, the bubble nucleation rate, is a constant which depends on the shape of the energy barrier. Both λ and H are time-independent during old inflation, and, hence, ϵ is also. The physical volume of false vacuum is $\mathcal{V}_F = p_F(t) R^3(t)$, where $R(t) = e^{Ht}$, as explained above. If $\epsilon < 3$, $\mathcal{V}_F$ increases forever and the transition never ends. If $\epsilon > 3$, the false vacuum disappears before $R(t)$ grows by e^{60}. Either outcome is disastrous.

The novel feature of "extended" approaches is that ϵ is time-dependent, beginning small enough to ensure sufficient inflation, but then growing to a large enough value to end the phase transition. In particular, if the gravitational constant, G, decreases monotonically during inflation, H also decreases and $\epsilon \propto 1/H^4$ increases. At first, the notion of time-varying G may seem radical and unattractive. However, the desired effect can be implemented by a simple modification of Einstein gravity in which a field is non-minimally coupled to the scalar curvature, $\mathcal{R}$. The non-minimally coupled field, ϕ, is completely distinct from the fields which govern the phase transition. Non-minimally coupled fields are quite natural, appearing in virtually every known unified theory that couples particles to gravity, including supersymmetry and superstring theories.

A surprising and important observation is that inflation can profoundly influence gravity by magnifying the effects of non-minimally coupled fields. Non-minimal couplings take the general form $f(\phi)\mathcal{R}$, where $f(\phi) \approx M_0^2 + \xi\phi^2 + \xi'\phi^4/M_0^2 + \ldots$, for $\phi \ll M_0$. By definition, G equals $1/f(\phi)$, which is constant in conventional (minimal) Einstein gravity. Typically, the coupling constants (ξ, ξ', etc.) and the initial value of ϕ are small, and so it is presumed that non-minimal couplings are negligible. Indeed, in standard big bang cosmology, $f(\phi)$ would remain nearly constant up to the present epoch and the modifications to Einstein gravity would remain small. However, if the universe is trapped in a false vacuum, a qualitatively different result occurs. At first, $f(\phi)$ is nearly constant and the universe begins to inflate just as in "old" inflation. At the same time, though, the false vacuum energy forces ϕ to greater values. Eventually, the non-minimal contributions grow to be non-negligible. *Inflation thereby changes the nature of gravity.* The non-minimal coupling terms cause $f(\phi)$ to increase, decreasing the effective gravitational constant and thereby H. In this way, inflation has brought about its own demise: as argued above, ϵ now begins to increase and the phase transition is completed by bubble nucleation without any need for fine-tuning.

The time-evolution of ϕ generates adiabatic density perturbations after the transition. "Adiabatic" means that the ratio of matter to radiation is spatially uniform. The perturbations arise because the value of ϕ varies spatially. The variations are caused by quantum fluctuations on subatomic scales that are stretched to wavelengths of astrophysical size during inflation. Variations in ϕ change local values of G thereby changing the local expansion rate. This results in fluctuations in the energy density after the transition.

The spectrum can be shown to be scale-invariant with an amplitude $\delta\rho/\rho = 2 \times 10^{-2} H^2/\dot{\phi}$, to be evaluated near the end of inflation[8]. In order to agree with the observed isotropy of the cosmic microwave background radiation (CMBR), $\delta\rho/\rho$ must be less than 10^{-4}. In "extended" inflation models, the constraint is satisfied if $\rho_F < (10^{17}\ \mathrm{GeV})^4$, a mild and simple condition. By contrast, in previous models, similar perturbations are produced but $\delta\rho/\rho \gtrsim 1$ unless several parameters are finely-tuned[7].

A key prediction of the "extended" approaches is additional, *non-adiabatic* density fluctuations occuring because the phase transition is completed by bubble nucleation. Initially, energy gained by converting false to true vacuum is contained in the bubble walls, only to be released into ordinary matter and radiation when the bubbles collide. Non-adiabaticity results because the intrinsic pressure of radiation separates it from the matter and forces it towards the vacuum in the bubble interior. Since all present models of galaxy formation based on the adiabatic fluctuations alone appear to be encountering difficulties, the prediction of a non-adiabatic component is a critical, new result.

A related concern is that too many big bubbles can produce inhomogeneities that destroy the isotropy of the CMBR. Since all bubbles grow at the same rate, bubbles nucleated later are smaller than those nucleated earlier. In the "extended" approaches, the number of bubbles nucleated per Hubble four-volume, ϵ, increases with time so that the resulting bubble distribution is weighted in favor of smaller bubbles. The first attempt at implementing "extended" inflation assumed a special non-minimal coupling in which $f(\phi) = M_0^2 + \xi\phi^2$. In this case, $\epsilon \propto t^4$. An acceptable bubble distribution[9],[10] results only for $\xi > .005$. This restricted range of parameters is somewhat disappointing. More significantly, $\xi > .005$ results in G varying with time today at a rate that conflicts with radar echo (time-delay) tests of Einstein gravity[11]. To avoid this conflict, a mechanism must be added to make G time-independent after inflation.

These problems are automatically resolved in an improved version, "hyperextended" inflation, which introduces a more general $f(\phi)$. In this approach, ϵ increases exponentially with time, much faster than in extended inflation. Consequently, the bubble distribution is overwhelmingly weighted in favor of the smaller bubbles nucleated near the end of inflation. For an extraordinarily wide range of parameters (including $\xi \ll .005$) and $f(\phi)$, an acceptable distribution is found whose form depends very insensitively on the parameter values. An added feature is a possible mechanism for making G time-independent after the phase transition. If $f(\phi)$ should have a maximum at some $\phi = \phi_m$ then inflation tends to drive ϕ towards ϕ_m, at which point ϕ (and, hence, G) reaches an equilibrium, constant value. Using this mechanism, complete models have been constructed which satisfy all constraints of inflationary cosmology and restore the universe to standard big bang expansion (with constant G) without any fine-tuning.

Several important lessons have been learned from extended and hyperextended inflation. The awkward failures and fine-tunings of previous inflation models are understood to be artifacts of strict, Einstein gravity. By introducing simple and natural modifications of Einstein gravity, a new source of inhomogeneities can be obtained via bubble nucleation that may influence the large-scale structure of the universe. Finally, in addition to solving the traditional cosmological puzzles, inflation can dramatically affect gravity, ultimately determining the gravitational field strength observed today.

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