HOW TO MINE ENERGY FROM A BLACK HOLE

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Abstract

We describe a process by which energy literally can be mined from a black hole. We argue that the only limit placed by fundamental considerations on the rate at which energy can be extracted from a black hole by this process is $dE/dt \lesssim 1$ in Planck units $G=c=\hbar=1$. This is far greater than the rate $dE/dt \sim 1/M^2$ at which the black hole spontaneously loses energy by Hawking radiation.

Black holes are not black--they radiate. This process was discovered by Hawking[1] in 1974. However, for realistic black holes, the rate is negligibly small. For a solar mass black hole the radiation comes out with a temperature of ~ 10^{-7}°K and the energy loss rate is only dE/dt ~ 10^{-20} ergs/sec. Nevertheless, this process is remarkable in that it allows the conversion of the mass of the black hole to energy. Does there exist a more efficient mechanism for this conversion? Is there some energy source associated with the black hole which could be tapped?

The early calculations of particle creation by black holes contained hints that vast amounts of energy near the black hole might be available for rapid extraction. The formal expression for the "out" state vector for particle creation from a Schwarzschild black hole described an outgoing thermal spectrum of particles appearing to originate from the white hole horizon of the extended vacuum Schwarzschild spacetime[2]. Some of these particles escape to infinity, giving rise to the Hawking radiation. However, most of the particles--in particular, essentially all of the particles in high angular momentum modes--get "reflected back" into the black hole by the large barrier appearing in the effective potential for radial motion. Thus, if this state vector were interpreted literally, it would suggest the existence of a huge energy density of particles just outside the black hole horizon. Indeed, the temperature and energy density of the particles would become infinite on the horizon. Thus, a literal interpretation of the state vector would suggest that if some means could be found to allow these particles to escape through the potential barrier to infinity, an enormous energy loss rate by the black hole could be obtained.

A number of authors, in effect, took this literal interpretation soon after Hawking's work. They predicted that the quantum stress energy of the particles is infinite on the horizon and its back reaction would convert the horizon to a singularity. However, this literal interpretation of the state vector clearly cannot be correct since the classical spacetime geometry is well behaved on the horizon. Hence, the stress-energy of a quantum field cannot become singular there for any physically reasonable incoming state, as subsequent calculations have explicitly shown[3]. Indeed, the regularized quantum field energy density near a large black hole is very small and negative, and inertial observers find the state of the field near the horizon to be essentially the vacuum[4]. Thus, it might appear that the formal expression for the "out" state vector gives completely misleading results near the horizon and that the "particles" it describes at the horizon are fictitious mathematical entities without any physical significance. The small, negative, true stress-energy of the quantum field near the horizon would appear to be of no use for energy extraction.

However, a remarkably similar situation occurs in flat spacetime. There it has been shown[5] that an accelerated observer would see the vacuum state of a field as a state in which he was surrounded by a thermal bath of particles with temperature equal to his acceleration divided by 2π, in Planck units. In exactly the same way, a stationary particle detector outside a Schwarzschild black hole would respond as though it were bathed by thermal particles at temperature $T = T_{bh}/\chi$, where T_{bh} is the temperature of the Hawking radiation at infinity, and $\chi = (1 - 2M/r)^{1/2}$ is the redshift factor. Thus, it turns out that the particles described by the "out" state vector of a Schwarzschild black hole are very real to stationary observers. We shall refer to this effective thermal bath as "acceleration radiation". In the case of flat spacetime, energy is required to maintain accelerated motion, and one cannot use acceleration radiation to extract net energy from the Minkowski vacuum. However, since no energy is needed to keep a detector stationary in the Schwarzschild geometry, the possibility now exists that energy can be extracted from this acceleration radiation.

Recently, we showed that indeed this can be done(6). If we lower an open box to near the horizon, hold it stationary there, close the box door, and remove it back to our laboratory far from the black hole, it will come back filled with thermal radiation at the temperature T_{bh}/x, where x is the redshift factor at the radius at which we closed the box. We must do work to lift the full box back to infinity, but because of the effective buoyancy force(7) produced by the temperature (and, hence, pressure) gradient of the acceleration radiation, we showed that a net energy

$$\varepsilon = T_{bh}\, s(T_{bh}/x)V \tag{1}$$

is gained in the process, where $s(T)$ is the entropy density of thermal radiation at temperature T and V is the volume of the box. We also showed how this process can be understood from the inertial point of view. In the inertial viewpoint, the energy is extracted from the black hole by the radiation of negative energy into the black hole by the walls of the box via the "radiation by moving mirrors" effect(8).

Thus, the acceleration radiation near the horizon of a black hole literally can be mined. By putting the high angular momentum modes of the acceleration radiation near the horizon into a box, we can bring them through the potential barrier to our laboratory. As our example given in [6] shows, even a modest amount of black hole mining is well beyond the capabilities of present technology. However, we argue, now, that fundamental considerations limit the energy extraction rate only by $dE/dt \lesssim 1$ in Planck units $G=c=\hbar=k=1$.

The energy extraction rate depends on two basic factors: (i) the time which elapses per "scoop" in the mining process and (ii) the energy gain per scoop. In calculating these factors, we initially will assume only that we can lower the bucket to a proper distance $D \ll M$ from the horizon. Note that in this approximation, we have $\chi \simeq D/M$.

There are two distinct contributions to the amount of time which elapses at infinity per scoop of the mining process: (a) It takes a finite amount of time to lower the box to near the horizon and bring it back. (b) The box must be held stationary for a finite amount of time at its minimum distance, D, from the horizon in order to fill up with acceleration radiation. However, contribution (a) really refers to the transport time of the mined energy to our laboratory at infinity rather than the time it takes to extract energy by mining. Furthermore, it is not necessary that we use only a single box and have to wait for it to go back and forth between the black hole and our laboratory. If we use a continuous "bucket brigade", the energy transport rate to infinity will not be limited by (a). Thus, we shall not include contribution (a) in our estimate of the time needed for energy extraction. Since the round trip time to go a proper distance D from the horizon varies for small D only as $t \propto M \ln(M/D)$--this being true even if the velocity of the box is much less than the speed of light, as must be the case to prevent additional energy from being radiated into the black hole by the walls of the box--our final formula for the limit on the energy extraction would be reduced only by a factor of $\sim\ln(M/D)$ if we included contribution (a).

There are two requirements which lead to a minimum time contribution to (b). First, the locally measured frequency of the ambient acceleration radiation is $\omega \sim T = T_{bh}/\chi \sim 1/M\chi$, so the box must be held stationary for at least proper time $\tau \sim \omega^{-1} \sim M\chi$ in order that the acceleration radiation properly establish itself. Second, the box must be opened for at least the light travel time across its shortest dimension. Since the temperature drops rapidly with height, there is no advantage to taking the proper height, L, of the box to be greater than $(\frac{1}{T}\frac{dT}{d\ell})^{-1} \sim M\chi$. This also yields the limit $\tau \gtrsim M$. The corresponding "time at infinity" is $t = \tau/\chi \sim M$. Thus, a minimum time ~M is required for one "scoop" in the mining process, independent of how close to the black hole we do our mining.

The energy gain per scoop is limited by D. At the proper distance D to the horizon, we have $T = T_{bh}/\chi \sim 1/D$. It appears reasonable to assume that the entropy density of thermal matter at this temperature is at least $s(T) \sim T^3 \sim 1/D^3$, i.e. at least the entropy of ordinary black body radiation. As mentioned above, we must take the height of the box to be $L \lesssim M\chi \approx D$. However, the horizontal dimensions of the box may be taken as large as M. Thus, by eq. (1) the energy mined from the black hole in one "scoop" is limited only by,

$$\begin{aligned} \varepsilon &\lesssim T_{bh}\, V\, s(1/D) \\ &\sim \frac{1}{M} M^2\, D\, \frac{1}{D^3} = \frac{M}{D^2} \end{aligned} \tag{2}$$

Thus, the energy loss rate from the black hole is limited only by,

$$\frac{dE}{dt} \sim \frac{\varepsilon}{t} \lesssim \frac{1}{M}\frac{M}{D^2} \sim \frac{1}{D^2} \tag{3}$$

independent of the mass of the black hole, as compared with the energy loss rate due to Hawking radiation,

$$\left(\frac{dE}{dt}\right)_{rad.} \sim \frac{1}{M^2} \tag{4}$$

Note, incidentally, that in two spacetime dimensions we lose the factor M^2 in the volume of the box and the factor $1/D^2$ in the entropy density of thermal radiation. Hence, we obtain $dE/dt \lesssim 1/M^2$ for the mining process, whereas, eq. (4) continues to hold for the Hawking radiation in two dimensions. Thus, in two dimensions no advantage is gained by mining(9). This is consistent with the fact that there is no angular momentum barrier in two dimensions, so all the acceleration radiation escapes to infinity as Hawking radiation anyway, and thus there is no need to mine it.

It is clear that the energy extraction rate in the mining process, eq. (3), can be much larger than the Hawking radiation rate, eq. (4), provided only that $D \ll M$. How small may D be? Although no known materials can withstand the huge acceleration needed to be held stationary just outside the horizon, the fundamental energy conditions on matter (in particular, the dominant energy condition) do not place any restrictions on how small D can be. Thus, it appears that the only fundamental limitation(10) on D is that it be larger than the Planck length, $D \gtrsim 1$. (There are many reasons why $D < 1$ should not be achievable.) For $D = 1$, eq. (3) becomes,

$$\frac{dE}{dt} \lesssim 1 \qquad (5)$$

In cgs units, the energy loss rate (5) is $dE/dt \lesssim 10^{59}$ erg/sec which is greater than the combined energy loss rate of all stars in the observable universe. Thus, if there exists a more practical process than the one described above for allowing the acceleration radiation near the horizon of a black hole to escape to infinity, black holes would have the potential to provide remarkably explosive energy outbursts.

References

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