IMPLICATIONS OF GRAVITY DATA OBTAINED FROM ANALYZING THE MOTION OF ARTIFICIAL SATELLITES

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ABSTRACT

THE 3rd, 4th AND 5th ZINAL HARMONICS OF THE EARTH'S EXTERNAL GRAVITATIONAL POTENTIAL OBTAINED BY ANALYZING THE MOTIONS OF ARTIFICIAL SATELLITES ARE CONSIDERED TO BE CAUSED BY A DENSITY VARIATION WITH RESPECT TO LATITUDE IN THE INTERIOR OF THE EARTH. THIS DENSITY VARIATION MAY CAUSE NON-HYDROSTATIC STRESS IN THE INTERIOR. THE TANGENTIAL AND RADIAL COMPONENTS ARE CALCULATED ON A SURFACE AT AN ARBITRARILY CHOSEN DEPTH OF 700 KILOMETERS. THE TANGENTIAL COMPONENT ATTAINS A MAXIMUM VALUE OF 14 BARS AT 65° N; THE RADIAL COMPONENT 51 BARS AT THE NORTH POLE.

The gravity of the earth is determined by the distribution of density within the earth; hence the best way to understand gravity is to obtain a knowledge of the internal structure of the earth. A satellite moving around the earth is under the attraction of the earth's gravity. The shape of the orbit of the satellite is determined by the characteristics of the gravitational field, and hence reflects the distribution of density in the interior.

For a given radial distribution of density, a constant rate of rotation, and hydrostatic stress at all points in the interior, the form of the surfaces of constant density is determinate and contains no harmonics other than the second, which arises from the ellipticity. External equipotential surfaces due to gravity for such a constant density surface should also contain no harmonics other than the ellipticity term. This result, however, does not hold if the stress in the interior of the earth is not exactly hydrostatic. Any departures of the external gravitational field from the main elliptical form are inconsistent with exact hydrostatic condition in the interior, and consequently prodide an estimate of how far this hypothesis is wrong.

Recent analyses (O'Keefe, et al, 1959, and Kozai, 1961) of the motion of artificial satellites have determined the coefficients of the 3rd, 4th and 5th order zonal harmonic components for the earth's gravitational potential. The coefficients of these components

are	A_3	A_4	A_5
Kozai (1961)	0.237	1.40	0.98
O'Keefe, et al, (1959)	0.25	1.12	0.2

with units of potential in (megameter)^2 (Kiloseconds)$^{-2}$.

If we assume that the second harmonic component arises solely from the ellipticity of the figure of the earth and that the A_n $(n > 2)$ are caused by density variations with respect to latitude in a thin shell of thickness ϵ, at the earth's surface, i.e., for depths greater than ϵ the density is a function of radius only, but for depths less than ϵ the density is a function of both radius and latitude. In general, the variation can be expressed in a harmonic expansion $\Sigma C_n P_n(\Theta)$, where $P_n(\Theta)$ is the n-th zonal spherical harmonic and D_n the density term corresponding to the measured A_3, A_4 and A_5. For an arbitrarily chosen ϵ of 100 kilometers, C_3, C_4 and C_5 have the following values:

$$C_3 = 1.95 \times 10^{-3} \text{ gm/cm}^3$$
$$C_4 = 2.42 \times 10^{-3} \text{ gm/cm}^3$$
$$C_5 = 5.49 \times 10^{-4} \text{ gm/cm}^3$$

The values of the sum $\sum_{n=3}^{5} C_n P_n(\Theta)$, which is the variation of the density with respect to latitude, are listed in table 1 and illustrated graphically in figure 1.

The variation of density with respect to latitude varies more rapidly and have greater positive and negative maximum values in the northern hemisphere than in the southern hemisphere. This implies the departure from a hydrostatic state is in general greater in the northern hemisphere. Furthermore, this variation term is positive between N 90° and N 60°, negative between N 60° and N 10°, positive again between N 10° and S 30°, and between S 30° and S 90° is negative again. A general relationship between this variation of density and the distribution of lands and oceans is evident; i.e. positive over most continental area and negative over most oceanic area. This indicates that the nonhydrostatic state of the earth may have some relationship with the distribution of continents and oceans.

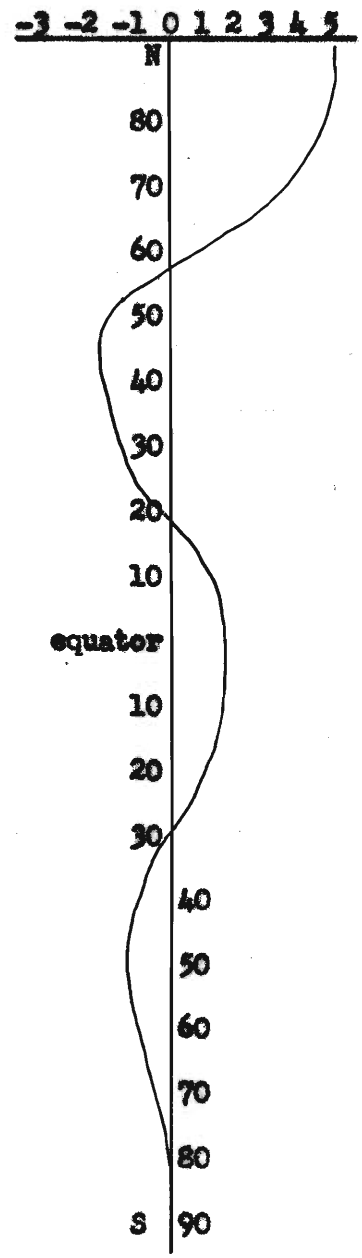
Table 1

Values of $\sum_{n=3}^{5} C_n P_n$ at different latitudes.

Latitude	C_nP_n (gm/cm^{3}x 10^{-3})
N. 90	4.92
85	4.74
80	4.29
75	3.55
70	2.61
65	1.21
60	0.41
55	-0.01
50	-1.16
45	-1.70
40	-1.88
35	-1.87
30	-1.55
25	-1.16
20	-0.62
15	-0.08
10	0.39
5	0.64
Equator	1.02
5	1.01
10	1.36
15	0.96
20	0.62
25	0.41
30	0.01
35	-0.03
40	-0.29
45	-0.42
50	-0.49
55	-0.62
60	-0.43
65	-0.34
70	-0.29
75	-0.20
80	-0.12
85	-0.14
S 90	-0.10

Figure 1

Values of $\sum_{n=3}^{5} C_nP_n$ at different latitudes.



The nonhydrostatic stress-components can be calculated by the classical method in elasticity (Love, 1927, p. 249-251). To simplify the calculations, it is assumed that the earth is an isotropic, elastic sphere, with radius equal to 6371 Km and elastic constants equal to 10^{12} units. We further assume that the interior of the earth is initially in a hydrostatic state and surface loading adds non-hydrostatic stress-components to the initial stress. Additional stress may be taken in connection with the corresponding strain, measured from the initial state as an unstrained state, by Hooke's law.

The loading on the surface at the depth of ϵ = 100 kilometers is in the direction towards the center of the earth, with magnitude of $\sum_{n=3}^{5} \epsilon \quad g\, C_n\, P_n\, (\theta)$, where g is the acceleration of gravity. For depths greater than ϵ the nonhydrostatic stress can be decomposed into two components: One in the radial direction, the other in the tangential direction. For the symmetrical earth, the tangential component is along the direction of the meridian. Calculation is made for these two components at an arbitrarily chosen depth of 700 kilometers. The results are listed in table 2 and 3 and illustrated graphically in figures 2 and 3.

Figures 2 and 3 show a striking correlation between the regular variations of the nonhydrostatic stress-components and the distributions of continents and oceans. The radial component attains a maximum value of 51 bars at the north pole and varies southwardly as a damped harmonic oscillation. The tangential component varies in a different way. It is zero at both poles and attained a maximum value of 14 bars at 65° N under the borders of the continents around the north pole. A much smaller relative maximum occurs about 70° S and appears to have the similar relationship with the border of the Antarctic continent. (Correlation between the tangential component and the continental borders running approximately in the north-sourh direction needs detail gravity data varying with longitude, but so far there have been none of these.) At depth of several hundred kilometers in the mantle where the hydrostatic pressure and temperature are high, small tangential component may cause plastic flow over a long period of time, or, if failure occurs, energy is released and might generate some deep focused earthquakes.

So far the present analysis of the recent gravity data has brought us a picture of s some of the internal structures of the earth. In addition it offers an explanation of some important geodynamic problems such as the hypothetical convection currents in the mantle. We may finish by remarking that much more analysis of the gravity data obtained from various measurements is to be done and that the more we understand gravity the more we will know about the interior of the earth.

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Table 2.

Radial stress-component Prr at depth 700 km and different latitudes

Latitude		Prr (bars)
N.	90	50.5
	85	48.3
	80	43.7
	75	36.0
	70	26.5
	65	14.1
	60	4.9
	55	- 3.6
	50	-11.3
	45	-16.3
	40	-18.4
	35	-17.9
	30	-15.5
	25	-11.2
	20	- 5.9
	15	- 0.9
	10	3.8
	5	7.7
Equator		10.2
	5	10.9
	10	10.5
	15	8.4
	20	5.9
	25	2.8
	30	- 0.2
	35	- 2.5
	40	- 4.6
	45	- 5.8
	50	- 6.0
	55	- 5.5
	60	- 4.5
	65	- 2.7
	70	- 1.0
	75	0.5
	80	1.8
	85	2.6
S.	90	3.0

Figure 2

Radial stress-component at depth 700 km and different latitudes

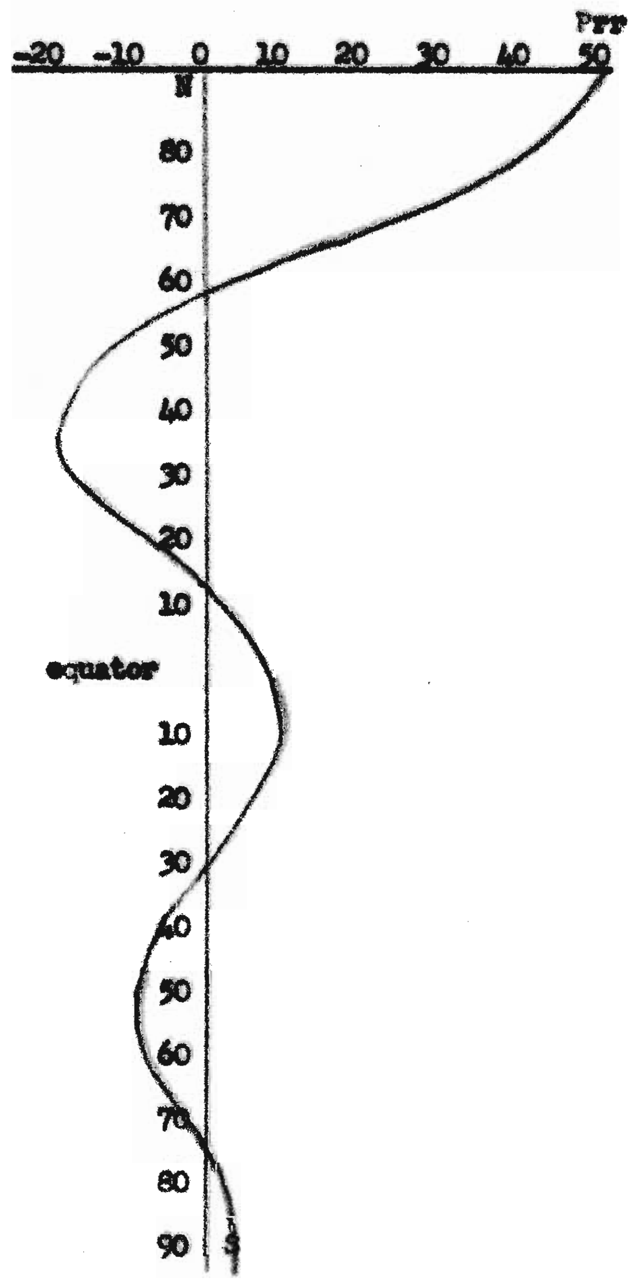

Table 3.

Magnitude of the tangential stress-component Pre at depth 100 km and different latitudes

Latitude	P_r (bars)
N. 90	0
85	0
80	4
75	8
70	13
65	14
60	13
55	11
50	8
45	4
40	0
35	0
30	4
25	6
20	7
15	7
10	5
5	3
Equator	1
5	1
10	3
15	4
20	5
25	6
30	5
35	4
40	0
45	0
50	1
55	2
60	2
65	2
70	2
75	2
80	0
85	0
S. 90	0

Figure 3.

Magnitude of the tangential stress-component Pre at depth 100 km and different latitudes