The Essence of Gravitational Waves and Energy

F. I. Cooperstock*
Department of Physics and Astronomy
University of Victoria
P.O. Box 3055, Victoria, B.C. V8W 3P6 (Canada)

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Abstract

We discuss the essential element of gravity as spacetime curvature and a gravitational wave as the propagation of spacetime curvature. Electromagnetic waves are necessarily localized carriers of spacetime curvature and hence are also gravitational waves. Thus electromagnetic waves have dual character and detection of gravitational waves is the routine of our every-day experience. Regarding the transferring energy from a gravitational wave to an apparatus, both Rosen and Bondi waves lack the essential characteristic of inducing a gradient of acceleration between detector elements. We discuss our simple invariant energy expression for general relativity and its extension. If the cosmological term is present in the field equations, its universal presence characteristic implies that gravitational waves would necessarily have an energy aspect in their propagation in every case.

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Electromagnetic waves are fields of electromagnetism that propagate at the speed of light. Similarly, gravitational waves are gravitational fields that propagate at (or at least near) the speed of light. By Einstein's general relativity, a gravitational field is simply spacetime curvature, an aspect of spacetime itself, which is generated by an energy-momentum tensor. Unlike all other fields in nature which live *in* spacetime, in essence the gravitational field *is* spacetime. Our simple act of moving, changes the distribution of mass density (and hence the energy-momentum tensor) in the universe. Relativity tells us that the information of this change can propagate at most at the speed of light and therefore

*cooperst@uvic.ca

any simple act of moving our bodies induces a flow of changing spacetime curvature. Since we have never actually measured the speed of this flow, we might not be prepared to call it with confidence, a gravitational wave by the above description. Some might be inclined to call the propagation a gravitational wave even if it is non-relativistic and if so, relativity would demand the existence of such disturbances and hence the existence of gravitational waves.

However, there is another indisputable source of gravitational waves that we know about exceedingly well. This source is an electromagnetic wave. The argument is very simple: where there is an electromagnetic wave, there is an energy-momentum tensor. By the field equations of general relativity, wherever there is an energy-momentum tensor, there is a non-vanishing Ricci tensor, hence a non-vanishing Riemann tensor which invariantly characterizes the presence of spacetime curvature. This field of spacetime curvature is necessarily localized within the region of the energy-momentum tensor (one cannot boost from the speed c) and it flows with speed c. So we draw the following conclusion: *electromagnetic waves have an intrinsic duality: they are necessarily also gravitational waves.* Thus the detection of gravitational waves is the routine of our everyday existence as we detect electromagnetic waves. It would be very valuable if a means could be found to explicitly extract the gravitational wave aspect from the electromagnetic aspect of an electromagnetic wave.

The emphasis on research in gravitational wave detection has focused upon efforts to register in an apparatus, flows of spacetime curvature through vacuum or through at most a non-relativistic medium. To date, this has not been achieved. Thus-far, the evidence that is presented for the existence of gravitational waves has been indirect, the observations of period changes in binary pulsar systems. This indirectness adds a layer of complexity that is less than satisfactory. Far more appealing would be a direct detection and for this, two basic kinds of detectors have been considered, Weber-type bars and laser interferometric systems. Naturally tied in to the question of detection is the issue of energy transfer from wave to detector. This presents an interesting challenge to us as we have long-argued that energy in general relativity is localized in the regions of the energy-momentum tensor [1]. By this view, a "pure" gravitational wave (i.e. one that is not carried by an energy-momentum tensor as in light) propagating through the vacuum would not carry energy and hence would be in direct contradiction with Feynman's "sticky bead" thought experiment. In this experiment, a bead is free to slide on a stick with a slight degree of friction. A gravitational wave impinges upon the system and forces the bead to slide, generating heat in the stick. This is presented as proof that a gravitational wave displays its reality by virtue of transferring some of its energy to the heating of the stick. In a paper to be published, we demonstrate the flaws in Feynman's argument and conclude that gravitational waves would not force Feynman's bead to slide. Here, we point out some of the basic features of the argument.

Generally, one encounters the following typical discussion in the texts: Two test particles are aligned perpendicular to the propagation direction of a gravitational wave. The wave confronts the particles and by the equation of geodesic deviation, one determines that the wave induces relative motion between the test particles. From this point, there is a leap of faith that one can simply transfer this knowledge to actual (i.e. non-test) matter, say a stick, held together by a stress tensor and a pair of beads free to slide along the stick with a small amount of friction. The standard argument is that the beads will be induced to slide along the stick, just as the test particles move relative to each other, while the role of the stick is ignored. In some of our earliest work, we noted that in general relativity, all stress-energy interacts with an incident gravitational wave. We had simulated the elastic properties of the stick with electromagnetic fields and we had found that to lowest order at least, the stick would respond to the wave in the same manner as did the beads and there would be no sliding, no rubbing, no heating. To go beyond this in all accuracy, we note that in the standard-bearer Rosen wave propagating in the z direction [2]

$$ds^2 = dt^2 - f^2(e^{2\psi}dx^2 + e^{-2\psi}dy^2) - dz^2 \tag{1}$$

$f = f(t-z)$, $\psi = \psi(t-z)$, there is no metric x or y dependence to bias a bead on the stick to move to the right or to the left so it does neither. Similarly, in the more complicated Bondi *et al* wave [3], their own analysis has revealed that their wave is plane-fronted with parallel rays, again lacking the required character to provide a bias for the shifting of a bead on the stick. In [3], these authors have focused their attention on acceleration whereas what is required for the energetics is a *gradient* of acceleration. It is easy to gain a sense of what is at play. The leap that others have made from the relative motion of two test particles in vacuum to deducing a rubbing of beads on a stick might not have been taken had they considered instead, a line of test particles. One could not say that any given particle moves either to the left or to the right when confronted by a wave. Rather, one can see that the spaces between the particles shrink and expand in a time sequence determined by the nature of the functions in the metric tensor.

The need for an agency of gradient of acceleration to bring about frictional heating is evident when we consider this same stick with beads in free-fall in the field of the earth. To first order, we are at the level where the beads and the stick fall with acceleration relative to the earth and at the same rate but there is no relative motion. This is at the level of the equivalence principle. To the next order, the effect of the beads falling in lines converging to the center of the earth is one of providing the agency for a gradient of acceleration, and there is rubbing. What is at work here is the detailed energy interplay between *two* bodies, the earth and the apparatus of stick plus beads, an interplay that is absent in the previous cases.

Regarding the issue of energy in general relativity, one that has evaded a satisfactory resolution for nearly a century, we have recently proposed a simple solution [4]. Our approach stems from a generalization of the invariant energy expression in special relativity. For a system with four-velocity u_i and four-momentum p^i, the invariant energy can be expressed by the inner product

$$E = p^i u_i. \tag{2}$$

Now we know that the energy including the contribution from gravity, for a stationary system in general relativity can be expressed by the well-known Tolman integral as

$$E = \frac{c^4}{4\pi G} \int R_0^0 \sqrt{-g} d^3x \tag{3}$$

where g is the determinant of the *four-dimensional* spacetime metric tensor and R_0^0 is the mixed time-time component of the Ricci tensor. It is well to dwell on this "odd couple" $\sqrt{-g}$ and d^3x. Their product does not produce a proper volume element. We cannot alter the first of these because it is an essential part of the stationary system energy integral. However we are free to extend our integration beyond three-space and integrate over time as well. In so-doing, we produce an invariant expression which we refer to as "spacetime energy". We suggest that it has been the fixation over nearly a century on our confining the search for a general relativistic expression for energy to the conventional three-space that has rendered this search so problematic. However the elements at play direct us to spacetime rather than to space. This should not be seen as an unnatural development as the essential domain of relativity is spacetime rather than space. Thus, guided by (2), the minimal requirements to generalize (3) to general relativity to be in accord with (2) and maintain invariance are to employ the full Ricci tensor, to use the four-velocity at each point of the source and to generalize from three-space with d^3x to four-space with d^4x,

$$E = \frac{c^4}{4\pi G} \int R_i^k u^i u_k \sqrt{-g} d^4x. \tag{4}$$

We have shown that this simplest extension which maintains invariance is also the maximal extension [4]. For a stationary system, (4) yields the Tolman integral times the time that the system has been observed. For a time-dependent system, the expression can acquire considerable complexity.

A final note concerns the issue of vacuum itself and how it meshes with the spacetime energy. Originally Einstein introduced the cosmological Λ term into his field equations as

$$R_i^k - 1/2\delta_i^k R + \Lambda\delta_i^k = (8\pi G/c^4)T_i^k \tag{5}$$

to render a static universe. This was discarded when the universe was seen to be dynamic. In recent times, it has been resurrected as the potential embodiment of "dark energy" to drive the apparent acceleration of the expansion of the universe. This new Λ term is more logically allowed to have the freedom to vary with the age of the universe [5]. In either case, taken to the other side of the field equations, it plays the role of a special energy-momentum tensor. Thus, if the Λ is present, it is everywhere and gravitational waves even in the absence of the usual energy-momentum tensor T_i^k, necessarily play an energy role everywhere.

References

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