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ORIGIN OF THE UNDULATIONS IN THE EARTH'S SATELLITE GRAVITATIONAL POTENTIAL

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INTRODUCTION

Knowledge about the earth's gravitational potential has increased significantly during the last few years, mainly through the studies on the motions of close artificial satellites. Analyses of the secular and long-period perturbations of satellite orbits have yielded determinations of the zonal harmonics up to the 14th order (Kozai, 1964). Meanwhile, improved determinations of the long-wavelength, longitudinal-dependent variations in the gravitational field have been made by careful analyses of different types of short-period perturbations of satellite orbits. Only recently, however, determinations based on optical technique (Kaula, 1963a, 1963b; Izsak, 1963a, 1964a, 1964b), and on Doppler technique (Guier and Newton, 1963; Guier, 1964) have shown good agreement on the general features of the geopotential. The results are also compatible with recent analyses of the surface gravity data (Kaula, 1961; Uotila, 1962). Izsak's analyses has yielded coefficients of significant magnitude for tesseral harmonics up to the 6th order; Figure 1 is the geoid corresponding to his most recent results (Izsak, 1964b).

That the figure of the geoid is not a rotational ellipsoid indicates that the interior of the earth is not exactly hydrostatic. Somewhere in or on the earth there must be a small departure in the actual density, call it the density anomaly, from that corresponding to a hydrostatic distribution. This density anomaly should have no relation with the crustal structures since the geoidal undulations show little, if any, correlation with the distribution of the land and sea or with any other large-scale crustal structures.

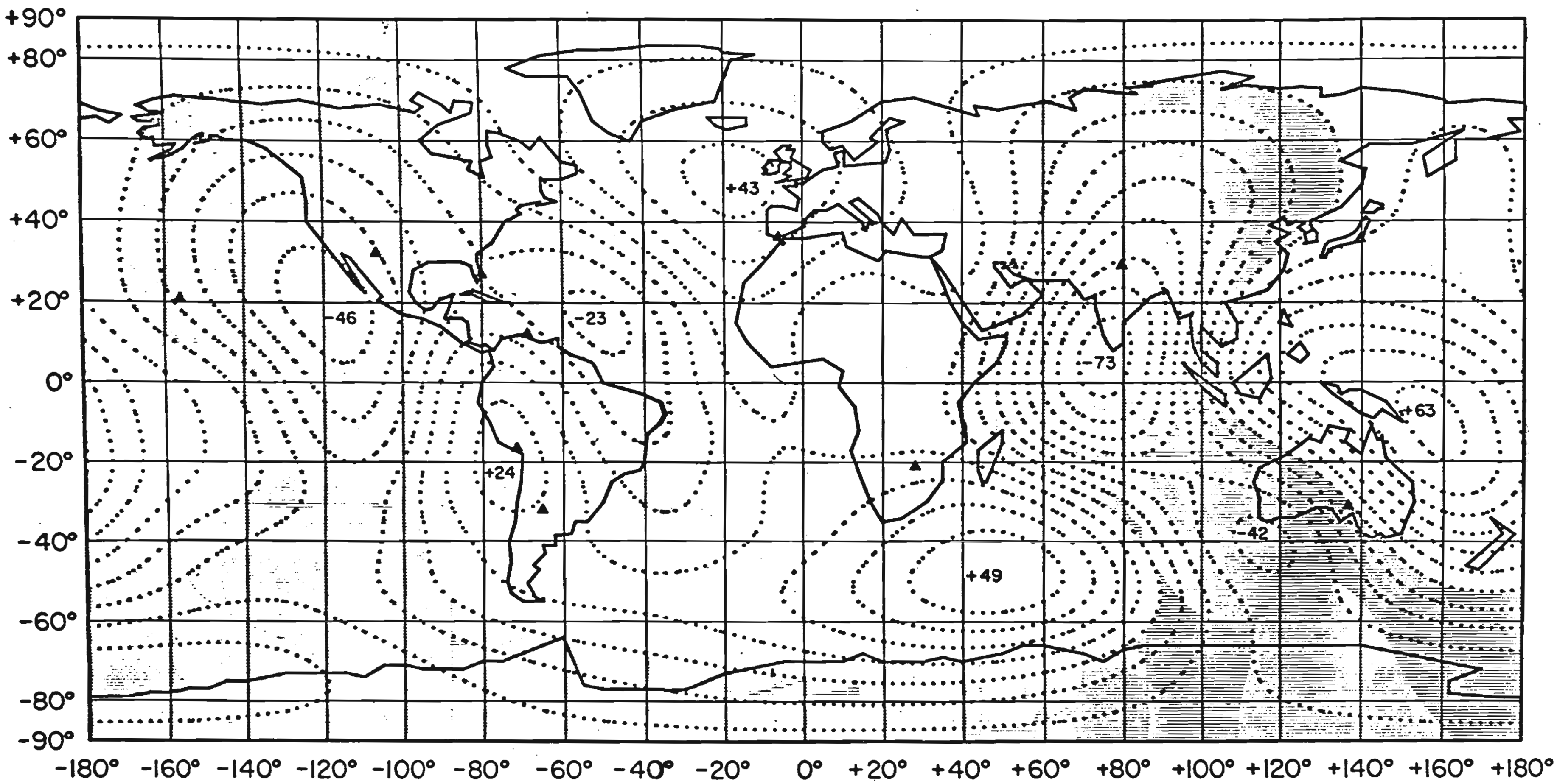

Fig. 1. Sixth-degree level curves of geoid heights at 10-m intervals determined from 26,244 Baker-Nunn observations of 11 satellites.

Recently some similarities between the patterns of distributions of the heat flow and the geoidal undulation were observed (Wang, 1963; Lee and MacDonald, 1963). It appears that the density anomaly related to the geoidal undulations may have its origin in a perturbation of temperature through uneven thermal expansion of the upper mantle.

Radiometric determinations of uranium, thorium, and potassium have shown that radioactivity of rocks varies through a wide range, even in the same type of rock. In an example given by Lovering and Morgan (1963) on the uranium and thorium abundances in eclogites, uranium varied from 0.018 to 0.24 p.p.m., thorium varied from 0.015 to 0.60 p.p.m., and the ratio Th/U was highly variable. It appears reasonable to assume that the radioactivity of the upper mantle is not laterally homogeneous. If the temperature perturbation is related to a laterally inhomogeneous distribution of radiogenic heat sources in the upper mantle where the dominant process of heat transfer is thermal conduction, it is possible to find a model in which the distributions of temperature and heat sources will give us the observed variations in the surface heat flow, and, through thermal expansion, the correct magnitude of density anomaly to account for the geoidal undulations. It is the purpose of this paper to present such a model. Whether the above proposed hypothesis is acceptable or not depends critically on whether the calculated results on the temperature and the heat sources in the model are reasonable or not.

THE DISTRIBUTION OF HEAT FLOW AND ITS CORRELATION WITH THE GEOIDAL HEIGHT

In recent years, numerous heat flow measurements have been made over the earth's surface. The distribution of the measurements, however, is very uneven; most of them, about 88 per cent, are in the oceans and concentrated in the East Pacific Rise, off the east coast of the United States and the west coast of Europe. The rest are scattered in North America, the British Isles, Japan, Hungary, Germany, Australia, and the southern tip of Africa. Large gaps exist in the continental regions; there are, for example, no measurements in South America or in most of Africa and Asia.

I have about 750 heat-flow determinations at hand, most of them from Lee's (1963) compilation. These data are averaged into the $5^\circ \times 5^\circ$, $10^\circ \times 10^\circ$ and $20^\circ \times 20^\circ$ squares as shown in Figures 2, 3, and 4, respectively. In Figure 3, the high and the very high heat flow appear to be closely associated with some major geological structures. For example, squares with very high heat flow occur in the regions of the North American Cordillera, the East Pacific Rise, the Mid-Atlantic Ridge, the Alps, the island arc of the western Pacific Ocean, and some volcanic areas. The geological structures are definitely related, in one way or another, to a higher rate of supplying heat.

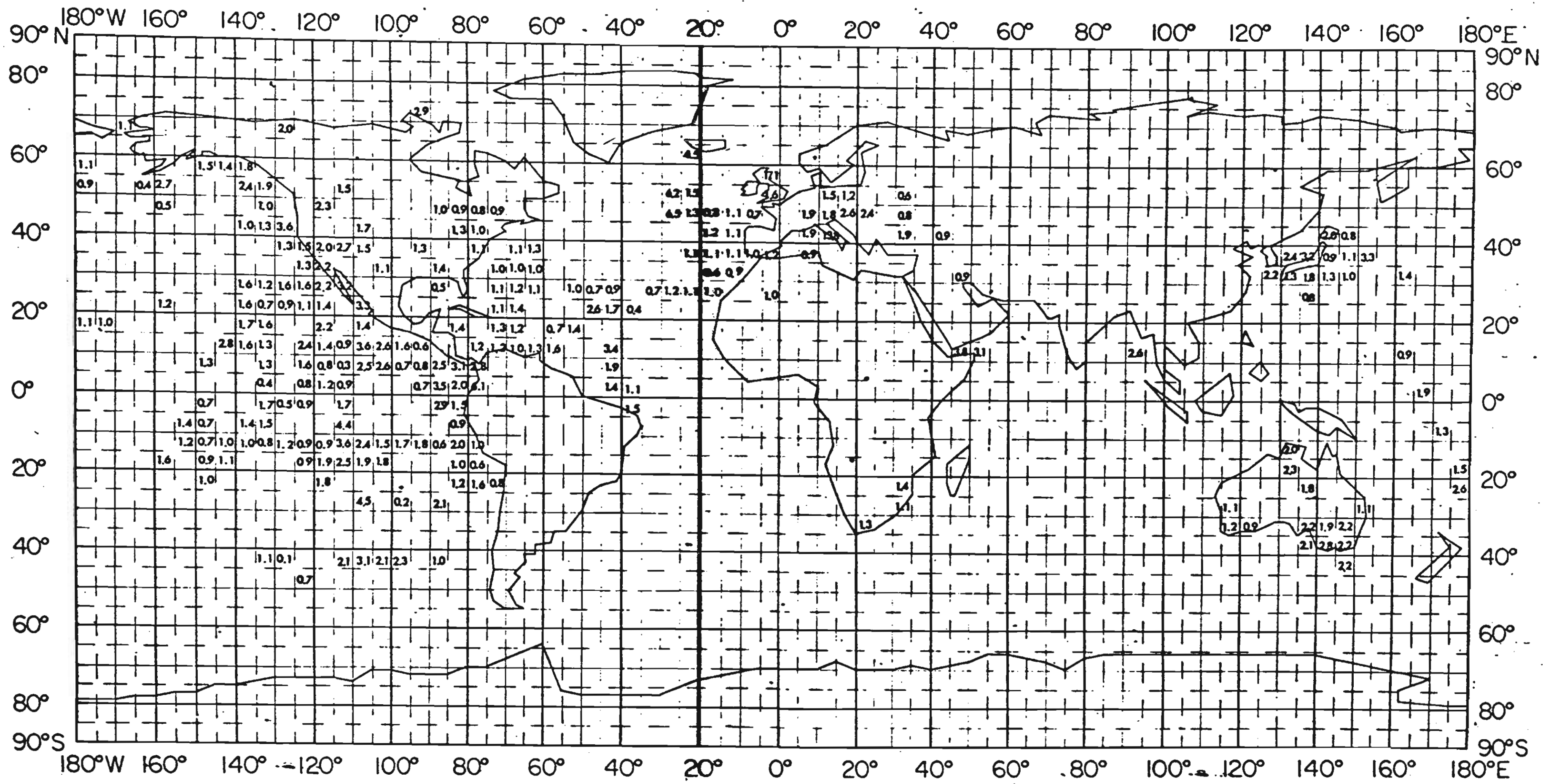

Fig. 2. Average Heat Flow Values in 5° x 5° Squares. In Units of 10^{-6} xcal/cm²sec.

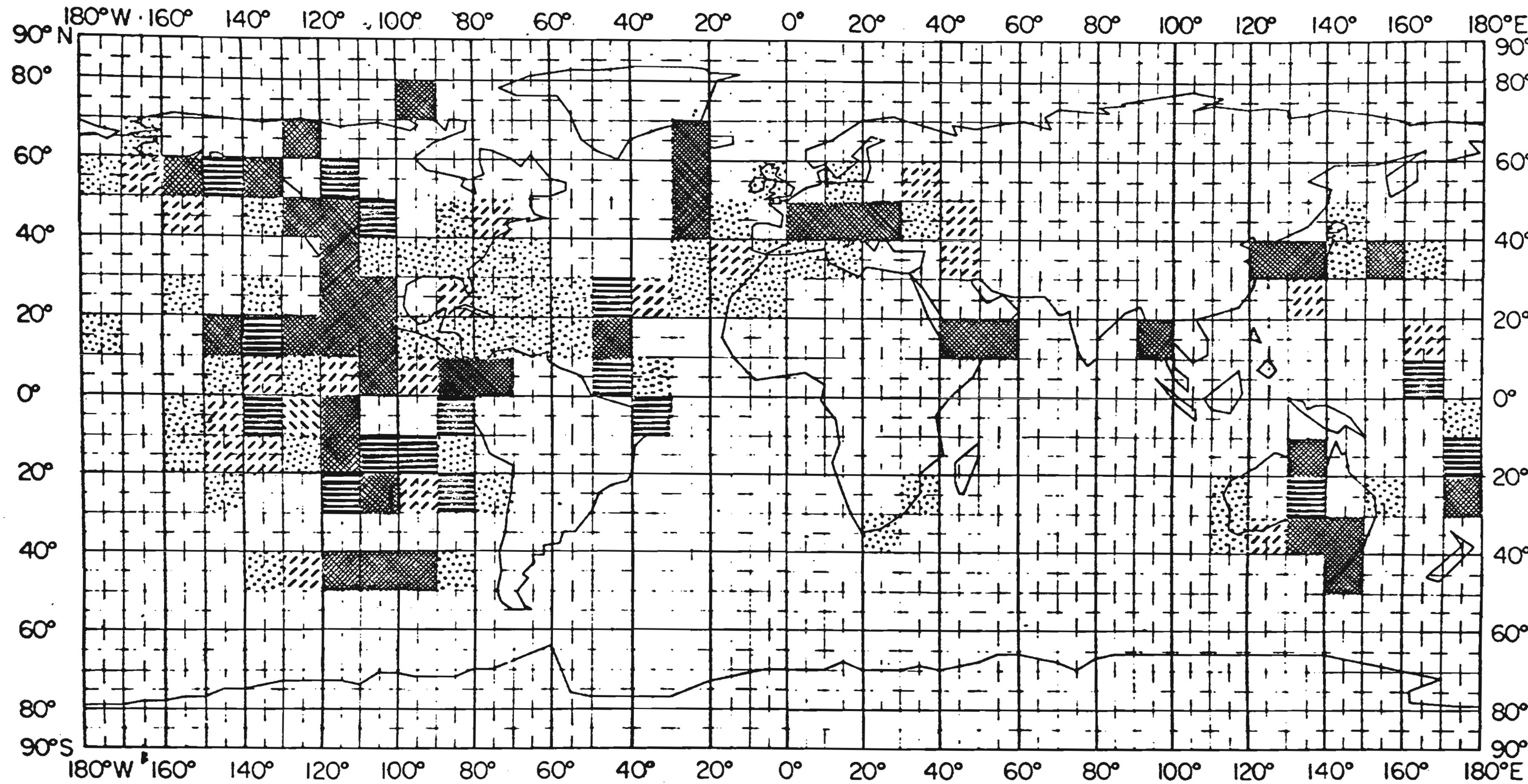

Fig. 3. Average values of heat flow for the $10^{o} x 10^{o}$ squares (in units of 10^{-6} $cal/cm^2 sec$)
2.0 and larger, 1.5 to 1.9, 1.0 to 1.4, 0 to 0.9.

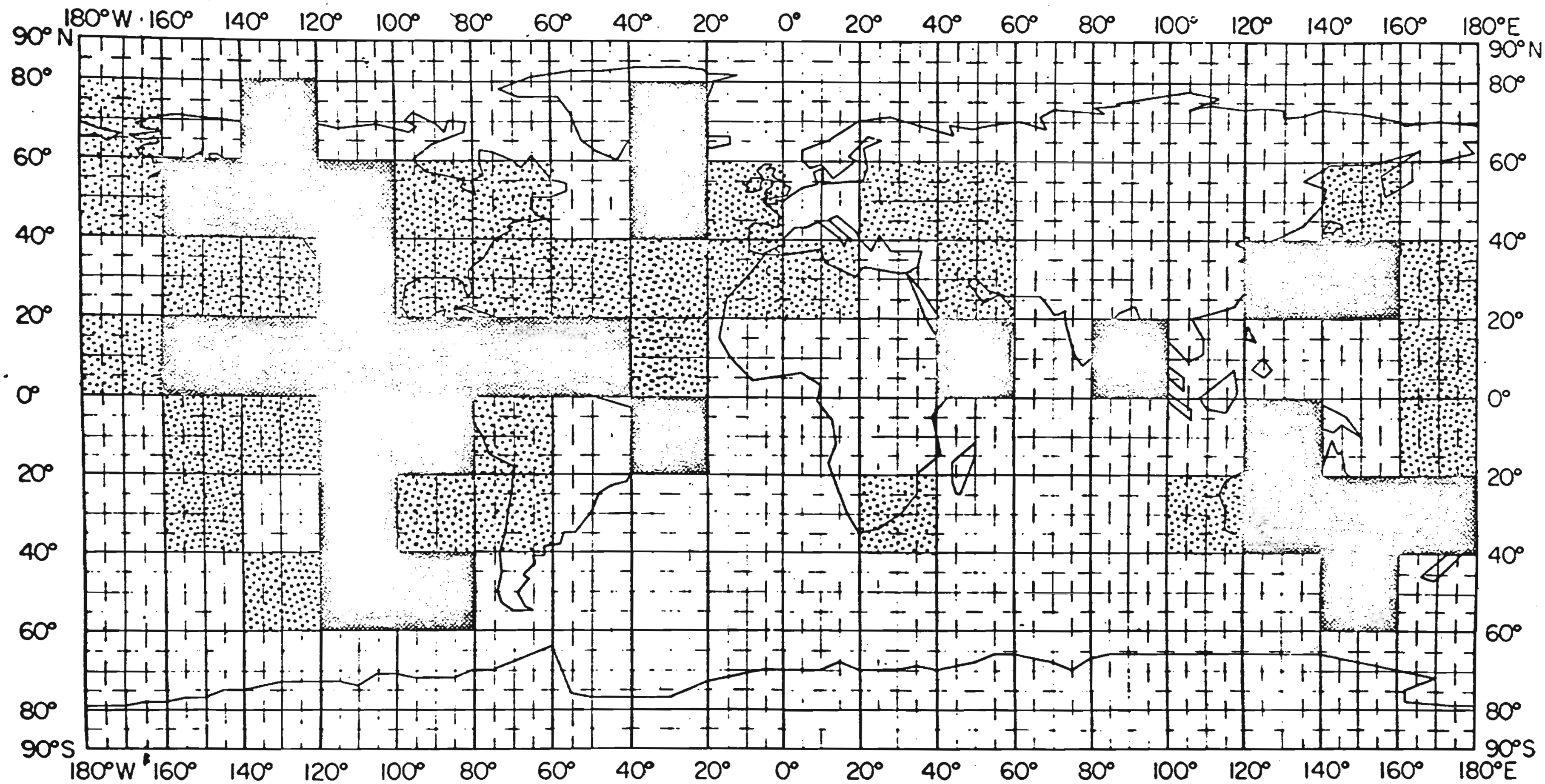

Fig. 4. Average values of heat flow for the 20°x20° squares. ▩ High heat flow, with average value larger than 1.5 cal/cm²sec. ⁘ Low heat flow, with average value smaller than 1.5 cal/cm²sec.

Figure 4 shows that the long-wavelength distribution of heat flow has no correlation with the distribution of the land and sea. Comparing this figure with Figure 1, it is seen that most of the squares with high heat flow correspond to squares with negative geoidal height, while most of the squares with low heat flow correspond to squares with positive geoidal height. The correlation is better when the extremely high heat flows are erased. To test this negative correlation, the spherical harmonic representation of heat flow by Lee and MacDonald (1963) is correlated with the satellite geopotential. Since the harmonic expansion of heat flow is made only up to the second order, for the purpose of correlation with the geopotential we choose harmonic coefficients for the latter only up to the second order so that both sets of data have similar wavelengths in their distributions. Using Lee and MacDonald's analysis with the extreme values deleted (1963, Table 2, column 4), and Izsak's (1963b) $C_{2,2} = 6.98 \times 10^{-7}$ and $S_{2,2} = -5.74 \times 10^{-7}$, the correlation coefficient is found to be -0.82. Figure 5 shows the distribution of the pairs of points and the regression line.

INTERPRETATION OF THE OBSERVED CORRELATION

The hypothesis proposed here is that the inhomogeneous distribution of radiogenic heat sources in the interior is responsible, through thermal conduction, for the fluctuations of the surface heat flow. The corresponding internal temperature, through thermal expansion, causes density anomalies which in turn produce the geoidal undulations on the earth's surface.

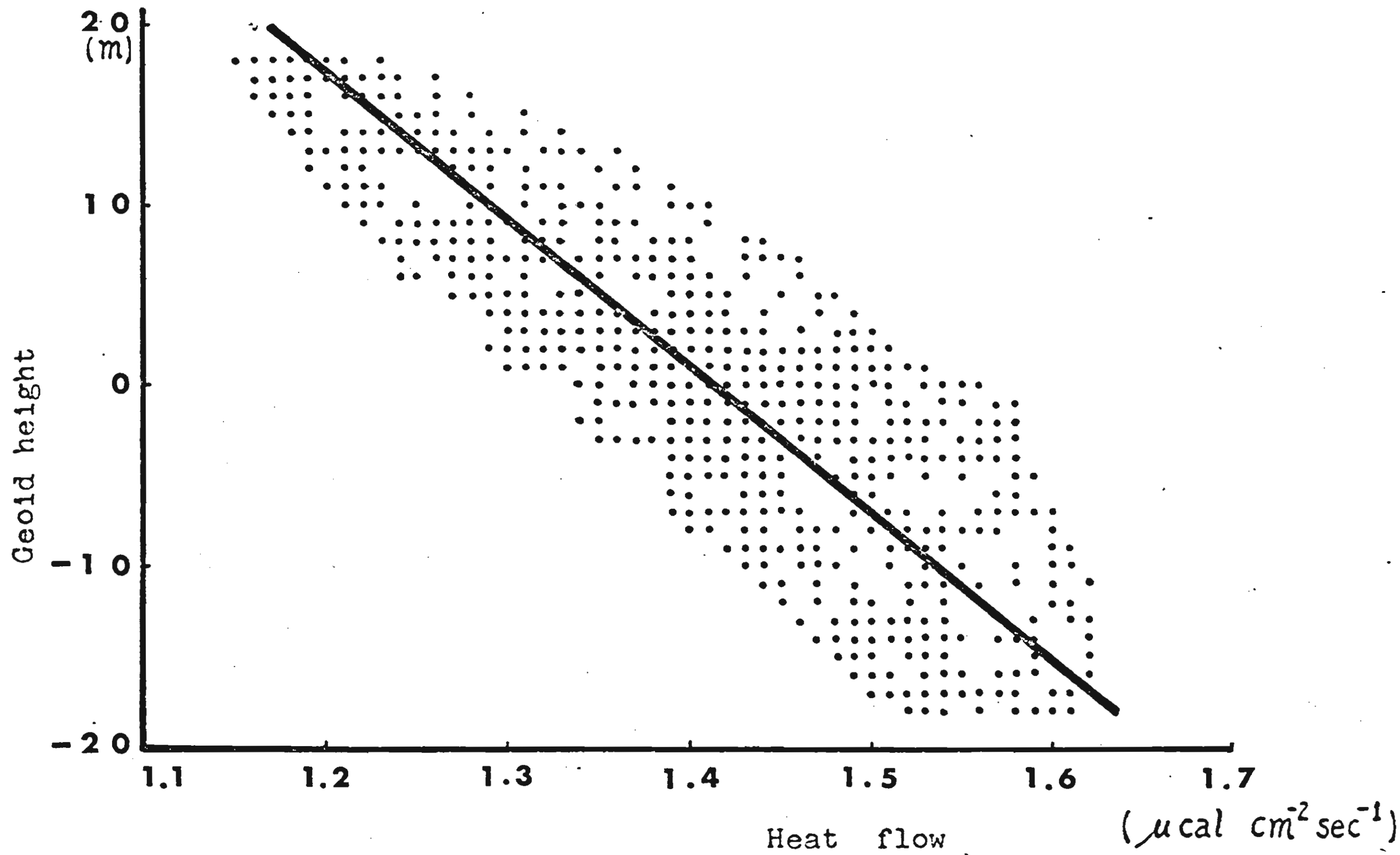

Fig. 5. Heat flow plotted against geoid height and the regression line.

Since we are only concerned with the deviations of heat flow and geoidal height from some reference levels, it is necessary to define the physical meaning for the latter. As the distribution of mass must be affected by temperature, the measured gravity on the surface of the earth must also be affected. For the present discussion, I define the reference state as the distribution of thermal properties and mass, one related to the other in the earth, which gives, as result of combined effects, the spherical and the elliptical terms in the geopotential and the average heat flow on the earth. The residual variations in both the heat flow and the geopotential are controlled by some factors in the interior that deviate from the reference state.

In the present treatment, thermal conductivity K and the diffusivity k are assumed to be uniform in the upper part of the earth. Since the basic physical equations controlling gravitational potential and the heat flow through thermal conduction then contain only linear, partial differential operators, it is justified, in the mathematical treatment of the residual physical quantities, to disregard the unknown quantities involved in the reference state. If the deviations in temperature and in the rate of heat generation per unit volume are denoted by ΔT and ΔA respectively, the temperature is then controlled by the conduction equation

$$k \nabla^2(\Delta T) - \frac{\partial}{\partial t}(\Delta T) = -\frac{k}{K}(\Delta A) \quad . \qquad (1)$$

We may assume that the pattern of the distribution of ΔA has been fixed since the crust and the upper mantle were formed. Although not all parts of the crust are the same age, it is assumed for mathematical simplicity that this took place some 3.5 billion years ago. Let us set $t = 0$ as the time at 3.5 billion years ago. The initial condition is then

$$\Delta T = 0, \text{ at } t = 0 .$$

The boundary conditions are

$$(1)\ \Delta T = 0, \text{ at } r = b$$

where b is the mean radius of the earth, and (2) ΔT is bounded at $r = 0$. It will be shown later that the distribution of ΔA is near the top of the mantle; the second term in Equation (1) can then be omitted since the time span is long enough for temperature to reach equilibrium. Owing to the radioactive decay, ΔA should be a decreasing function of time; but since uranium, thorium and potassium 40 all have very long half lives (in the order of billions of years), the decrease in ΔA is very slow. Besides, thermal equilibrium has been reached in the top portion of the mantle and excess of heat was conducted to the surface of the earth and dissipated; so the present temperature variation ΔT is mainly controlled by the present distribution of ΔA. It is therefore justified to take ΔA as independent of time.

Assuming a very simple model for the distribution of ΔA such that

$$\Delta A = \sum_{n} (\Delta A_n) = \sum_{n,m} (p_{n,m} \cos m\lambda + q_{n,m} \sin m\lambda)\, P_n^m(\sin\beta), \quad \text{for } c < r \le b \qquad (2)$$

and $\qquad = 0, \qquad$ for $r < c$

we have

$$\Delta T = \frac{1}{K} \sum_{n,m} \frac{r^n \Delta A_n}{2n+1} \left[\int_r^b \left(\frac{1}{r'^{\,n-1}} - \frac{r'^{\,n+2}}{b^{2n+1}} \right) dr' + \left(\frac{1}{r^{2n+1}} - \frac{1}{b^{2n+1}} \right) \int_c^r r'^{\,n+2}\, dr' \right] . \qquad (3)$$

$p_{n,m}$ and $q_{n,m}$ are related to the observed variation of surface heat flow by

$$\Delta F = - K\, \partial(\Delta T)/\partial r \big|_{r=b} . \qquad (4)$$

The above two equations give ΔT in terms of c and the observed heat flow. The value of c is determined by the condition

$$\Delta\rho/\rho = -\alpha\, \Delta T , \qquad (5)$$

such that $\Delta\rho$ corresponds to the density anomaly causing the residuals in the satellite geopotential. In Equation (5) ρ is the density for the reference state and α is the coefficient for the thermal expansion of rock. The external potential corresponding to this density anomaly is (Jeffreys, 1959)

$$- \sum_{n} \frac{2\pi\alpha\, \rho\, G\, (b^2 - c^2)}{2n+1} \left(\frac{b+c}{2b} \right)^{n+1} (\Delta T_n) . \qquad (6)$$

Adopting $K = 0.006$ cal/cm sec C°, $\alpha = 5 \times 10^{-5}$/C° for eclogite (Birch, 1952) and Lee and MacDonald's heat flow coefficients (1963, Table 2, column 4), it is found, by trying various values of c, that in order to make Equation (6) the same magnitude as that of the residuals in the satellite geopotential, c is equal to 6271 km; that is, the distribution of ΔA is within the outer 100 km of the earth. Figures 6 and 7 give the corresponding distributions of ΔT and ΔA. Since rocks cannot have negative radioactivity, the fluctuation of ΔA in Figure 7 suggests that the average rate of heat production in the upper mantle should be at least 2×10^{-14} cal/cm^3 sec. This rate is much higher than those of peridotite and dunite, but is lower than those of basalt and some eclogites (Lovering and Morgan, 1963). An eclogitic upper mantle is hence in this respect more favored than one made of peridotite or dunite.

CONCLUSIONS

On the basis of the observed correlation between geoidal height and heat flow, the geoidal undulations have their origin in the temperature perturbations within the outer 100 km of the mantle, having a maximum amplitude of about one hundred degrees. The corresponding fluctuations of heat sources indicate that an eclogitic upper mantle is more adequate than a peridotitic or a dunitic one.

Since the mantle under the continents may be different from that under the oceans, the temperature curves given in Figure 6 do not represent the actual variations of temperature in the mantle. A more realistic picture cannot be given until a definite answer about the temperature distribution in the assumed reference state is found. The temperature curves in Figure 6 imply, however, that isothermal surfaces in the upper mantle cannot be simple geometric surfaces; they have their ups and downs.

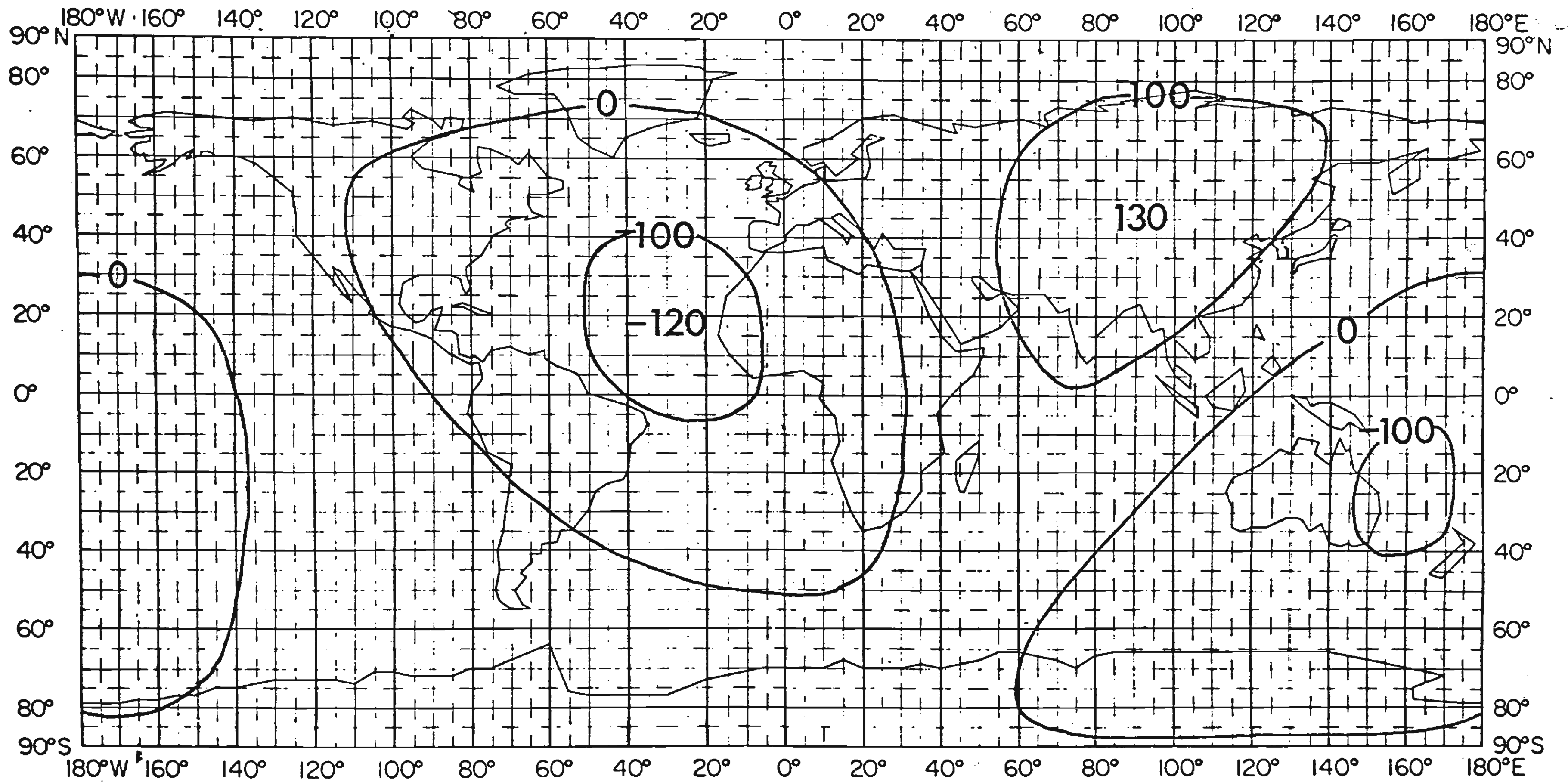

Fig. 6. Temperature perturbations with respect to the reference state (in degrees Centigrade) at depth 50 km.

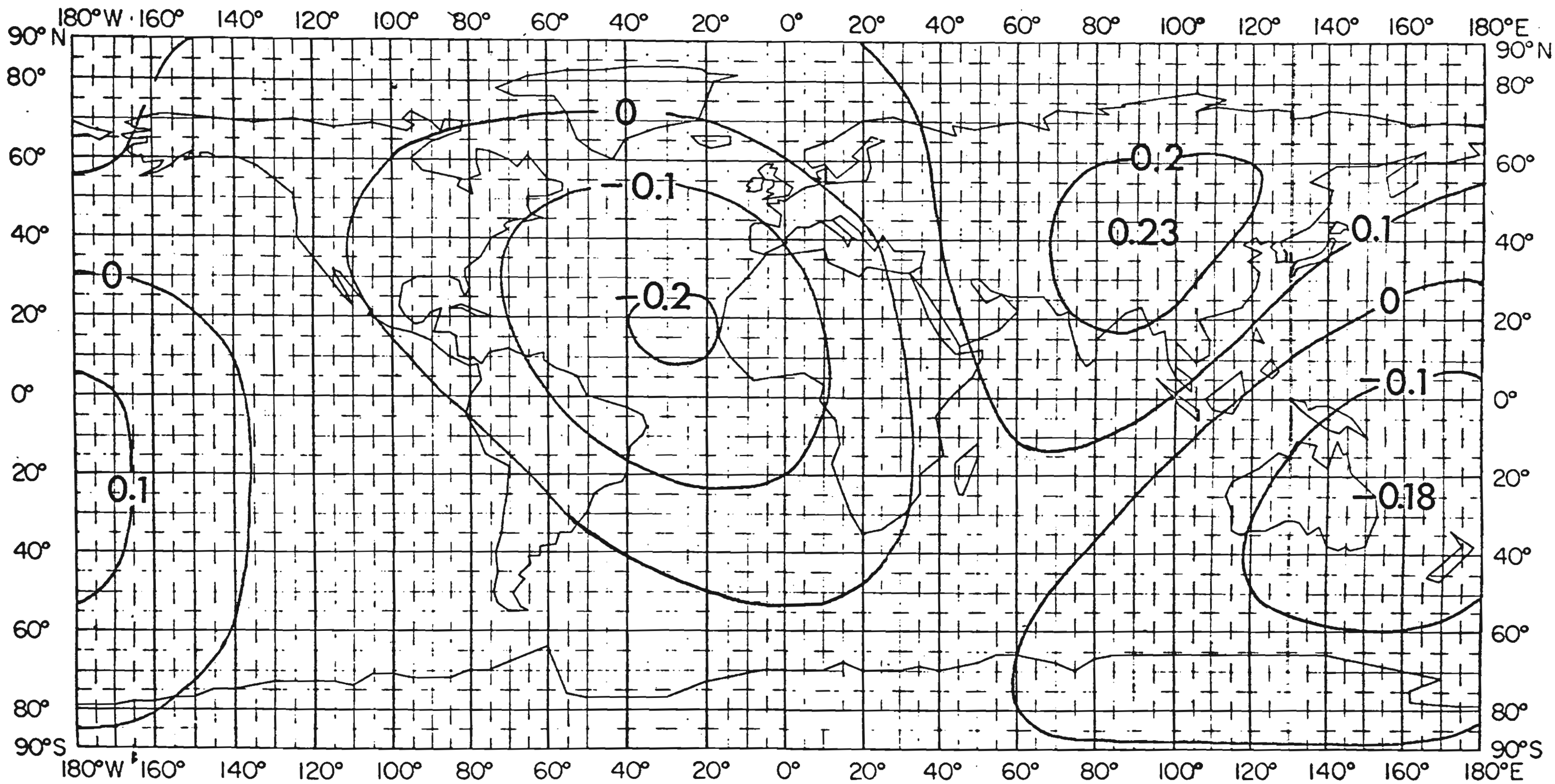

Fig. 7. Variations of heat sources A in unit of 10^{-13} $cal/cm^3 sec$ with respect to the reference state.

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Wang, C. Y., On the distribution of surface heat flow and the second-order variations in the external gravitational field, _Smithsonian Astrophys. Obs._, _Special Rept._, No. 134, 13, pp., 1963.

Abstract. The negative correlation coefficient, -0.82, between Izsak's (1963b) satellite geoid and Lee and MacDonald's (1963) heat-flow distribution suggests a correlation between the geoidal undulations and the highs and lows of heat flow, in the sense that depressions on the geoid correlate with regions of high heat flow while rises on the geoid correlate with regions of low heat flow. This implies that the depressions on the geoid are related to hotter and lighter material in the interior of the earth and the rises on the geoid are related to colder and heavier material. Assuming that the density anomaly $\Delta\rho$, which is responsible for the geoidal undulations, is related to a temperature perturbation ΔT by $\Delta\rho/\rho = -\alpha\Delta T$, where α is the coefficient for the thermal expansion, and that ΔT is caused by an inhomogeneous distribution of radiogenic heat sources, the location of this distribution of heat sources is found within the outer 100 km of the mantle, and the corresponding temperature variation has a maximum amplitude of about 100 degrees. The corresponding fluctuation of radiogenic heat is about $\pm 2 \times 10^{-14}$ cal/cm^3 sec, implying that the upper mantle contains far more heat sources than does peridotite or dunite. An eclogitic upper mantle seems, in this respect, more likely.

BIOGRAPHY

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