A NEW TEST OF GENERAL RELATIVITY[*]

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In the study of gravitation, the struggle for each new experimental result is arduous but the rewards are great, since we have so little evidence concerning the relativistic properties of the gravitational interaction. In the past, these experiments have been carried out in the laboratory of the solar system, because only there have we been able to disentangle the gravitational from the astrophysical effects. However, the recent discovery [1] of a pulsar (PSR 1913+16) in a binary system has provided us with another system relatively free of astrophysical complications. It is the purpose of this essay to show that this system may make possible a new test of general relativity, one that is unique in three ways. First, it is the only new test that has a chance of being accurately carried out on this system. Second, it is the first test of general relativity that probes its structure beyond the first post-Newtonian level of approximation. Third, it is based upon the effects of gravitational radiation.

Basically, the test consists of a measurement of the effects on a binary system of its loss of energy and angular momentum by gravitational radiation. The calculation of the magnitude of the predicted observable effects is based upon two assumptions: The validity of general relativity, and the relative insignificance of competing effects such as mass loss and tidal interactions. With regard to the first assumption, similar calculations carried out within other viable theories of gravity will usually include uncertainties which arise from the fact that in most theories other than general relativity, the trajectory of a massive body depends upon its self-gravitational energy [2,3]. The validity of the second

assumption will be discussed later.

Consider a binary system with members of mass m_p (pulsar) and m_c (companion), and define $M \equiv m_p + m_c$. At any time, their relative motion is completely described by the six standard elements i (angle of inclination between the orbital plane and the plane of the sky), Ω (angle from the reference direction to the ascending node), ω (angle from the ascending node to the periastron point), ϵ (eccentricity of the orbit), a (semi-major axis of the orbit), and τ (time of initial periastron passage). In place of the semi-major axis, we may employ the orbital period $P \equiv 2\pi a^{3/2} M^{-1/2}$, using units in which $G = c = 1$.

Under the influence of perturbing forces or mass loss, these elements will change slowly with time. In particular, the emission of gravitational radiation by the binary system is accompanied by radiation-reaction forces which decrease both the period and eccentricity of the orbit at rates first calculated by Peters [4]. The accompanying change in the element τ is not separately observable.

The raw data consists of pulse arrival times t_r, which yields the observed pulsar period Δt_r as a function of time. It is related to the properties of the orbit by the equation [5-8]

$$\Delta t_r = \Delta t_{cm}\{1 + K[\cos(\omega + \theta) + \epsilon\cos\omega] + \beta\cos f + \gamma\} . \tag{1}$$

The "center-of-mass" pulse period Δt_{cm} depends upon the unknown velocity of the binary system. The angular position (true anomaly) θ is a well-known function of ϵ and $(t - \tau)/P$. The term representing the first-order Doppler shift is proportional to $K \sim v/c \sim 10^{-3}$. The terms involving the constants $\beta \sim (v/c)^2$ and $\gamma \sim (v/c)^2$

represent the combined effects of second-order Doppler shift and gravitational redshift.

From observations over only ten days, Hulse and Taylor [1] were able to obtain the results Δt_{cm} = 0.059030 ± 0.000001 sec, K = 199 ± 5 km/sec, P = 27908 ± 7 sec, ε = 0.615 ± 0.010, and ω = 179 ± 1 degrees. In terms of these observables, one obtains the mass function $f_p = M^{-2}(m_c \sin i)^3$ = 0.13 ± 0.01 $M_\odot$. Using these data, one can then predict that gravitational radiation induces the following rates of change of orbital period and eccentricity [7]:

$$\frac{1}{P}\frac{dP}{dt} = -0.96 \times 10^{-9} \frac{(M/M_\odot)^{4/3}}{\sin i} \left[1 - 0.51 \frac{(M/M_\odot)^{-1/3}}{\sin i} \right] \text{year}^{-1} \quad (2)$$

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{dt} = \frac{0.342}{P}\frac{dP}{dt} \quad . \quad (3)$$

More recently, Taylor has announced [9] a measurement of the periastron shift, $d\omega/dt$ = 4.0 ± 1.5 degrees/year. Within our working hypothesis that all orbital perturbations are of general-relativistic origin, one obtains the well-known theoretical prediction $d\omega/dt = 2.10 (M/M_\odot)^{2/3}$ degrees/year, which provides a value M = 2.63 ± 1.5 $M_\odot$ for the total mass of the system. With this result plus that for the mass function, one has two equations relating the relevant unknowns m_p, m_c, and i. The table below indicates how the period decrease and masses vary over the presently allowable range of inclination angle, using M = 2.63 $M_\odot$. The main feature to note is the relative insensitivity of the prediction for the period decrease to our present lack of knowledge of the inclination angle.

i	m_p	m_c	$-P^{-1}dP/dt$
21.5°	0	$2.63\ M_{\odot}$	0
25.0	$0.35\ M_{\odot}$	2.28	1.1×10^{-9} year^{-1}
30.0	0.70	1.93	1.8×10^{-9}
50.0	1.37	1.26	2.4×10^{-9}
70.0	1.60	1.03	2.2×10^{-9}
90.0	1.66	0.97	2.2×10^{-9}

A third equation will be available to finally determine the masses and inclination angle when the coefficient β in equation (1) can be measured, since it depends on m_p and m_c in an independent way [1,5-8]. The time required for ω to change sufficiently to allow such a measurement has been estimated to be a few years [8].

Note that $-P^{-1}dP/dt \sim 10^{-9} d\omega/dt \sim (v/c)^3 d\omega/dt$. This reflects the fact that the effects of gravitational radiation occur at the 2½ post-Newtonian order in an expansion in powers of gravitational potential $\sim$ (orbital velocity)^2, while all previous tests of general relativity have involved effects of at most first post-Newtonian order [3,10].

The first question that arises in connection with the predictions $P^{-1}dP/dt = 2.9\ \varepsilon^{-1}d\varepsilon/dt \cong -2 \times 10^{-9}$ year^{-1} is whether such slow changes are indeed measurable. An analysis by Blandford and Teukolsky [8] indicates that the r.m.s. error in a determination of $P^{-1}dP/dt$ carried out over T years will be roughly $10^{-8}T^{-5/2}$ if the pulsar remains stable enough for pulse number to be determined and if pulse arrival times can be measured with an accuracy of 10^{-3}

seconds. A measurement of $P^{-1}dP/dt$ to 10% accuracy will then take roughly five years, at which time the prediction will also be more definite because the individual masses will have been determined, as indicated above. However, a similar analysis implies that the error in a determination of $\epsilon^{-1}d\epsilon/dt$ will be roughly $10^{-5}T^{-3/2}$, which makes it unobservable over realistic time spans.

The second question that arises in connection with this new test of general relativity is whether other effects can produce a comparable change in the orbital period. The three candidates are tidal interactions, mass loss, and acceleration of the binary system. A limit on the radius R_c of the companion can be obtained from the contribution of its quadrupole moment to the periastron advance [11-13], $d\omega/dt \geq (d\omega/dt)_0 = 21(M_\odot/M)^{5/3}(m_p/m_c)k_c(R_c/10^5 km)^5$ degrees/year, where the dimensionless structure constant $k_c \gtrsim 10^{-2}$. The constraints that this be no greater than the observed periastron advance, plus the absence of eclipses in the system, definitely rule out a main-sequence star [1,12,13]. In addition, other calculations [14] indicate that even an evolved stellar companion which has lost its outer layers would produce too large a periastron advance. It therefore seems safe to assume that the companion is a collapsed star (white dwarf, neutron star, or black hole). Only in the case of a rapidly rotating white dwarf would there be any contribution to the periastron advance [12], but this possibility appears unlikely on evolutionary grounds [14].

An induced tidal bulge which lags in orbital phase by an angle δ will produce a change in orbital period given by $P^{-1}dP/dt \cong \delta(2\pi/360)(d\omega/dt)_0$, for $\delta \ll 1$. The phase lag

$\delta \lesssim 10\,\nu R\Omega/m$ in a star with kinematic viscosity ν and angular velocity Ω. If the companion is a white dwarf, it therefore must have a viscosity $\nu \gtrsim 10^{14}$ cm^2/sec in order to produce $P^{-1}dP/dt \sim 10^{-9}$ year^{-1}. Electron and ion viscosities are far less than this in white dwarfs [15], but it is harder to rule out magnetic viscosity. Of course, this effect is negligible for neutron stars with any lag angle because of the R^5 dependence of $(d\omega/dt)_0$.

Any loss of mass from the binary system induces a period increase $P^{-1}dP/dt \cong -2M^{-1}dM/dt$. The major loss of mass is likely to occur as relativistic emission from the pulsar, whose period will thereby increase at a rate $d(\Delta t)/dt$ given by $c^2 dM/dt = -4\pi^2 I(\Delta t)^{-3}d(\Delta t)/dt$. For an expected neutron star moment of intertia $I < 7 \times 10^{44}$ g cm^2, binary mass $M = 2.6\,M_\odot$, and the observed [9] upper limit $d(\Delta t)/dt < 1 \times 10^{-13}$, one obtains $P^{-1}dP/dt < 2 \times 10^{-10}$ year^{-1}, a factor of ten less than the general-relativistic contribution.

Finally, the gravitational field of the Galaxy will change the center-of-mass velocity of the binary system, but this produces the negligible effect $P^{-1}dP/dt \sim 10^{-11}$ year^{-1}.

In summary, it is presently impossible to definitely rule out all other causes of a change in orbital period, although it seems likely that they will prove to be negligible compared to gravitational radiation. Nevertheless, if this effect can be observed and if it agrees with our prediction $P^{-1}dP/dt = -(3 \pm 2) \times 10^{-9}$ year^{-1}, one would be justified in feeling that general relativity had passed one of its most probing tests.

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SUMMARY

The emission of gravitational radiation by the recently discovered binary pulsar system will cause its orbital period P to decrease at a rate which can now be predicted to be $P^{-1}dP/dt = -(3 \pm 2) \times 10^{-9}$ year^{-1} if the only orbital perturbations are of general-relativistic origin. It is shown that other sources of period change are probably less important. The accuracy of this prediction as well as the possibility of its verification will improve greatly over the next few years. This is the first observation that can test general relativity beyond the post-Newtonian approximation.