What have we learned from two-dimensional models of quantum black holes?

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Abstract

The two-dimensional black hole provides a theoretical laboratory in which the quantum nature of black holes may be probed without the complications of four-dimensional dynamics. It is therefore natural to ask, what have we learned from this model? Much recent work has focused on the semi-classical limit where the black hole is similar to the Schwarzschild solution. However, in this essay, I demonstrate that the *exact* two-dimensional quantum black hole is non-singular. Instead the singularity is replaced by a surface of time reflection symmetry in an extended space-time. The maximally extended space-time thus consists of an infinite sequence of asymptotically flat regions connected by timelike wormholes, rather analogous to the Reissner–Nordström space-time. The implications of this to the apparent loss of quantum information arising from black hole evaporation are also briefly discussed.

Black holes are manifestations of General Relativity infringing upon the realm of the Quantum—and without the harmonious marriage of these two diverging physical descriptions of nature, our understanding of black holes will necessarily be incomplete. One could, for instance, ponder the true nature of the black hole singularity—a point which signals the effective breakdown of classical physics. History has taught us to expect that this "blemish" of General Relativity would be "washed away" by quantum effects. What would replace it however still remains a mystery.

Another unanswered question concerns the apparent loss of quantum information arising from the process of black hole evaporation [1,2]. The semi-classical analysis of Hawking indicates that the emitted radiation is thermal, and does not record any information about the quantum states of the matter that formed the black hole. So when the black hole disappears completely, what happens to this missing information [3]? Is it lost forever (together with a cherished principle of quantum mechanics), or does it reappear in a way only apparent in an exact quantum treatment of the problem?

A deeper understanding of quantum effects in General Relativity is clearly needed to shed light on these perplexing issues, but a consistent quantum theory of gravity still eludes us. One promising candidate is string theory. In the past few years, our understanding of string theory has seen important advances. In particular, it has been realized that consistent string theories may be constructed in target space dimensions much lower than the critical dimension, and that these target spaces admit curved background solutions. Thus a unique opportunity has opened up for us to investigate what string theory can reveal of quantum gravity.

The prototype of such a solution of string theory is a black hole metric in two dimensions [4,5], which has been the focus of much recent attention. This model provides a setting which is non-trivial enough to accommodate the quantum behavior of black holes, yet simple enough to be fruitfully addressed using the powerful methods of conformal field theory. While some may argue against the unphysical nature of such models, it is

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hoped that they would nevertheless provide some intuition as to what might be expected of four-dimensional quantum black holes. After all, one has to learn to walk before running.

In this essay, I shall reflect on, from my *personal* viewpoint, some of the lessons that we have learned from the two-dimensional black hole. In particular, I shall describe what this model tells us about the fate of the black hole singularity; and briefly discuss a possible resolution to the information-loss puzzle arising from this.

The two-dimensional target space background is specified by the metric; and a dynamical conformal factor of the geometry, ϕ, known as the dilaton. Because of the absence of transverse string oscillations in a single spatial dimension, a string is dynamically restricted to its tachyonic ground state, and its classical motion is completely specified by its center-of-mass coordinates. In this case, the tachyon is actually massless, and it resembles a point particle.

The dynamics of strings in a curved space-time is governed by conformal invariance of the world sheet, which is imposed by the vanishing of the β-functions of the metric, dilaton and tachyon [6]. However, these equations are only known perturbatively in the inverse string tension α', and so conformal invariance is only imposed order by order. The two-dimensional black hole of ref. [4] was found by setting the tachyon to zero and solving the lowest order β-functions, which have the form of Einstein's equations coupled to a massless scalar field ϕ.

The resulting black hole solution may be written as [7]

$$\mathrm{d}s^2 = -\frac{x-1}{x+1}\,\mathrm{d}t^2 + \frac{\mathrm{d}x^2}{4(x^2-1)}\,, \qquad \phi = \phi_0 + \ln|x+1|\,, \tag{1}$$

which is clearly seen to be asymptotically flat for $x \to \pm\infty$. There is an event horizon at spatial coordinate $x = +1$, and a curvature singularity at $x = -1$. The black hole space-time exterior to the horizon may be parametrized by $x = \cosh 2r$, in which case the metric reduces to the more familiar form [5] $\mathrm{d}s^2 = -\tanh^2 r\,\mathrm{d}t^2 + \mathrm{d}r^2$. I shall denote this region by I. The black hole space-time interior to the horizon will be called II, and
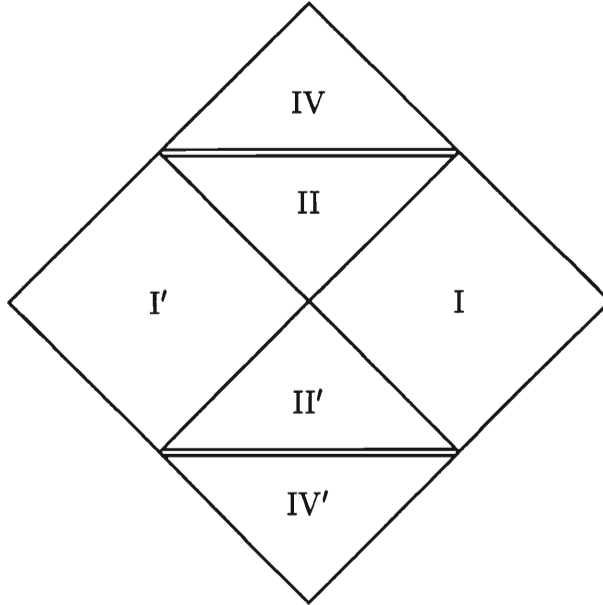
FIG. 1. Causal structure of the two-dimensional semi-classical black hole. Regions I, I' are asymptotically flat space-times exterior to the black hole and white hole horizons. Regions II, II' are inside the horizons, while IV, IV' are asymptotically flat regions each containing a naked singularity. The curvature singularities are marked by the double lines.

may be parametrized by $x = \cos 2r$. The causal structure of these two regions and their analytic continuation to regions I' and II' is sketched in Fig. 1. It is identical to that of the four-dimensional Schwarzschild black hole.

The exterior black hole space-time can be transformed by string duality into a new region with metric $ds^2 = -\coth^2 r\, dt^2 + dr^2$. It is an asymptotically flat space-time exposed to a naked singularity at $r = 0$. This region, which will be referred to as IV, is also described by (1) for the parametrization $x = -\cosh 2r$. Regions I, II and IV thus patch together to form the extended two-dimensional black hole space-time given by (1), whose causal structure is shown in Fig. 1.

Witten has found an exact conformal field theory description of this black hole [5], which ensures that conformal invariance is obeyed non-perturbatively to all orders in k, the

Kač–Moody level. This description is in the form of a Wess–Zumino–Witten model based on the group SL(2,ℝ) gauged by SO(1,1), which maps to a non-compact $1+1$-dimensional gravitational background in which strings propagate. In the semi-classical approximation $k \to \infty$, the above black hole solution is recovered. However, $k = 9/4$ for a bosonic string background, so $1/k$ is quite large and corrections due to this should not be neglected.

The effective space-time background for general k is [8,7]

$$ds^2 = 2(k-2)\left[-\beta(x)\,dt^2 + \frac{dx^2}{4(x^2-1)}\right] , \qquad \phi = \phi_0 + \tfrac{1}{2}\ln\left|\frac{x^2-1}{\beta(x)}\right| , \qquad (2a)$$

where

$$\beta(x) = \left(\frac{x+1}{x-1} - \frac{2}{k}\right)^{-1} . \qquad (2b)$$

It reduces to (1) for $k \to \infty$ (up to an overall scale factor), and is believed to be an exact string background to all orders in α'. Indeed, (2) has been checked to solve the β-function equations up to the four loop level [9].

The exact geometry describes the exterior of a black hole for $x > +1$, with an horizon at $x = +1$. This coincides with region I of the $k \to \infty$ limit. Region II will again denote the black hole interior, where $-1 < x < +1$, except that now $x = -1$ is a coordinate singularity. The curvature singularity is located at $x_c \equiv -(k+2)/(k-2) < -1$. The regions for which $x_c < x < -1$ and $x < x_c$ will be called III and IV respectively. Region IV is an asymptotically flat space-time containing a naked singularity, as in the previous case. Region III is however a new region not present in Fig. 1 [10].

Observe that region III is an area of *Euclidean* signature embedded in an otherwise Lorentzian space-time. This fictitious region is the outcome of choosing the unphysically extended spatial coordinate x. By transforming to suitable coordinates [10], it can be shown that $x = -1$ actually corresponds to a perfectly regular surface of time reflection symmetry. Two copies of region II are glued together at this surface to form a wormhole bridging asymptotically flat space-times isometric to region I. The final result is an infinite chain of black hole space-times I connected by time-like wormholes II, whose causal
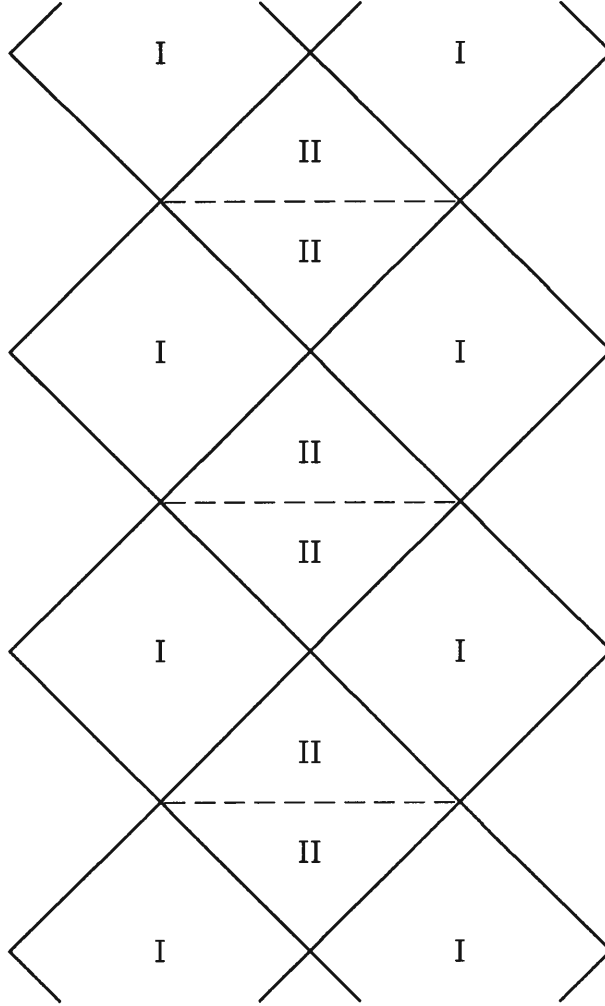
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FIG. 2. Causal structure of the maximally extended exact black hole geometry. This consists of an infinite sequence of asymptotically flat regions I linked by wormholes II. The dashed lines represent $x = -1$ surfaces.

structure is shown in Fig. 2 [10]. It is reminiscent of the Reissner–Nordström black hole in four-dimensions, except that there are *no singularities* present in this case. There is also a disjoint naked singularity space-time IV, whose existence is inferred by duality but is otherwise presumed unphysical.

The reason for the appearance of a wormhole rather than a curvature singularity may be traced down to the negative curvature of region II [10]. The non-attractive character of

gravity there prevents a singularity from forming, but supports a wormhole. Note that this feature strictly results from the inclusion of *all* the higher-order quantum corrections, which happily reinforces the general belief that quantum effects dominate near the singularity of the semi-classical approximation.

"Singularities are repositories for our ignorance, and appear to provide promising scapegoats for any conceptual problems we encounter with black holes," to quote Wilczek [11]. That was the traditional rôle of these otherwise unwanted objects. But now that the exact two-dimensional black hole has been found to be non-singular, it is time to readdress these puzzles, like that of information loss Wilczek was referring to.

The study of black hole evaporation within the context of the two-dimensional semi-classical black hole was pioneered by Callan, Giddings, Harvey and Strominger [12], who investigated the formation and subsequent evaporation of the black hole using a semi-classical analysis which includes the effects of gravitational back reaction. While soluble to a remarkable extent, the approximations upon which this model is based prevent it from convincingly probing the real mysteries surrounding black holes. The next logical step is to study the formation and evaporation of the exact black hole. Ideally, one would like to find a conformal field theory *exactly* representing this process, probably by perturbing the conformal field theory associated with the static black hole background (2) to one which has a non-trivial propagating tachyon. However, this seems like a very difficult task at present.

A reasonable approximation is to study the scattering of a tachyon in this fixed background, which would be valid for sufficiently large black holes where the back reaction of the tachyon on the metric may be neglected. While this approximation will not be able to address the endpoint of an evaporating black hole, it still does strongly suggest a possible explanation for the loss of information. It is clear from the exact geometry of the two-dimensional black hole that infalling matter tunnels to another universe. It cannot be recovered without violating causality. However, no information is actually lost—it is

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merely transferred from one universe to another. Quantum theory is thus well-defined in the larger sense which includes all the universes—the "multiverse" [3].

The idea that a black hole opens the gates to another universe is not new—a closed baby universe scenario resembling a bag of gold was proposed some years ago by Wheeler [13] and by Dyson [14]. However, it would appear that this is the first time a concrete model substantiating this picture has been found, apart from a few differences. In the model I have presented here, the baby universe is asymptotically flat and isometric to the parent universe. And there is no preventing it from having offspring universes of its own. Every time matter collapses to form a black hole, it induces a "big bang" by which a new universe is created from this white hole.

Ultimately, we are interested in four-dimensional black holes, and it would be very pleasing if the two-dimensional example described in this essay carried over directly. To check this, we have to represent the known black hole solutions of General Relativity within a string framework, and look for any qualitative differences near the singularies.

If we are lucky, we might even be able to visit other universes in the foreseeable future!

References

[1] S. W. Hawking, "Particle creation by black holes" *Commun. Math. Phys.* **43** (1975) 199.

[2] S. W. Hawking, "Breakdown of predictability in gravitational collapse" *Phys. Rev. D* **14** (1976) 2460.

[3] J. Preskill, "Do black holes destroy information?" Caltech preprint CALT–68–1819 (1992).

[4] G. Mandal, A.M. Sengupta and S.R. Wadia, "Classical solutions of 2-dimensional string theory" *Mod. Phys. Lett. A* **6** (1991) 1685.

[5] E. Witten, "String theory and black holes" *Phys. Rev. D* **44** (1991) 314.

[6] C.G. Callan, D. Friedan, E.J. Martinec, and M.J. Perry, "Strings in background fields" *Nucl. Phys.* **B262** (1985) 593.

[7] I. Bars and K. Sfetsos, "Conformally exact metric and dilaton in string theory on curved spacetime" *Phys. Rev. D* **46** (1992) 4510.

[8] R. Dijkgraaf, E. Verlinde and H. Verlinde, "String propagation in a black hole geometry" *Nucl. Phys.* **B371** (1992) 269.

[9] A.A. Tseytlin, "On the form of the black hole solution in $D = 2$ string theory" *Phys. Lett. B* **268** (1991) 175;
I. Jack, D.R.T. Jones, and J. Panvel, "Exact bosonic and supersymmetric string black hole solutions" Liverpool preprint LTH 277 (1992).

[10] M.J. Perry and E. Teo, "Non-singularity of the exact two-dimensional string black hole" Cambridge preprint DAMTP R93/1 (1993).

[11] F. Wilczek, "Quantum purity at a small price: easing a black hole paradox" Institute for Advanced Study preprint IASSNS–HEP–93/12 (1993).

[12] C.G. Callan, S.B. Giddings, J.A. Harvey and A. Strominger, "Evanescent black holes" *Phys. Rev. D* **45** (1992) 1005.

[13] B.K. Harrison, K.S. Thorne, M. Wakano and J.A. Wheeler, "Gravitational theory and gravitational collapse" University of Chicago Press (1965), pages 75–79 and Fig. 15.

[14] F.J. Dyson, "Breakdown of physics in gravitational collapse?" unpublished (1975).