THE TILTED UNIVERSE

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Summary

The simplest interpretation of the CMBR dipole anisotropy is that it arises due to our motion with respect to the cosmic rest frame. However, the existence of a superhorizon-sized isocurvature perturbation can give rise to a dipole anisotropy intrinsic to the CMBR. In this case the cosmic rest frame and the CMBR rest frame *do not* coincide, and when viewed from the CMBR rest frame the Universe appears "tilted": matter streams uniformly from one side of the Universe to the other. The intrinsic dipole model provides an explanation for the puzzling observation that most of the matter within a $100h^{-1}$ Mpc cube centered on our galaxy has a large velocity (of order $600\,\mathrm{km\,s^{-1}}$) with respect to the CMBR.

The Cosmic Microwave Background Radiation (CMBR), along with the expansion of the Universe and the abundances of the light elements, provide the foundation for the highly successful hot big-bang cosmology.[1] An observer moving with respect to the CMBR measures a direction-dependent black-body temperature:

$$T(\hat{\mathbf{n}}) = T_0 \frac{(1-v^2)^{1/2}}{(1-\mathbf{v}\cdot\hat{\mathbf{n}})} = T_0 \left[1 + v\cos\theta + (-0.5+\cos^2\theta)v^2 + \cdots\right], \tag{1}$$

where $\mathbf{v}$ is the observer's velocity and $\cos\theta = \hat{\mathbf{n}}\cdot\hat{\mathbf{v}}$. Thus, the dipole temperature anisotropy, discovered in 1977, provides evidence for motion of the earth (at about $350\,\mathrm{km\,s^{-1}}$) with respect to the rest frame defined by the CMBR.[2] Taking into account the motion of the solar system within the galaxy, it follows that the Milky Way (and the Local Group) are moving relative to the CMBR with a speed of about $600\,\mathrm{km\,s^{-1}}$ in the direction of the constellation Hydra.[3]

According to the standard kinematic interpretation of the dipole anisotropy, the cosmic rest frame and the CMBR rest frame coincide, implying that our galaxy has a "peculiar velocity" of about $600\,\mathrm{km\,s^{-1}}$. Peculiar velocities—that is, motions with respect to the cosmic rest frame—arise due to the inhomogeneous distribution of matter in the Universe:

$$\mathbf{v}_P(\mathbf{r},t) = -\frac{H_0}{4\pi}\int \frac{\delta(\mathbf{r}',t)(\mathbf{r}-\mathbf{r}')d^3r'}{|\mathbf{r}-\mathbf{r}'|^3}, \tag{2}$$

where $\mathbf{v}_P(\mathbf{r},t)$ is the peculiar velocity at position $\mathbf{r}$ and time t, $\delta(\mathbf{r},t) = \rho(\mathbf{r},t)/\langle\rho\rangle - 1$ is the density contrast (assumed to be small), and $H_0 = 100h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is Hubble's constant. A peculiar velocity that is not "supported" by gravitational effects decays as $R(t)^{-1}$; $R(t)$ is the cosmic-scale factor.

With reasonable assumptions about the primeval inhomogeneity, the *rms* of the peculiar velocity averaged over a sphere of radius r decreases with r; for the Harrison-Zel'dovich (scale-invariant) spectrum $v_{\mathrm{RMS}} \propto r^{-1}$ for $r \gg 10h^{-1}\,\mathrm{Mpc}$. If the kinematic interpretation of the dipole anisotropy is correct, the rest frame defined by the distant galaxies coincides with that of the CMBR. This notion has received weak support from a marginally significant determination of the anisotropy of the x-ray background radiation (XRB) which indicates that the XRB dipole coincides with that of the CMBR.[4] (The origin of the XRB is believed to be sources—QSO's, AGN's, hot gas, etc.—at high red shift.)

To the surprise of most cosmologists, early measurements of the peculiar velocities of "distant" galaxies indicated that their rest frame did not coincide with that of the CMBR.[5] Because the sample of galaxies was small and relatively nearby, and because peculiar velocities are very difficult to measure, requiring very reliable distance determinations, many discounted the so-called Rubin-Ford effect. However, more recent measurements, consisting now of a sample of more than 1000 galaxies throughout a cube of nearly $100h^{-1}\,\mathrm{Mpc}$ on a side (centered on the Milky Way), indicate a similar effect.[6] While it is not easy to characterize the data in a simple way, a general feature does seem to emerge: Throughout the volume sampled, galaxies are moving with respect to the CMBR at about the same velocity as the Milky Way (about $600\,\mathrm{km\,s^{-1}}$), with an *rms* scatter in their peculiar motions (about $200\,\mathrm{km\,s^{-1}}$) that is only about one third of the bulk motion. Since the volume-averaged peculiar velocity should decrease with increasing volume size this is a very perplexing result. On the theoretical side, none of the models for structure formation under serious consideration—e.g., hot or cold dark matter with inflation-produced curvature perturbations, hot or cold dark matter with cosmic strings as seeds, or a baryon-dominated Universe with $\Omega_0 \sim 0.1$—seem to be able to account for such a large peculiar motion on such a big scale.[3,7]

Perhaps the resolution of this conundrum requires abandoning the simple kinematic interpretation of the dipole anisotropy. Could it be that the rest frame of the CMBR *does not* coincide with the rest frame of the expansion? This would occur if the dipole anisotropy were intrinsic to the CMBR itself.[8] Then, the large, uniform component of the measured peculiar motions merely reflects the intrinsic dipole of the CMBR—not motion with respect to the cosmic rest frame—and the true peculiar motions of galaxies in the local $100h^{-1}$ Mpc cube are only of order $200\,\mathrm{km\,s^{-1}}$, which can easily be explained by the inhomogeneous distribution of matter. *If the CMBR dipole anisotropy is intrinsic, then viewed from the rest frame of the CMBR the Universe appears to be tilted (!), with galaxies moving from one side of our Hubble volume to the other with speeds of order* $600\,km\,s^{-1}$.

How can an intrinsic CMBR dipole arise? First, if it is to be uniform across our Hubble volume it must involve a superhorizon-sized density perturbation. Second, it must involve an isocurvature perturbation.[9] To see this, consider a superhorizon-sized curvature perturbation (in a flat Universe) of wavenumber $\mathbf{k}$ (length scale $L \sim |\mathbf{k}|^{-1}$) whose horizon-crossing amplitude is δ_{HOR}. The horizon-crossing amplitude corresponds to the fluctuation in the Newtonian potential; the horizon-crossing epoch is defined by $RL \sim H^{-1}$, and occurs at a time $t_{\mathrm{HOR}} \sim H_0^{-1}(L/H_0^{-1})^3 \sim 10^{10}\,(L/H_0^{-1})^3$ yrs. The temperature anisotropy that arises is given by the Sachs-Wolfe formula:[10]

$$\frac{\delta T(\hat{\mathbf{n}})}{T_0} = \frac{1}{2}\left[\sqrt{R}(\mathbf{x}\cdot\nabla)[\delta_{\mathrm{HOR}}e^{-i\mathbf{k}\cdot\mathbf{x}}] + \delta_{\mathrm{HOR}}e^{-i\mathbf{k}\cdot\mathbf{x}}\right]_R^E, \tag{3}$$

where "E" denotes the emission event, spatial position $(1-\sqrt{R_E})\mathbf{x}$ and scale factor $R_E \simeq 10^{-3}$, "R" denotes the reception event, spatial position $\mathbf{r} = \mathbf{0}$ and scale factor $R_R = 1$, and $\mathbf{x} = 2H_0^{-1}\hat{\mathbf{n}}$. The first term in Eq. (3) arises due to the relative peculiar motion between the last-scattering surface and the observer; it has the appearance of a dipole anisotropy of order $(H_0^{-1}/L)\delta_{\mathrm{HOR}}$. The second term arises due to the potential difference between the last-scattering surface and the observer; if we expand $e^{-i\mathbf{k}\cdot\mathbf{x}} = 1 - i\mathbf{k}\cdot\mathbf{x} - (\mathbf{k}\cdot\mathbf{x})^2/2! + \cdots$ the $\mathcal{O}(\mathbf{k}\cdot\mathbf{x})$ term cancels precisely the would-be dipole anisotropy. *Even in the presence of a superhorizon-sized curvature perturbation the rest frames defined by the expansion and the CMBR coincide.* The lowest-order CMBR anisotropy is $\mathcal{O}[(H_0^{-1}/L)^2\delta_{\mathrm{HOR}}]$ and quadrupole in character.[11]

The situation for isocurvature perturbations is dramatically different. An isocurvature perturbation corresponds to a spatial variation in the form of the equation of state, rather than a "true" perturbation of the energy density. For definiteness, consider an isocurvature axion perturbation in a flat, axion-dominated Universe (for simplicity ignoring baryons). Such a perturbation corresponds to $\delta\rho_a \neq 0$ and $\delta\rho_a + \delta\rho_\gamma = 0$: The fluctuations in the radiation-energy density (and hence temperature) "compensate" for those in the axion-energy density.

Once the Universe becomes matter dominated, the compensating temperature fluctuation is related to the horizon-crossing amplitude of the axion perturbation,

$$\frac{\delta T}{T} = -\frac{1}{3}\delta_{\rm HOR}e^{-i\mathbf{k}\cdot\mathbf{r}}. \tag{4}$$

The CMBR anisotropy that arises due to an isocurvature mode is given by the Sachs-Wolfe formula—which leads to a vanishing dipole anisotropy and a quadrupole of $\mathcal{O}[(H_0^{-1}/L)^2\delta_{\rm HOR}]$—*plus an additional term due to the "compensating" fluctuations in the radiation.* The net effect is an intrinsic dipole

$$\frac{\delta T(\hat{\mathbf{n}})}{T_0} \sim \hat{\mathbf{k}}\cdot\hat{\mathbf{n}}\left(\frac{H_0^{-1}}{L}\right)\delta_{\rm HOR}, \tag{5}$$

and a quadrupole which is smaller by a factor of $\mathcal{O}(H_0^{-1}/L)$. Provided that $(H_0^{-1}/L)\delta_{\rm HOR} \sim 2\times 10^{-3}$, a superhorizon-sized isocurvature perturbation can account for the CMBR dipole—and can lead to the impression that the Universe is *tilted.* To be consistent with the lack of a quadrupole anisotropy at the level of 3×10^{-5},[2] the wavelength of the fluctuation today must be greater than about $100H_0^{-1}$.

What about the motivation for this scenario? The axion is an excellent candidate for the ubiquitous dark matter known to exist in the Universe;[12] inflation is a very attractive early-Universe paradigm, and isocurvature axion perturbations arise automatically.[13] On scales that were smaller than the Hubble radius at the start of inflation—which, if inflation is to solve the horizon/flatness problems, includes all scales presently smaller than H_0^{-1}—the amplitude $\delta_{\rm HOR}$ is independent of scale and related to the Hubble constant during inflation. For scales that were superhorizon-sized at the start of inflation, $\delta_{\rm HOR}$ is determined by the pre-inflationary fluctuations that existed in the axion field; these, by their nature, cannot be calculated.

The current limits to the quadrupole anisotropy constrain $\delta_{\rm HOR}$ to be less than about 3×10^{-5} for the first category of modes, so we must turn to the second. For these modes $\delta_{\rm HOR}$ is unknown and could be large—even greater than unity. What about H_0^{-1}/L? It can be written as e^{-P}, where $P = p + q$, e^p is the ratio of the wavelength of the perturbation at the start of inflation to the inflationary Hubble radius, and q is the number of e-folds by which inflation exceeds the minimum required to solve the horizon/flatness puzzles. Within our present knowledge of inflation it is difficult to make definite statements about P—other than to say that it must be greater than unity, perhaps much greater; thus, it is not implausible that $(H_0^{-1}/L)\delta_{\rm HOR} = e^{-P}\delta_{\rm HOR}$ could be $\mathcal{O}(10^{-3})$. The axion example provides one scenario that leads to an intrinsic CMBR dipole; doubtless there are others.[14]

How can one test for a tilted Universe? If the bulk of the CMBR dipole is intrinsic, then (to within $200\,\mathrm{km\,s^{-1}}$ or so) we are in the cosmic rest frame, and should be at rest with respect to distant galaxies. Provided that the XRB arises from objects at high red shift, it should appear isotropic in our rest frame rather than in that of the CMBR. ROSAT may well have the sensitivity to definitively determine the anisotropy of the XRB. Since the bulk of our motion with respect to the CMBR is not due to the inhomogeneous distribution of matter, the peculiar velocity computed from the local distribution of matter, cf. Eq. (2), has no reason to be aligned with the CMBR dipole. As the IRAS and other high-quality catalogues of galaxies are completed and used to compute $\mathbf{v}_P$, one should be able to determine whether or not there is such an alignment.[15] Finally, if the CMBR dipole is a kinematic effect, then associated with it is a kinematic quadrupole of $\mathcal{O}(v^2)$, cf. Eq. (1), which is aligned with the dipole. Such a relationship between the dipole and quadrupole anisotropies would not exist in a "tilted Universe." Whether one can both achieve the required sensitivity and separate the kinematic quadrupole from other contributions to the quadrupole anisotropy remains to be seen.

While it is likely that the simplest interpretation—a kinematic dipole—is correct, the intrinsic CMBR dipole—or tilted Universe—explanation is intriguing and deserves consideration. It neatly resolves the puzzle of the large peculiar velocities seen out to distances of $\mathcal{O}(50h^{-1}\,\mathrm{Mpc})$ and can be tested decisively.

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References

1. For a review of the standard cosmology see, e.g., E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990), or S. Weinberg, *Gravitation and Cosmology* (Wiley, NY, 1972).
2. G.F. Smoot, M.V. Gorenstein, and R.A. Muller, *Phys. Rev. Lett.* **39**, 898 (1977); E.S. Cheng et al., *Astrophys. J.* **232**, L139 (1979). The most up-to-date measurements of the dipole anisotropy and limits to the quadrupole anisotropy are given in P.M. Lubin et al., *Phys. Rev. Lett.* **50**, 616 (1983); *Astrophys. J.* **298**, L1 (1985); D.J. Fixsen et al., *Phys. Rev. Lett.* **50**, 620 (1983); A.A. Klypin et al., *Sov. Astron. Lett.* **18**, 104 (1987); G.F. Smoot et al. (COBE Collaboration), *Astrophys. J. Lett.* **371**, L1 (1991).
3. Aside from the dipole anisotropy, the CMBR temperature is isotropic to a few parts in 10^5 over angular scales from a few arc minutes to 90° (quadrupole). In contrast to the dipole anisotropy, whose standard interpretation is kinematic, anisotropies on other angular scales (of amplitude 10^{-5} or so) are expected to arise due to the density inhomogeneities that are believed to have initiated the formation of structure in the

Universe. In fact, based upon the structure we observe and current theoretical models it is surprising that additional anisotropies have not yet be detected. For a recent review of anisotropy limits and a discussion of the anisotropies expected to arise in various models of structure formation see, e.g., J. Silk, *Physica Scripta*, in press (1991).

4. E. Boldt, *Phys. Rep.* **146**, 215 (1987).
5. V.G. Rubin et al., *Astron. J.* **81**, 687 (1976); G. deVaucouleurs and W.L. Peters, *Nature* (London) **220**, 868 (1968).
6. A. Dressler et al., *Astrophys. J.* **313**, L37 (1987); D. Lynden-Bell et al., *ibid* **326**, 19 (1988); M. Aaronson et al., *ibid* **302**, 536 (1986); C.A. Collins et al., *Nature* **320**, 506 (1986); D.S. Mathewson et al., in preparation (1991).
7. It should be said that the Great Attractor, a phenomenological model which involves a large ($few \times 10^{16} M_\odot$) mass concentration at a distance of about $40h^{-1}$ Mpc, can account for much of the peculiar-velocity data; see, D. Lynden-Bell, *Q. Jl. astr. Soc.* **28**, 186 (1987); D. Lynden-Bell et al., *Astrophys. J.* **326**, 19 (1988). Whether or not such a mass concentration actually exists has yet to be determined.
8. Gunn has mentioned an intrinsic CMBR dipole as the explanation for the large peculiar motion in our neighborhood; see J.E. Gunn, in *The Extragalactic Distance Scale (ASP Conf. Series, Vol. 4)*, edited by S. van der Bergh and C.J. Pritchet (BYU Print Service, Provo, UT), p. 357. Ostriker has emphasized the difficulty that all conventional scenarios of structure formation have explaining the "high Mach number" (around 3) implied by measurements of the peculiar-velocity field in the local $100h^{-1}$ Mpc neighborhood: large coherent component (about $600\,\mathrm{km\,s^{-1}}$) and small random component (*rms* dispersion about the coherent component of only about $200\,\mathrm{km\,s^{-1}}$); see J.P. Ostriker and Y. Suto, *Astrophys. J.* **348**, 378 (1990).
9. This issue is treated in more detail by M.S. Turner, *Phys. Rev. D*, submitted (1991) [FERMILAB-Pub-91/43A].
10. R.K. Sachs and A.M. Wolfe, *Astrophys. J.* **147**, 73 (1967).
11. L.P. Grishchuk and Ya.B. Zel'dovich, *Sov. Astron.* **22**, 125 (1978).
12. For a discussion of the dark-matter problem and particle dark-matter candidates see e.g., M.S. Turner, *Physica Scripta*, in press (1991).
13. See e.g., D. Seckel and M.S. Turner, *Phys. Rev. D* **32**, 3178 (1985).
14. Paczynski and Piran have discussed a toy model based upon the Bondi-Tolman cosmology wherein they introduce an entropy gradient to produce an intrinsic dipole [*Astrophys. J.* **364**, 341 (1990)].
15. Preliminary work in which the inhomogeneity of the matter is determined from the distribution of galaxies in the IRAS catalogue indicate that the two vectors differ in direction by $\mathcal{O}(10°)$ [M. Rowan-Robinson et al., *Mon. Not. R. astr. Soc.* **247**, 10p (1990); E. Bertschinger et al., in preparation (1991)]. Whether or not this difference is significant remains to be seen.