CORRELATIONS BEYOND THE HORIZON

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Abstract

It is a fundamental feature of quantum field theory that correlations between observable quantities occur over all spacetime regions. In particular, in cosmological models with horizons, such correlations will be present in regions which "lie outside of each other's horizon". Such correlations may play an important role in processes occurring in the early universe.

One of the most striking features of standard cosmological models in general relativity is the presence of horizons: There exist pairs of events p, q whose pasts have vanishing intersection. In many arguments concerning phenomena that may have occurred in the early universe, it is customary to assume that if p and q "lie outside each other's horizon", then physical processes at p must be independent of those at q, since no causal communication of any kind is possible between p and q. Thus, in particular, it is normally assumed that physical quantities measured at p should be entirely uncorrelated with similar quantities measured at q.

A good example of the type of argument where it is assumed that there do not exist any "correlations beyond the horizon" is the one used to estimate production of monopoles in the early universe. (Similar arguments also apply to production of strings and other "topological defects".) Consider a field theory containing a Higgs scalar field coupled to an SU(2)-Yang-Mills field, with the minimum of Higgs potential having topology S^2. In a standard Robertson-Walker cosmological model, one expects the state of the field in the very early universe to be locally in thermal equilibrium at high temperature. However, as the temperature drops below the scale set by the potential, the Higgs field at any given point p should "settle into" a minimum of the potential at some "randomly chosen" direction in field space, $\hat{v}$, (i.e., at a direction depending sensitively on initial conditions) on the spherical potential minimum surface. At points spatially very nearby to p, one would expect the field to "settle into" the potential minimum at a direction very close to $\hat{v}$. (Otherwise, the energy stored in the spatial derivatives of the field would be very large; the thermalizing interactions should allow this to occur only with negligible probability.) However, if q is outside of the horizon of p, it is assumed that the direction $\hat{w}$ at q is completely uncorrelated with $\hat{v}$. This leads to a picture where the field breaks up into domains of horizon size (or smaller) such that the field direction, $\hat{v}$, is correlated within each domain, but the different domains are uncorrelated. (Similar behavior is predicted and observed to occur in condensed matter systems which are cooled rapidly.) One then can estimate the frequency at which the relative alignment of neighboring domains is such as to produce configurations with a non-zero winding number, corresponding to the production of a monopole.

The estimates of monopole production obtained by this argument would appear to yield a highly reliable lower limit to monopole production, since the only crucial ingredient in the analysis is the seemingly very natural assumption that the field directions $\hat{v}$ and $\hat{w}$ at points p and q lying outside each other's horizons are uncorrelated. Since an unacceptably high rate of monopole production is obtained, this gives rise to a serious "monopole problem", which, in order to solve, one must either abandon the field theory model or appeal to mechanisms such as inflation.

In this essay, I wish to point out that the existence of "correlations beyond the horizon" is a fundamental aspect of any quantum field theory. The assumption made in the above argument that the field directions, $\hat{v}$ and $\hat{w}$, at p and q are strictly uncorrelated is incorrect. The crucial issue with regard to this argument and others is not whether correlations beyond the horizon exist -- they do -- but whether they are large enough to substantially alter any of the conclusions. I shall not argue here that these correlations plausibly are large enough to solve the monopole problem or substantially alter the conclusions of other previous arguments which have explicitly or implicitly invoked the lack of correlations beyond the horizon. Indeed, the simple model calculation which I shall describe below yields rather small correlation effects. I also shall resist any temptation to speculate upon possible implications of correlations beyond the horizon for such issues as how to account for the observed homogeneity and isotropy of the universe. The point I do wish to make is that -- at the very least -- it is far from obvious, *a priori*, that the relevant correlations beyond the horizon in a quantum field theory model will be small, and the neglect of such correlations in analyzing any phenomenon must be justified by quantitative estimates rather than by a simple appeal to a lack of causal communication. Indeed, because of the fundamental and universal nature of these correlations, it would be surprising if they did not play some important role in our understanding of the nature of the early universe.

Let O_1 and O_2 be two open spacetime regions. In quantum field theory, for each of these regions we can construct local algebras of observables $\mathcal{A}_1$, $\mathcal{A}_2$ generated by field operators smeared with test functions with support in O_1 and O_2, respectively. Let $A_1 \in \mathcal{A}_1$ and $A_2 \in \mathcal{A}_2$. We say that the observable A_1 is correlated with observable A_2 in state Ψ if

$$\langle\Psi| A_1 A_2 |\Psi\rangle \neq \langle\Psi| A_1 |\Psi\rangle\langle\Psi| A_2 |\Psi\rangle \tag{1}$$

The existence of correlations implies that the measured values of A_1 and A_2 are not independent, i.e., a specification of the observed value of A_1 affects the probabilities that would be assigned for the possible observed values of A_2. Correlations can occur even when O_1 and O_2 are spacelike related (so that A_1 and A_2 commute) and, indeed, the presence of correlations in this case underlies the Einstein-Podolsky-Rosen phenomenon. However, the existence of correlations cannot be used to communicate information between spacelike separated regions.

The Reeh-Schleider theorem (see, e.g., [1]) asserts that for the vacuum state Ψ_0 -- and, more generally, for a dense set of all states [2] -- of any quantum field theory satisfying the Wightman axioms (or other similar axioms [2]), given any open spacetime region O, the states obtained by applying to Ψ_0 observables in the local algebra, $\mathcal{A}$, associated with O span a dense subspace of the Hilbert space of all states. It is an immediate corollary of this theorem that given any two open regions, O_1, O_2 -- no matter how small and/or widely separated -- there exist observables $A_1 \in \mathcal{A}_1$ and $A_2 \in \mathcal{A}_2$ such that eq. (1) is satisfied. In other words, correlations of at least some observables of a quantum field exist over *all* pairs of spacetime regions.

A good concrete illustration of this completely general property of quantum fields is given by the vacuum state of the free massless scalar field in Minkowski spacetime. We have $\langle 0| \phi(x) | 0\rangle = 0$ for all x, but we have

$$\langle 0|\, \phi(x_1)\, \phi(x_2)\, |0\rangle = \frac{1}{(2\pi)^2\ \sigma} \qquad (2)$$

where σ denotes the squared geodesic distance between x_1 and x_2. From this equation it follows immediately that it always is possible to choose test functions f_1, f_2 with supports arbitrarily near x_1 and x_2, respectively, such that eq. (1) holds, with $A_1 = \phi(f_1)$, $A_2 = \phi(f_2)$. Indeed, considerable further insight into the nature of these correlations inherent in the Minkowski vacuum state can be obtained by expressing it in the "Rindler representation" where one sees that the vacuum state corresponds to a thermal state in the two "Rindler wedges", with perfect correlation between the particle content in the two wedges; see [3] for further discussion.

The Reeh-Schlieder theorem has been proven only in the context of flat spacetime quantum field theory, although some generalizations to curved spacetime have been given [4]. However, the Hadamard condition on states [5] in linear quantum field theory in curved spacetime -- necessary for a state to have a nonsingular expected stress-energy -- requires a local singularity structure of the two-point function with leading behavior as in eq. (2), so, at the very least, in linear field theory some correlations over spacelike separations similar to those occurring in flat spacetime case always must be present. Indeed, the strength and generality of the Reeh-Schleider theorem in flat spacetime is such that it seems inconceivable that similar correlations could fail to be present for essentially all states and over essentially all regions in any curved spacetime, including cosmological spacetimes with horizons. Thus, as already indicated above, the real issue is not whether such correlations are present, but how large their effects plausibly might be in physical processes.

In order to illustrate the existence of correlations beyond the horizon, I now shall describe the results of a simple model calculation involving a linear, conformally invariant scalar field in the conformal vacuum state of a flat ($k = 0$) Robertson-Walker cosmological model with horizons; details of the calculations will be given elsewhere [6]. Consider the hypersurface Σ_t at proper time t after the "big bang", and let h denote the horizon radius at that time. Thus, the intersection of Σ_t with the boundary the future of a point (or, more precisely, a TIF) on the big bang singularity is a sphere of radius h. Let $\rho_y(x)$ be following "cone-shaped" function on Σ_t centered about point $y \in \Sigma_t$,

$$\rho_y(x) = \begin{cases} 1 - |x-y|/h & \text{if } |x-y| \leq h \\ 0 & \text{if } |x-y| > h \end{cases} \tag{3}$$

where $|x-y|$ is the distance between x and y on Σ_t. (This particular functional form of ρ_y is chosen entirely for simplicity; the key feature is that ρ_y vanish when $|x-y| > h$.) Consider, now, the one-parameter family of field configurations on Σ_t of the form $\alpha\rho_y(x)$ where α is a constant. In the conformal vacuum state, it is easily shown that within this family of field configurations, the most probable value of α is zero. Now, consider instead, the one-parameter family of field configurations of the form

$$\xi(x) = \rho_0(x) + \alpha\rho_y(x) \qquad (4)$$

where $\rho_0(x)$ is given by eq. (3) with $y = 0$. I pose the following question: What is the most probable value, α_p, of α in this case? The sign and magnitude (compared with 1) of α_p can be interpreted as giving a measure of the "bias" on the expected results to be obtained by observers measuring the field within a ball of radius h at y caused by an observation of the field to be in configuration ρ_0 by observers in a ball of radius h about the origin. Indeed, this question is closely analogous to the following question in the monopole production analysis: What effect does the formation of a horizon-sized domain in direction $\hat{v}$ at the origin have on the probability distribution for the alignment of a domain formed at y. A qualitatively similar "bias" occurring in that analysis would affect the relative alignment probabilities assigned to different field domains, thereby affecting the monopole production probabilities.

If $|y| > 2h$, then the supports of ρ_0 and ρ_y on Σ_t do not overlap, and if $|y| > 4h$, then no event in the support of ρ_0 could have had causal contact with any event in the support of ρ_y. In the latter case, a complete absense of correlations normally would be assumed, in which case the most probable value of α would be taken to be $\alpha_p = 0$. However, in this model we can calculate α_p for all y from the formula for the ground state wavefunctional in the conformal vacuum state. The results of an exact, analytic calculation yield [6],

$$\begin{aligned}\alpha_p = \{&6 + [6\lambda^4 - 10\lambda^2] \ln\lambda + [-4\lambda^4 - 10\lambda^3 + 20\lambda + 20 + 6/\lambda] \ln(\lambda+1) \\ &+ [\lambda^4 + 5\lambda^3 + 5\lambda^2 - 10\lambda - 20 - 8/\lambda] \ln(\lambda+2) \\ &+ [-4\lambda^4 + 10\lambda^3 - 20\lambda + 20 - 6/\lambda] \ln|\lambda-1| \\ &+ [\lambda^4 - 5\lambda^3 + 5\lambda^2 + 10\lambda - 20 + 8/\lambda] \ln|\lambda-2|\}/[40 \ln 2 - 10]\end{aligned} \quad (5)$$

where $\lambda = |y|/h$. In particular, α_p remains nonvanishing even when $|y| > 4h$.

For $|y| > 2h$, α_p is positive but its magnitude is quite small. In particular, for $|y| = 2h$, we have $\alpha_p \sim 10^{-2}$, and for $|y| = 4h$ we have $\alpha_p \sim 10^{-3}$. (Although it is not immediately apparent from this formula, it is not difficult to show that α_p decreases to zero as λ^{-4} as $\lambda \to \infty$.) Clearly, a "biasing" of this magnitude would have negligible effect on quantitative estimates of phenomena such as monopole production. However, the model calculation is far too simple and special to provide a reliable quantitative estimate of the correlations beyond the horizon occurring in a nonlinear field theory, especially since it is far from clear what the initial field correlation functions at the Planck time may have been in a more realistic model. What the model calculation does reliably illustrate is that such correlation effects always will be present at some level.

In summary, it is a fundamental and universal property of quantum field theory that correlations between observable quantities occur over all spacetime regions. I feel that it would be rather surprising if the existence of correlations beyond the horizon did not play an important role in accounting for some basic phenomena occurring in the early universe.

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