Euclidean gravity and holography

Daniel Harlow[a] and Edgar Shaghoulian[b]

[a] *Center for Theoretical Physics*
Massachusetts Institute of Technology, Cambridge, MA 02139, USA

[b] *David Rittenhouse Laboratory, University of Pennsylvania*
209 S.33rd Street, Philadelphia PA, 19104, USA

`harlow@mit.edu, eshag@sas.upenn.edu`

Abstract

We discuss a recent proposal that the Euclidean gravity approach to quantum gravity is correct if and only if the theory is holographic, providing several examples and general arguments to support the conjecture. This provides a natural mechanism for the low-energy gravitational effective field theory to access a host of deep ultraviolet properties, like the Bekenstein-Hawking entropy of black holes, the unitarity of black hole evaporation, and the lack of exact global symmetries.

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Corresponding author: Edgar Shaghoulian

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Introduction

The Euclidean gravity path integral is unreasonably powerful, producing the Bekenstein-Hawking entropy $S = A/4$ of black holes [1], the Gibbons-Hawking entropy $S = A/4$ of cosmological horizons [2], the Ryu-Takayanagi formula of AdS/CFT [3], the Page curve [4] of unitary black hole evaporation [5–8], the lack of exact global symmetries in quantum gravity [9–15], and many other beautiful results. But for nearly half a century it has been a black box: it produces results believed to be correct – in some cases substantiated by microscopic details of the theory [16–22] – but there is no general understanding of its validity. The most challenging aspect of the problem is to explain how infrared data, when inputted into the machine of the Euclidean gravity path integral, produces results about the deep ultraviolet of the theory.

In this essay we give a streamlined presentation of a proposal from [23] that says the Euclidean gravity path integral is only valid in the context of holographic theories. More precisely, our conjecture is as follows: *The Euclidean path integral in a gravitational effective field theory produces correct results if and only if the theory admits a holographic UV completion, in which case the computed quantities are those of the holographic theory.*

By a holographic theory we mean one where the fundamental description is a non-gravitational theory at some asymptotic boundary [24–27]. In particular the fundamental description is not based on a local Lagrangian living in the gravitational spacetime. We will now provide evidence for this conjecture from various lines of thought. We will focus on the validity of the Bekenstein-Hawking entropy, but similar comments apply to other outputs of Euclidean gravity as well [23].

Non-holographic theories

A crucial piece of our conjecture is that the Euclidean gravity path integral is *incorrect* for theories with a non-holographic UV completion.

One set of such examples is the CGHS/RST [28,29] or Jackiw-Teitelboim [30–32] model of gravity in $1+1$ dimensions minimally coupled to any conformal field theory with central charge $c = c_L = c_R$. These two cases are similar, so we choose to focus on the latter, whose action is

$$S = \int_M d^2x\sqrt{-g}\,(\Phi_0 R + \Phi(R+2)) + 2\int_{\partial M} dt\sqrt{-\gamma}\,(\Phi_0 K + \Phi(K-1)) + S_{CFT}(\psi_i, g), \quad (1)$$

where Φ_0 is a constant, Φ is a dynamical "dilaton" field, $g_{\mu\nu}$ is a dynamical metric, R is its

Ricci scalar, ψ_i are CFT matter fields, and γ and K are the induced metric and extrinsic curvature at the asymptotically-AdS boundary ∂M.

Since this theory is renormalizable, canonical quantization leads to a well-defined but non-holographic quantum theory. With two asymptotic boundaries and no matter the resulting theory is equivalent to the quantum mechanics of a particle moving in an exponential potential [33], while with a nontrivial matter CFT it can be solved by Weyl transformation to flat space [6,7,34]. This theory has black hole solutions, but the entropy of these black holes does *not* obey the Bekenstein-Hawking formula: the renormalizable bulk theory can have an arbitrarily large number of low-energy excitations near the black hole horizon, conflicting with the Euclidean gravity approach.

Another example is pure Einstein gravity in 2+1 dimensions with a negative cosmological constant:

$$S = \frac{1}{16\pi G}\int_M d^3x\sqrt{-g}(R+2) + \frac{1}{8\pi G}\int_{\partial M} d^2x\sqrt{-\gamma}(K-1). \tag{2}$$

This theory can be canonically quantized [35–37]. In that case the number of microstates is again incompatible with the Bekenstein-Hawking formula since the quantization of the moduli space at fixed genus leads to a continuous spectrum and the sum over spatial genus is divergent [36,37].

Holographic theories

To further motivate our conjecture, we first recall that in ordinary quantum field theory on a spatial manifold Σ the Euclidean path integral representation of a thermal partition function is derived by inserting complete sets of states into a thermal trace

$$Z(\beta) = \mathrm{Tr}\left(e^{-\beta H}\right), \tag{3}$$

which leads to a path integral on the manifold $\mathbb{S}^1 \times \Sigma$. Applying this algorithm to a renormalizable gravitational field theory such as Jackiw-Teitelboim gravity coupled to conformal matter therefore only includes manifolds which are topologically of the form $\mathbb{S}^1 \times \Sigma$ for some Σ. As originally explained by Hawking, by time-translation symmetry the on-shell action of any gravitational field theory on such a manifold will be proportional to β, giving a vanishing thermal entropy

$$S(\beta) = (1-\beta\partial_\beta)\log Z = 0 \tag{4}$$

at leading order in the gravitational constant. Therefore no time-translation-preserving saddle-point approximation to a Euclidean path integral derived from canonical quantization

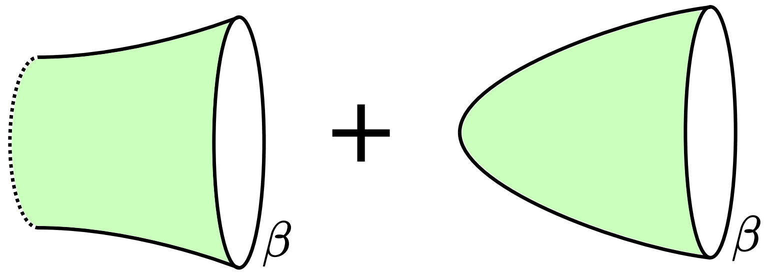

Figure 1: Two contributions to the Euclidean gravity path integral with a boundary thermal circle of circumference β. On the left some cycle of the transverse directions contracts at the dotted line, while on the right it is the thermal circle which contracts. The contribution on the left is what is obtained from canonical quantization of gravitational effective field theory, and gives no contribution to the entropy at leading order in the gravitational coupling. The contribution on the right leads to the black hole entropy; it should be included only in effective theories which are UV-completed into a holographic description.

of a gravitational field theory can ever lead to the Bekenstein-Hawking formula. There is however a standard proposal for how to fix this: instead of only including topologies of the form $M = \mathbb{S}^1 \times \Sigma$, include all topologies M such that $\partial M = \mathbb{S}^1 \times \partial\Sigma$, where $\partial\Sigma$ is the topology of the spatial boundary [1], even though most of these topologies are not generated by canonical quantization of the gravity variables. In particular the boundary circle $\mathbb{S}^1$ is allowed to contract somewhere inside M, which invalidates Hawking's argument that $\log Z \propto \beta$. The Euclidean Schwarzschild geometry has such a point where the thermal circle contracts to zero size, and evaluating its action leads directly to the Bekenstein-Hawking formula. We illustrate geometries of both types in figure 1.

Why though are we allowed to include geometries where the circle contracts? We believe that the reason is holography: if the true microscopic description lives at the asymptotic boundary, then so does the true thermal circle! Therefore we should really only require a product spacetime topology $\mathbb{S}^1 \times \partial\Sigma$ at the boundary, and it is thus plausible to include geometries where the boundary thermal circle contracts in the interior of the spacetime. In fact in AdS/CFT it is necessary to include them, as one is otherwise unable to obtain the correct high-temperature entropy [38]. On the other hand, in any theory where we view the gravitational field theory description as fundamental – as happened in our non-holographic examples and we expect would happen in any asymptotic safety scenario – then results which rely on Euclidean topologies with no canonical interpretation need not be correct (and indeed they aren't in our examples).

There remains the challenge of the introduction: how does our proposal explain why the low-energy path integral knows about deep ultraviolet data, like the microstates of a black hole? In many contexts the holographic description provides a beautiful explanation: consistency between cutting and gluing the boundary Euclidean path integral in different ways provides what are known as "crossing" relations [39], and these relations typically introduce relationships between infrared and ultraviolet data. The rules of the Euclidean gravity path integral – in particular the sum over bulk saddles and Witten exchange diagrams – produces bulk data that manifestly satisfies these relations, and thus allows the low-energy gravitational path integral to access the deep ultraviolet. For example, for thermodynamically-stable black holes in AdS/CFT there is a high-temperature/low-temperature duality in the boundary theory which relates geometries where the thermal circle contracts to geometries where it doesn't [17, 19, 40, 41]. This is most familiar in the context of AdS_3, where it reduces to modular invariance in the boundary theory [17] and implies $Z(\beta) = Z(1/\beta)$ [42]. Since we *do* expect the low-energy gravitational theory to know the partition function on spacetimes where the thermal circle doesn't contract, this ensures a reliable calculation of the Bekenstein-Hawking entropy within the low-energy theory. Thus by allowing topologies where the thermal circle contracts, we manifestly restore a duality between low and high temperature which was not apparent from the canonical point of view. In general we suspect that any time the low-energy gravity theory seems to inexplicably know some kind of ultraviolet information, there is a crossing relation lurking in the fundamental boundary description which is responsible.

Discussion

We have argued that Euclidean gravity is correct if and only if the theory under consideration is holographic. All known non-holographic theories violate the predictions of Euclidean gravity, while all holographic theories perfectly corroborate Euclidean gravity. Remarkably, in many instances the holographic description also explains why the *low-energy* gravitational theory accesses deep ultraviolet data. This correspondence should help elucidate situations where Euclidean gravity techniques are used but holography is poorly understood, such as our universe.

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