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What is the Effective Stress-Energy of Particles Created from the Vacuum?

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Summary

When spontaneous particle creation occurs in a strong gravitational field, it seems clear on physical grounds that the particle creation must "back react" on the gravitational field. It is generally believed that in the semiclassical approximation this effect can be described by assigning an effective stress-energy to the created particles which acts as a source of the gravitational field via Einstein's equation. In this essay, I discuss an axiomatic approach for determining the renormalized value of this effective stress-energy.

In the past several years, a considerable amount of progress has been made in our understanding of quantum processes occurring in a strong gravitational field. A satisfactory quantum theory of the gravitational field itself still does not exist[1]. However, the framework of a semiclassical theory describing other quantum fields present in a strong gravitational field does exist and has been used to investigate particle creation effects. Here the gravitational field is described in an entirely classical manner as curvature of the geometry of spacetime, in accordance with the notions of general relativity, while the quantum fields (e.g., scalar, Dirac, or Maxwell fields) which are present in spacetime are described in accordance with the principles of quantum field theory. It is not believed that this theoretical framework can provide an exact description of nature, since it cannot be entirely consistent to have quantum fields (described in probabilistic terms) interact with a classical gravitational field (with definite, determined values). Rather, this semiclassical theory is viewed as an approximation to the true - as yet unknown - quantum theory of gravitation interacting with other fields. Such a semiclassical framework is analogous to the situation in atomic physics where, for the description of a wide range of phenomena, it is a good approximation to describe the electromagnetic field in an entirely classical manner while treating the electrons quantum mechanically. On dimensional grounds it is generally believed that quantum effects of gravity should be important at least when the spacetime curvature becomes comparable to the Planck length $(\hbar G/c^3)^{1/2} \simeq 10^{-33}$ cm. However, for less extreme spacetime curvature, one hopes that the semiclassical approximation will be valid at least in many situations.

If the gravitational field has suitable asymptotic behavior in the past and future, a description of the quantum fields in terms of particles will be possible in these asymptotic regimes. One may then ask about particle creation: If the field is initially in the vacuum state, how many particles will be present at late times? More generally, what is the S -matrix? It turns out that a few simple assumptions within the semiclassical framework described above uniquely lead to an expression for the S -matrix in a manner which is very nearly free of any mathematical difficulties(2). Thus, one can make well defined, unambiguous predictions concerning particle creation in a strong gravitational field.

The most remarkable application of these ideas is, of course, Hawking's discovery[3] that particle creation near a Schwarzschild black hole will result in a steady rate of emission of particles with an exactly thermal spectrum[2], [4]. This result is particularly striking in view of the analogies that had previously been discovered between black hole physics and thermodynamics[5], [6]. In the absence of any experimental or observational confirmation of the predictions of the semiclassical theory, it is the beauty of Hawking's result as well as the simplicity, naturalness, and good mathematical behavior of the theory which gives one confidence that this approach is on the right track.

In the particle creation calculations referred to above, the spacetime geometry (i.e., gravitational field) is taken to be that appropriate to some classical physical situation, e.g., the gravitational collapse of a body to form a black hole. The particle creation is then calculated in this fixed spacetime geometry. However, on physical grounds it is clear that the quantum particle creation must "back react" on the spacetime geometry. In particular, for the case of a black hole, the particle creation calculations show a flux of energy coming from the black hole. By conservation of energy one would expect this energy flux to be balanced by a decrease in the mass of the black hole (i.e., a decrease in the energy of the gravitational field). The determination of the nature and magnitude of the "back reaction" effect is of great interest and importance, both in the black hole context and in the cosmological context where the "back reaction" or particle creation may have an important effect on the dynamics of the universe. It is also needed to check the validity of the particle creation calculations, since if the effect of the "back reaction" is large, it must be taken into account in these calculations.

In what framework can one analyze this "back reaction" effect? It is conceivable that one will need a complete quantum theory of gravitation in order to describe it, since to describe it in the semiclassical framework involves having a quantum field act as a source for a classical field. If, for example, the quantum field source has a probability of ½ of being "very small" and a probability of ½ of being "very large," it would not seem reasonable to try to describe the gravitational field to which it gives rise as a "medium-sized" classical field. Thus, in particular, Hawking has expressed the view that "back reaction" can be described only in the context of quantum gravity. However, there is clearly <u>some</u> domain of validity to associating a classical gravitational field to a quantum source. After all, ordinary matter is, of course, in reality of a quantum nature but it certainly makes sense to assign it a classical gravitational field. More generally, if the gravitational field is not so strong that the effects of quantum gravity should be of direct importance, it seems reasonable to expect the approximation of a classical gravitational field to be valid whenever the expected quantum fluctuations in the source are negligible compared with the expected value of the source itself. It is, of course, not obvious whether this domain of validity extends to cases where particle creation effects are important. However, the example of particle creation near a black hole suggests that this may be the case since at least at large distances from the black hole the created particles are thermally distributed and hence should satisfy the above criterion. I shall assume in what follows that the semiclassical approach to "back reaction" has a nontrivial range of validity.

In classical general relativity, the source of the gravitational field is the stress-energy tensor $T_{\mu\nu}$ of the fields present in spacetime. The gravitational field (described by the spacetime metric) is related to $T_{\mu\nu}$

via Einstein's equation,

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{1.1}$$

where $G_{\mu\nu}$ is the Einstein tensor. In quantum theory, observables are described as operators acting on the Hilbert space of states of the system. Hence, in quantum theory, the stress-energy tensor should become an operator. A natural procedure for treating the "back reaction" effect in the semi-classical approximation then suggests itself: we require that the classical Einstein tensor be set equal to the expected value of the stress-energy tensor in the given quantum state,

$$G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle \tag{1.2}$$

More precisely, the structure of the theory is as follows: For each (suitably well behaved) classical spacetime geometry, there should exist a stress-energy operator for each field of interest. A spacetime together with a quantum state of the field which satisfies Eq. (1.2) is considered to be a solution of this semiclassical Einstein theory. This solution is to be taken seriously if the characteristic radii of curvature of the spacetime are much greater than the Planck length and if the expected fluctuations in $T_{\mu\nu}$ in this state are negligible compared with $\langle T_{\mu\nu} \rangle$. In the limit where the field can be described classically (i.e., a large number of appropriately distributed particles and negligible particle creation), the theory should reduce to classical general relativity.

It is natural to postulate that the stress-energy operator is given in terms of the quantum field operator by the same formula by which the classical stress-energy tensor is related to the classical field. It is here, however, that a serious problem arises. The quantum field operator does not exist as an operator defined at each point of spacetime; only "smeared" fields make sense mathematically, i.e., the field is an operator-valued distribution on spacetime. When one performs operations which are linear in the field, this distinction is basically just a technical point; for example, it leads to no difficulties in the derivation of the $\mathcal{S}$ -matrix[2]. However, nonlinear operations on distributions, such as taking products, have no obvious mathematical meaning. Since the stress-energy is quadratic in the field, the formula for the stress-energy operator involves a product of distributions and hence must be viewed as only a formal expression. Therefore, it is not surprising that when one naively attempts to calculate expectation values of the stress-energy operator, one gets infinite answers. Thus, some sort of renormalization prescription must be given.

In flat spacetime there is a completely satisfactory solution to this renormalization problem: normal ordering. One can view this prescription as renormalizing the energy of the vacuum state to zero. However, in curved spacetime, when particle creation takes place there is no invariant vacuum state and thus there is no natural analogue of normal ordering. Furthermore, even if no particle creation occurs (e.g., in a stationary spacetime) it is not at all clear that normal ordering is correct, since vacuum polarization effects may cause the stress-energy of the vacuum state to be nonzero. Thus, the problem of renormalizing the stress-energy tensor in curved spacetime is a nontrivial one, as has been further demonstrated by the considerable amount of effort that has gone into attempting to solve it.

A number of proposals for renormalizing $T_{\mu\nu}$ are discussed by DeWitt[7]. More recently, dimensional regularization[8] and zeta-function[9] techniques have been developed and further work has been done on the "point-splitting" method[10], [11]. However, all the prescriptions that have been given thus far are either applicable only to a very restricted class of spacetimes (e.g., the "adiabatic regularization" scheme of Parker and Fulling[12]) or have ambiguities (e.g., the direction dependent terms in the "point-splitting" approach), or, at the very least, have features which are ad hoc. The extent to which the different procedures agree or disagree has not been fully investigated.

Recently[13], I have taken a different approach toward the stress-energy renormalization problem. Rather than develop a particular renormalization scheme, I have attempted to list all the conditions which the renormalized $T_{\mu\nu}$ should satisfy in order to yield a viable theory of "back reaction" in the semiclassical framework described above. Taking these conditions as axioms, I have then investigated their self-consistency and the extent to which they determine the renormalized $T_{\mu\nu}$.

The axioms are the following (see Ref. [13] for a much more complete discussion):

(1) The formal mode sum expression for $T_{\mu\nu}$ is valid for calculating the matrix elements of $T_{\mu\nu}$ between orthogonal states. (Here the formal expression yields unambiguous finite results; the divergences occur for expectation values.)

(2) Normal ordering is valid in Minkowski spacetime.

(3) Expectation values of the renormalized stress-energy satisfy $\nabla^\mu \langle T_{\mu\nu} \rangle = 0$.

(4) Causality: (a) For fixed "in" state, $\langle T_{\mu\nu} \rangle$ at point p depends only on the spacetime geometry to the causal past of p. (b) For fixed "out" state, $\langle T_{\mu\nu} \rangle$ at p depends only on the spacetime geometry to the causal future of p.

(5) $\langle T_{\mu\nu} \rangle$ contains no "local curvature terms." (A local curvature term, such as $\nabla_\mu \nabla_\nu R$, would drastically alter the character of the semi-

classical equation $G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle$ and make it difficult, if not impossible, to yield general relativity in the classical limit. The precise mathematical condition expressing this notion and a full discussion of the motivation for it is given in Ref. [13].)

The main result which has emerged(13) from this investigation is that the above five axioms admit at most one solution for the renormalized $T_{\mu\nu}$. Furthermore, while I have not succeeded in proving that the axioms are self-consistent (so that a solution does exist), I have found a "point-splitting" type of prescription which satisfies at least the first four axioms(13). Thus, axioms 1 - 4 are self-consistent. At the present time, the stumbling block to further progress is the mathematical technicality of axiom 5 which makes it very difficult to check for this and other proposed prescriptions for $T_{\mu\nu}$.

It is my hope that the remaining issue in this approach - namely, the question of existence of a prescription for $T_{\mu\nu}$ which satisfies all five axioms - can be resolved in the near future. If it turns out that no prescription can satisfy all the axioms, I feel that the validity of the semiclassical approach to "back reaction" will be very much in doubt; it will probably be necessary to quantize the gravitational field first. On the other hand, if - as I believe will happen - a solution of the above system of axioms is found, then one will have a firm reason for believing that it is the correct prescription. One will then be able to proceed with calculations of the "back reaction" effect for black holes and in cosmological contexts. If this occurs, there is a good chance that the insight obtained from these calculations will lead us to a better understanding of nature and perhaps, eventually, to a complete quantum description of gravity.

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SOME SPECULATIVE SUGGESTIONS FOR DEALING WITH THE GRAVITON AS AN ELEMENTARY PARTICLE

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SUMMARY

Several speculative results are presented for dealing with the graviton and other spin two particles from the point of view of elementary particle wave equations and their symmetry and transformation properties.

One possible approach to the understanding of gravitation is to utilize the particle approach and to deal with the graviton, the quantum of the gravitational field, as an elementary particle. One then tries to apply ideas and techniques that are succesful with other particles to the graviton, hoping that the graviton has some similarity with the more conventional elementary particles. The basis of this approach is that when one deals with gravitons, one is dealing with particles having a definite mass (usually taken to be zero) and a definite spin (usually taken to be two).

The purpose of this article is to present several speculative ideas based upon the elementary particle with definite mass and spin picture of gravitation. All but the most essential details will be omitted, the intention being to stimulate further work along these lines rather than to present final results.

Chiral Symmetry

The particle aspects of the free graviton are particularly emphasized by looking at its Hamiltonian wave equation, known to be, in the space-inversion invariant form (the units are $\hbar = c = 1$),

$$\vec{\alpha}\cdot\vec{p}\,\Psi = i\frac{\partial\Psi}{\partial t} \tag{1}$$

with $\vec{p} = -i\vec{\nabla}$, $\vec{\alpha} = \frac{1}{2}\begin{pmatrix}\vec{S} & 0\\ 0 & -\vec{S}\end{pmatrix}$ and $\vec{S}$ a representation of the spin two spin matrices. The wave function Ψ has ten components. Defining the matrix $\gamma_5 \equiv \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$, one sees that

$$[\gamma_5, \vec{\alpha}\cdot\vec{p}]_- = 0 \tag{2}$$

so that $\gamma_5\Psi$ or, indeed, the general form

$$\begin{aligned}\Psi' &= e^{i\gamma_5\theta}\,\Psi\\ &= \{\cos\theta + i\gamma_5 \sin\theta\}\,\Psi\end{aligned} \tag{3}$$

is still a solution of Eq. 1 with the same Lorentz transformation properties as Ψ itself. This invariance is the chiral symmetry[1] for the free graviton. What is most important is the extension of symmetry properties to the interacting case. For spin one-half fermions, the chiral symmetry was postulated for the four fermion weak coupling with great success in the theory of beta decay. For the photon, the chirality transformation represents a generalization of the duality transformation[2] ("rotation" of electric and magnetic fields into one another) which leaves the Maxwell action as well as the stress tensor components invariant. One could certainly impose this symmetry on the interaction of the various massless particles with one another, by permitting only the $1 \pm \gamma_5$ projections of the wavefunctions to participate in the interaction.

The "Rest-Frame" of a Graviton

A Hamiltonian wave equation has proved extremely useful to study transformations into representations which exhibit some special feature of the system such as the non-relativistic limit, closely related to the rest-frame of the particle.

Suppose that the graviton wave equation were unitarily transformed to a "rest-frame" representation, that is, to a representation that has the same matrix structure (matrix equivalent) as massive particle rest-frames. Is it possible?, and what are the results? The result should be[3]

$$p\beta\Psi_R = i\frac{\partial}{\partial t}\Psi_R \tag{4}$$

where $\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\vec{\alpha}$ has been rearranged by a unitary transformation to $\frac{1}{2}\begin{pmatrix} 0 & \vec{S} \\ \vec{S} & 0 \end{pmatrix}$. One would then have an exact matrix equivalent of say the rest-frame of an electron, and a wavefunction Ψ_R in which the ten independent components have been rearranged in a possibly suggestive way. Clearly, going directly from Eq. 1 to Eq. 4 with a unitary transformation is not possible because $\vec{\alpha}\cdot\vec{p}$ and β have different eigenvalues. However, since Ψ, as the wavefunction of a spin-two massless particle must contain only helicity components ± 2, that is $(\vec{\alpha}\cdot\hat{p})^2\Psi = \Psi$, one may augment $\vec{\alpha}\cdot\vec{p}$ with matrices that give one or zero when acting on Ψ, but which combine into a matrix with the same eigenvalues as β. The postulated change is[4]

$$\vec{\alpha}\cdot\vec{p} \longrightarrow \vec{\alpha}\cdot\vec{p}\left\{\frac{7}{3} - \frac{4}{3}(\vec{\alpha}\cdot\hat{p})^2\right\} + p\beta\left\{1 - 4(\vec{\alpha}\cdot\hat{p})^2\right\}\left\{1 - (\vec{\alpha}\cdot\hat{p})^2\right\} \tag{5}$$

and then the unitary transformation to Eq. 4 may be performed. To have some feeling for the significance of the resulting wavefunction Ψ_R, one may look at the analogous results for spin zero with mass and for the photon. In the former case the resulting wavefunction has the form $\begin{pmatrix}\phi_+\\ \phi_-\end{pmatrix}$ where ϕ is the Klein-Gordon wavefunction and $\pm$ refer to the positive and negative frequency parts (annihilation and creation parts in a second-quantized theory). In the case of the photon, the wavefunction analogous to Ψ_R has the form $\begin{pmatrix}\vec{E}_-\\ -i\vec{B}_-\end{pmatrix}$ where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, respectively, and $\pm$ has the same meaning as above.

Spin Two Characteristic Equation

To deal with spin two particles, the algebra of the spin two spin matrices needs to be utilized. This algebra includes the usual commutation relations $[S_i, S_j]_- = i\,\epsilon_{ijk} S_k$ as well as other relations that serve to reduce products involving more than four spin two matrices to four or fewer. One example of such relations is the characteristic equation for spin two:

$$\{\vec{S}\cdot\hat{\vec{e}} - 2\}\{\vec{S}\cdot\hat{\vec{e}} + 2\}\{\vec{S}\cdot\hat{\vec{e}} - 1\}\{\vec{S}\cdot\hat{\vec{e}} + 1\}\,\vec{S}\cdot\hat{\vec{e}} = 0$$

where $\hat{\vec{e}}$ is a unit vector with commuting components. The result is $(\vec{S}\cdot\hat{\vec{e}})^5 = 5(\vec{S}\cdot\hat{\vec{e}})^3 - 4\,\vec{S}\cdot\hat{\vec{e}}$. A very important problem arises when the spin two particle is charged and there is a constant, external magnetic field. Then, one needs to evaluate $(\vec{S}\cdot\vec{\pi})^5$ where $\vec{\pi} = \vec{P} - q\vec{A}$, $\vec{A} = \frac{1}{2}B(-y, x, 0)$ and q is the charge. Of course, this kind of analysis does not apply directly to the graviton, but a knowledge of the behavior of all spin two particles will probably be essential for a complete understanding and synthesis of the four kinds of interaction found in nature. The result in the special case $P_3 = 0$ (the eigenvalue) is

$$(\vec{S}\cdot\vec{\pi}_\perp)^5 = 5\alpha\,(\vec{S}\cdot\vec{\pi}_\perp)^3 - 4(\alpha^2 - \tfrac{q}{2}q^2B^2)\,\vec{\alpha}\cdot\vec{\pi}_\perp \tag{6}$$

where $\alpha \equiv \pi^2 - 2q\,\vec{S}\cdot\vec{B}$ is a remarkable operator that commutes with $\vec{S}\cdot\vec{\pi}$, π^2, P_3, $\vec{S}\cdot\vec{B}$. Eq. 6 is unexpected in that all the complicated combinations of spin matrices and elements of $\vec{\pi}$ have disappeared, leaving only α which has well-defined real eigenvalues since $\pi_\perp^2$ resembles the simple harmonic oscillator Hamiltonian. A further result of Eq. 6 is that its use in a rotationally invariant linearization of the non-relativistic Schrodinger equation leads, in the weak magnetic field approximation (neglecting q^2B^2), to a Hamiltonian $H = \frac{\alpha}{2m}$ for mass m so that the spin-two charged particle is suggested to have a g-factor of two like an electron and as is required for independent reasons.

The reduction of $(\vec{S}\cdot\vec{\pi}_\perp)^5$ may also be used to carry out a Melosh transformation[5] of the spin-two charged particle in almost complete analogy to the result for spin one-half quarks. Such a result may prove useful for the interacting systems to which constituent quark-current quark transformations must ultimately be applied, again with the spin two multiplet partners of the graviton playing their appropriate roles.

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